Optimal Sizing of Photovoltaic Arrays under Pool-based Power Market with Considering Uncertainty of Solar Radiation Using PSO

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Abstract- In this paper a multi-objective formulation for optimal sizing of photo-voltaic resources in distribution systems for maximizing net present worth of system is analyzed. The proposed system in this paper consists of upstream network i.e. $63^{Kv}/20^{Kv}$ substation as main grid and photovoltaic cells as DG to supply load. The objective function considered in this paper, consists of net present worth of distribution company (DISCO) and of photovoltaic owners (PV-Owner). In this paper the Monte Carlo technique is used for considering uncertainty of solar radiation. The implemented technique relies on particle swarm optimization method (PSO) and weight coefficient method (WCM). Simulation results on 33-bus distribution test system under pool-based power market operation are presented to show evidence of the effectiveness proposed procedure.

Key Words: Optimization, Photovoltaic Arrays, Pool Electricity Market, Monte Carlo, particle swarm optimization

I. INTRODUCTION

Many types of power market models and transactions there are in order to reach a competitive electricity environment. Three basic models based on types of transactions are expected such as: pool model, bilateral contract model and hybrid market model [1]. Pool market is a centralized market environment that cleans the market for both buyers and sellers. Electric power sellers/buyers offer bids to the pool for the value of power that they are desire to exchange in the market. Many studies have been investigated the optimization of solar energy in distribution system. In [2] Unit sizing and cost analysis of stand-alone hybrid wind/PV/fuel cell power generation systems is presented. The size optimization of a PV/wind hybrid energy conversion system with battery storage using simulated annealing is analyzed in [3]. In mentioned paper stand alone mode is considered and the uncertainly of solar radiation and wind speed is not considered. Economic analysis of standalone and grid connected hybrid energy systems, is discussed by [4].Study evaluation of utilizing solar and wind energy with hydrogen as a storage device to cover the electricity demand is investigated. The method of analyzing in this paper is basically on costs of system and simulated using Homer software.

[5] Studied the Multi-objective design of PV-wind-diesel-hydrogen-battery systems. Minimizing, simultaneously, the total cost throughout the useful life of the installation, pollutant emissions (CO2) and unmet load are considered.

[6], investigates the Multi-objective Particle Swarm Optimization for the optimal design of photovoltaic grid-connected systems. The objective function describing the

economic benefit of the proposed optimization process is the lifetime system's total net profit which is calculated according to the method of the Net Present Value (NPV). The second objective function, which corresponds to the environmental benefit, equals to the pollutant gas emissions avoided due to the use of the PV. The uncertainty of solar radiation is not considered in this study. [7], investigates the Dynamic modeling and sizing optimization of stand-alone photovoltaic power systems using hybrid energy storage technology. In [8] Cost benefit analysis of a photovoltaic-energy storage electrification solution for remote islands is presented. Optimal sizing of hybrid wind/Photovoltaic/battery considering the uncertainty of wind and photovoltaic power using Monte Carlo have been studied by [9]. In mentioned research, a micro-gird consists of hybrid wind/photovoltaic/battery in stand-alone mode based on cost optimization has been analyzed. In restructured power market place, under open access and competition, the various transactions can be occur between buyers and sellers directly; these transactions are classified as bilateral, multilateral and ancillary services transactions [10]. In hybrid model several features of the previous two models are combined. Under hybrid power market model, several transactions are expected between buyers and sellers for more flexible and economic market operation. These transactions require to be assessed ahead of their scheduling time to check their feasibility with respect to system operating conditions. Infeasible transactions can alter the economic schedule, cause congestion and threaten system security and stability of the network. So the idea of DG optimal sizing in electricity market model needs to be addressed. In this paper, an algorithm for optimal sizing of photovoltaic arrays and battery in interconnected mode is described. In this study, uncertainty in solar radiation in state of grid-connected with main upstream network is considered. The objective function is maximizing the total net present worth (NPW) of the generation system i.e. sum of net present of photovoltaic

arrays-owners and distribution company over its 10-year lifetime. Uncertainty of solar radiation is considered with using Monte Carlo method. For probabilistic analysis of solar radiation, 15 years data of solar radiation are used. The data has been used as every four-hour. For every four-hour data, a best probability density function (PDF) of solar radiation is determined. For a year, month and day Weibull distribution is known as suitable distribution for solar radiation. Using Monte Carlo and Particle Swarm Optimization (PSO), best sizing of photovoltaic arrays, is determined.

II. POOL-BASED POWER MARKET

A pool market is a centralized market place that clears the market for the buyers and sellers. Electric power sellers/buyers submit bids to the pool for the value of power that they are desire to trade in the market [11-13]. In pool power market, power producers present suggestions for n-chronological sequence and remain into the game even if they don't manage to find a market share for a long time. They not are aware with each other's payoffs and offers and costs at the end of each round are aware only with the market-clearing price and with their own market portion and profits. Their income are evaluated in terms of profit, with the one they obtained in the preceding round and if it is better they reward the last randomly selected action using increasing its probability in the distribution of probability of potential actions. Or else, they de-crease that probability value. New chosen values from the adjusted probability distribution clean the next offer.

III. OPF EQUATIONS AND CONSTRAINTS

Power flow equations corresponding to both active and reactive power balance for all the buses are expressed by following equations [14]:

$$P_i = \sum_{j=1}^{Nb} V_i V_j [G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)] \qquad (1)$$

$$Q_i = \sum_{j=1}^{Nb} V_i V_j [G_{ij} \sin(\delta_i - \delta_j) - B_{ij} \cos(\delta_i - \delta_j)] \qquad (2)$$

The equality constraints are, as follows:

Active and reactive power injections at bus-*i* are expressed by the following equations:

$$Q_i = Q_{gi} + \theta * Q_{PV-i} - Q_{di} \qquad (3)$$

$$P_i = P_{gi} + \theta * P_{PV-i} - \rho * P_{di} \qquad (4)$$

Where, ρ is the demand variation factor for both pool and bilateral demand. It represents the change in the operating point of the system. Also θ is decision variable with value {0, 1}. In the presence of PV this value are equal to one and otherwise, they are zero.

Inequality constraints are, as follows:

These constraints are power generation, voltage and angle limits, expressed by following equations:

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \qquad (5)$$

$$\delta_i^{\max} \le \delta_i \le \delta_i^{\max} \qquad (6)$$

$$|V|_i^{mix} \le |V|_i \le |V|_i^{\max} \qquad (7)$$

IV. PHOTOVOLTAIC ARRAYS

PV technology is identified as most environment friendly technologies. It requires only sunlight and no other energy fuel. Being modular in design, the capacity can be increased to meet additional demand. It is easy to dismantle and reconfigure these systems for other applications. PV systems require little maintenance. These components can be manufactured and assembled locally. PV system has several advantages as follows [15]:

(1) Environment friendly as they do not emit gaseous and liquid pollutants

(2) Without any noise

(3) Generate DC electricity that can be easily stored in batteries

(4) Zero fuel usage

(5) Can be easily transported, assembled and installed in remote areas

Another advantage of PV is low maintenance cost. However the PV energy cost is still higher than the utility energy price. Therefore, the PV applications have been limited to remote locations not connected to the utility lines .The PV out-put depends on climatic conditions. The combination of PV with battery will, therefore, form a very reliable distributed system, where the battery acts as a back-up during low PV out-put.

Photovoltaic power output is proportional with solar radiation. In this paper, the photovoltaic array with rated power of 1 (kW) is considered. The output power of photovoltaic array without considering uncertainties can be calculated using equation (8) [16].

$$P_{PV} = A_{pv}x^2 + B_{pv}x + C_{pv} \qquad (8)$$

where x is solar radiation [W/m2] and P is power generation [W] and A_{pv}, B_{pv} and C_{pv} are constants, which can be derived from measured data. In this study the corresponding values are A_{pv}=55, B_{pv}=-12.5 and C_{pv}=1.11

By using the above formula, solar power generation at any solar radiation can be predicted. This is also useful in estimating the suitable solar photovoltaic panels for many required load.

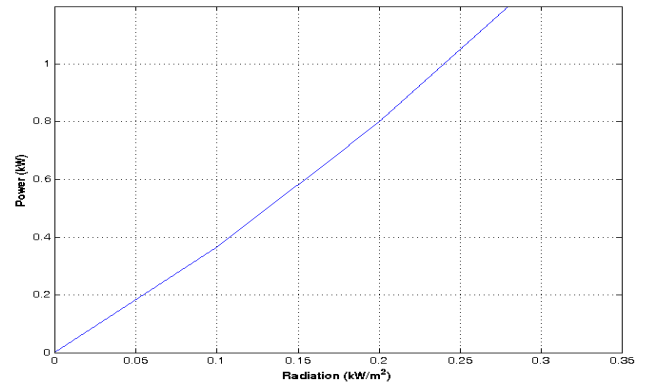The power-radiation curve of PV, employed in this study, is based on the quadrate function shown in Fig.1.

Figure 1.Power-radiation curve of PV

V. ECONOMIC ANALYSIS OF SYSTEM

According to the proposed structure for distribution system, including PV units, the objective function is suggested, as follows:

Maximize:

$$\text{NPW} = \omega_1 \times \text{NPW}_{\text{DISCO}} + \omega_2 \times \text{NPW}_{\text{PV-Owner}}$$
(9)

With

$$0 \leq w_i \leq 1 \quad and \quad \sum_{i=1}^{2} w_i = 1$$
(10)

where, ω_1, ω_2 are weighting factors for DISCO (Distribution Company) and PV owner respectively.

NPW is the Net Present Worth of the total benefit of overall system. It includes the net present worth of Distribution Company (DISCO) and Photovoltaic Owner. The Net Present Worth of benefits of Distribution Company is presented by *NPW_DISCO* and is calculated as follows:

$$\text{NPW}_{\text{DISCO}} = \text{PWR}_{\text{DISCO}} - \text{PWC}_{\text{DISCO}}$$
(11)

Where *PWR_DISCO* is present worth of revenues of Distribution Company and *PWC_DISCO* is present worth of costs of Distribution Company during *n* years of long term programming.

Also the Net Present Worth of benefits of PV Owner is presented by *NPW_PV-Owner* and is calculated as follows:

$$\text{NPW}_{\text{PV-Owner}} = \text{PWR}_{\text{PV-Owner}} - \text{PWC}_{\text{PV-Owner}}$$
(12)

Where *PWR_PV-Owner* is present worth of revenues of Distribution Company and *PWC_PV-Owner* is present worth of costs of Distribution Company during *n* years of long term programming.

The DISCO provides the necessary power of customers from the PV and Utility Grid. In fact, the main purpose is maximizing the net present worth of PV Owner and DISCO. In this paper, PSO is used for the optimization procedure.

In this section, all of the benefits and costs of DISCO and PV Owners are presented.

A. Income of DISCO

The revenue of DISCO is the amount of monetary income received from selling electricity to customers (load), as follows [17]:

$$R_{DISCO} = \sum_{k=1}^{T} \sum_{i=1}^{M} P_{k,i,sell}^{e} \times \rho_{k,i,sell}^{e}$$
(13)

B. Costs of DISCO

The cost function for DISCO can be determined by the following equation [18]:

$$C_{DISCO} = C_F + C_{S-M\&O} + C_{Buy}$$
(14)

Where, the first term of (14) is the total cost of repairing and upgrading the network feeders and expressed by the following equation [19]:

$$C_F = \sum_{i=1}^{TN} \sum_{j=1}^{M} C_{ij} \sigma_{ij}$$
(15)

The second term is the total cost of the maintenance and operation and repairing of substations expressed as follows [19]:

$$C_{S-M\&O}$$
$$= \sum_{i=1}^{TN} \sum_{u=1}^{M} C_{i,u} \sigma_{i,u} + 8760 \sum_{t=1}^{T} \beta (\sum_{i=1}^{TN} \sum_{u=1}^{TUi} C_e . P_{i,u})$$
(16)

The last term is the total cost used for buying electricity from main grid, photovoltaic owner in order to supply the local load, and it is expressed by the following equation [17]:

$$C_{Buy} = \sum_{k=1}^{T} P_{k,buy}^{e,UP} \rho_{k,buy}^{e,UP} + \sum_{k=1}^{T} P_{k,buy}^{e,PV} \rho_{k,buy}^{e,PV}$$
(17)

It is notable that $\sigma_{i,u}$ and $\sigma_{i,j}$ are decision variables for the repair. It is equal to one in case of repair, otherwise it is zero.

C. Income of photovoltaic owner:

The revenue of photovoltaic owner is the amount of the monetary income received from selling electricity to DISCO on peak. It is expressed by the following equation [17]:

$$R_{DG} = \sum_{k=1}^{T} \sum_{i=1}^{N_{DG}} P_{k,i,sell}^{e,PV} \times \rho_{k,sell}^{e,PV}$$
(18)

D. Costs of Photovoltaic Owner:

The cost function for photovoltaic owner can be calculated by the following equation:

$$C_{PV} = C_{r\&m} + C_{P(PV)}$$
(19)

The first term is the repair and maintenance cost and expressed by the following equation [19]:

$$C_{r\&m} = 8760 \sum_{i=}^{T} \beta \sum_{i=1}^{M} (C_{r\&m}.pf.S_{DGi})$$
(20)

The second term is the production cost. It is considered as a quadratic function of produced power, as follows [20]:

$$C_{P(PV)} = \sum_{i=1}^{N_{PV}} C_{PV,i}$$
(21)

where, $C_{PV,i}$ represents the generation cost of PV expressed by the following equation [21]

$$C_{PV,i} = a' + b' \times P_{PV_i}$$
(22)

As described in objective function, the NPW is deference between present worth of revenues and costs. Each component of market i.e. DISCO and PV has costs and incomes which have been discussed earlier in section 5A to 5D. Sum of costs of PV during n years of life time which analyzed in detail in section 5D, are converted to present worth as $PWC_{PV\text{-}Owner.}$

On the other hand the sum of incomes of PV which presented in section 5C are converted to present worth as $PWR_{PV\text{-}Owner}$ and the differences between these two items are inserted into objective function with ω_2 weighting coefficient. The value of this factor is dependent to decision of operator and is between 0 and 1. In this study this factor is considered equal 0.6.

Similarly for DISCO the sum of incomes and sum of costs which addressed in section 5A and 5B during n years of life time are converted to PWR_{DISCO} and PWC_{DISCO} and finally their difference is inserted into objective function with ω_1 weighting coefficient which has been considered as 0.4 in this study.

VI. PROPOSED METHOD

In this research, the numbers of photovoltaic array and battery are optimized for supplying the load. Maximization of Net Present Worth is considered as objective function and satisfies defined pool market indices as constrain. Uncertainty in power output of photovoltaic arrays is also considered. The algorithm of presented method is represented in Fig.2.
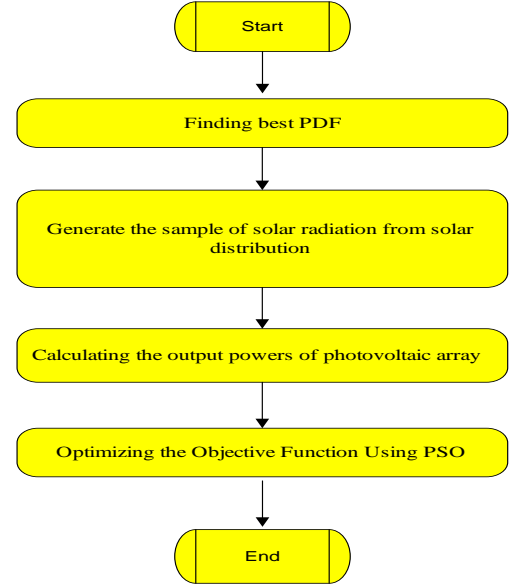
Figure 2.The algorithm of presented

1. Monte Carlo Method:
Monte Carlo method is a set of computational algorithms based on repeated random sampling to subtract their results. The Monte Carlo method is a broadly used apparatus in various courses, counting engineering, finance, biology, computer graphics, physics, chemistry, operations research and management science [22].

Monte Carlo approach works as a pattern described follows:

1- Determination the boundary of data.

2- Data generation from probability distribution of input.

3- Deterministic calculation on the data.

2. Best Distribution of Solar Radiation
In order to clarify the best probability density functions (PDF) of solar radiation, 15 years data are used. The data has been used as every four-hour period, i.e. one year is separated

to 2190 samples of solar radiation and each four-hour sample have 15 samples of all 15 years, so the time step is four-hour. The purpose is getting the PDF model for every four-hour of solar radiation. In order to compute the PDF for solar radiation, various representation of solar radiation for a year, month and day is investigated. For a year, month and day Weibull distribution is known as suitable distribution for solar radiation. In this research, a distribution style for every time step solar radiation data is considered, that for each time step the parameters of distribution are different from other hours. The algorithm of getting the most suitable distribution for each time step solar radiation is shown in Fig.3.
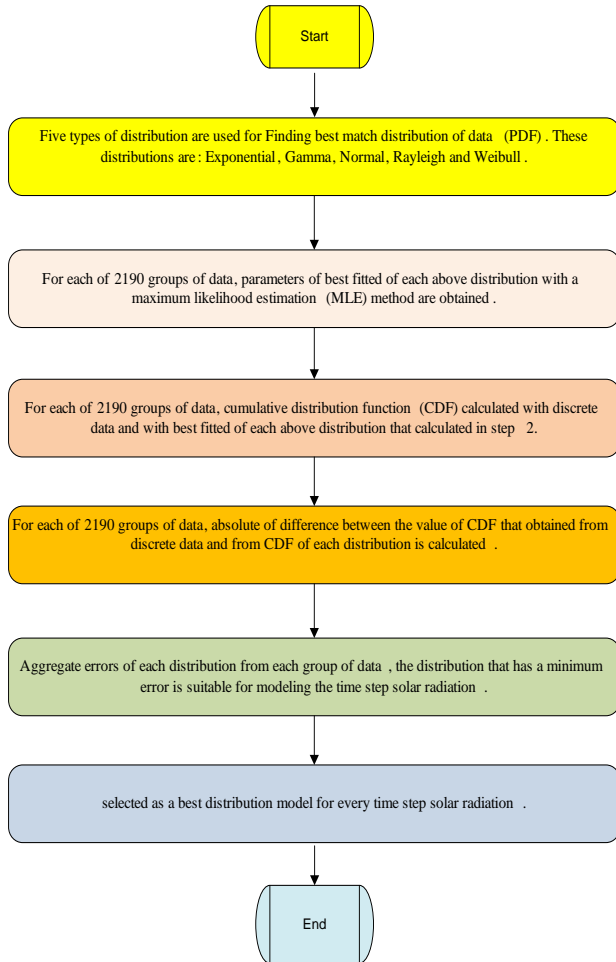


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┌─────────────┐
│    Start    │
└─────────────┘
        │
        ▼
┌──────────────────────────────────────────────────┐
│ Five types of distribution are used for Finding    │
│ best match distribution of data  (PDF) . These     │
│ distributions are : Exponential , Gamma , Normal , │
│ Rayleigh and Weibull .                             │
└──────────────────────────────────────────────────┘
        │
        ▼
┌──────────────────────────────────────────────────┐
│ For each of 2190 groups of data, parameters of     │
│ best fitted of each above distribution with a      │
│ maximum likelihood estimation  (MLE) method are    │
│ obtained .                                         │
└──────────────────────────────────────────────────┘
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        ▼
┌──────────────────────────────────────────────────┐
│ For each of 2190 groups of data, cumulative        │
│ distribution function  (CDF) calculated with       │
│ discrete data and with best fitted of each above   │
│ distribution that calculated in step  2.           │
└──────────────────────────────────────────────────┘
        │
        ▼
┌──────────────────────────────────────────────────┐
│ For each of 2190 groups of data, absolute of       │
│ difference between the value of CDF that obtained  │
│ from discrete data and from CDF of each            │
│ distribution is calculated  .                      │
└──────────────────────────────────────────────────┘
        │
        ▼
┌──────────────────────────────────────────────────┐
│ Aggregate errors of each distribution from each    │
│ group of data , the distribution that has a        │
│ minimum error is suitable for modeling the time    │
│ step solar radiation  .                            │
└──────────────────────────────────────────────────┘
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│ selected as a best distribution model for every    │
│ time step solar radiation  .                       │
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┌─────────────┐
│     End     │
└─────────────┘
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Figure 3.The algorithm of finding the suitable and best distribution for each time step solar radiation

In this paper, Particle Swarm Optimization (PSO) is considered as optimization algorithm.

3. PSO-based optimization method

Particle Swarm Optimization, as an optimization tool, consists of a characteristic called particle. Each particle in order to move to optimum position, changes its position with time [23], [24]. Particles move around in a multi dimensional search space, during flight. Each particle according to its own experience, and the experience of neighboring particles, adapt its position. Other characteristic in the PSO approach is called swarm. A swarm consists of a set of particles, neighboring the particle and its history experience. X represents a vector that shows various positions of particle. So, the dth particle in a k-dimensional space is represented as:

$$x_d = (x_{dk}, x_{dk}, ...; x_{dk}) \qquad 23$$

In order to achieve to optimum position, the best previous position is recorded as

$$qbest_d = (qbest_{d1}, qbest_{d2}, ...; qbest_{dk}) \qquad 24$$

In Eq. (23), the "$sbestd$" is represented as index of the best particle among all the particles in the swarm. Another characteristic defended in the PSO approach is called velocity. The velocity of the dth particle is expressed as (25):

$$v_d = (v_{d1}, v_{d2}, v_{d3}, ..., v_{dk}) \qquad 25$$

In order to search the better velocity and position, in the next iteration, velocity and position of each particle may be obtained by using current velocity and position as expressed by (26) and (27).

$$v_{dk}^{r+1} = w.v_{dk}^{r+1} +$$
$$c_1 * rand(qbest_{dk} - x_{dk}^r) + c_2 * Rand(sbest_{dk} - x_{dk}^r) \qquad 26$$
$$x_{dk}^{r+1} = x_{dk}^r + v_{dk}^{r+1}$$
$$d = 1,2,...,D \quad k = 1,2,...,m \qquad 27$$

That D indicates the number of particles in a group; m is the population size in a particle, r is the number of iterations (generations), w indicates the weight factor, c_1, c_2 are the acceleration constants.

Moreover in above equations the $Rand (), rand ()$ are the uniform random values in the range [0, 1], x_{dk}^r is the position of the kth member in the dth particle at iteration r and finally v_{dk}^r represents the velocity of the kth member in the dth particle at iteration r.

It is notable that $V_d^{min} \le v_{dk}^r \le V_d^{max}$ .The parameter V^{max}

determines the resolution, or fitness, showing which regions

are to be searched between the present position and the target position. If $V^{\max}$ is very high, particles might fly past good solutions. Similarly if $V^{\max}$ is too small, particles may not explore sufficiently beyond local solutions. In many experiences with PSO, $V^{\max}$ was often set at 12–25% of the dynamic range of the variable on each dimension. The parameters c_1 and c_2 represent the weighting of the stochastic acceleration terms. High values result in abrupt movement toward, or past, target regions. On the other hand, low values allow particles to roam far from the target regions before being tugged back. Hence, according to past experiences, the acceleration constants c_1 and c_2 were often set to be 2.0.

Suitable choice of the inertia weight w can supply a balance between global and local explorations. In general, the inertia weight w is adjusted according to the following equation:

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{iter_{\max}} \times iter \qquad 28$$

In this equation $iter_{max}$ represents the maximum number of iterations, and $iter$ indicates the current number of iterations.

In this optimization problem with n type candidate of energy source for planning, a particle in order to represents the added capacity in planning horizon will be defined by (29) and (30).

$$q_{i,d} = [(q_{d1}^1,...,q_{d1}^T)(q_{dn}^1,...,q_{dn}^T)...(q_{dn}^1...,q_{dn}^T)] \qquad 29$$

$$q_i^t = [q_{d1}^t, q_{d2}^t,...,q_{dn}^t] \qquad 30$$

Where qd,i is the dth particle vector that represents the added capacity of different type energy source of the ith energy source in planning horizon. Also, q_{dn}^t, as a member of the dth particle, relates to the added capacity of the nth type of the energy source at period t. It should be noted that the planning horizon of energy source planning is split into T2-year periods. For example in a 6-year horizon, T will be equal to 3, i.e. the horizon is covered three 2-year periods. As described before, q_i^t is vector, representing the added

capacity of each energy source type (n type unit) of the ith energy source such as (PV, Upstream network=main utility=DISCO).

According to the above mentioned explanation, the dimension of a swarm will be $d \times (n.T)$, and the population size of each particle will be $n.T$ .For example, if the number of energy source type is 2(n=2), the number of periods in planning horizon is 3(T =3) and number of particles is assumed 2(d=2), the dimension of the swarm will be $2 \times (3 \times 2)$ .therefore the population size of each particle will be 6.

So, the swarm can be shown as follows:

$$S = \begin{bmatrix} q_{i,1} \\ q_{i,2} \end{bmatrix} = \overbrace{\begin{bmatrix} q_{11}^1, q_{11}^2, q_{11}^3 & q_{12}^1, q_{12}^2, q_{12}^3 \\ q_{21}^1, q_{21}^2, q_{21}^3 & q_{22}^1, q_{22}^2, q_{22}^3 \end{bmatrix}}^{\text{population size}}$$

After preparing particles and swarm, the fitness value should be calculated. In this study, the fitness value is equal to the value of objective function, shown in Eq. (9). In order to comparing of particles with together and also speeding up the convergence of the iteration procedure, the fitness value is normalized between [0, 1] based on the following equation that is given by (31):

$$f'^{(i)} = \frac{f(i) - f_{\max}}{f_{\max} - f_{\min}} \qquad 31$$

Where $f(i)$ is the fitness value, $f_{\min}$, $f_{\max}$ represent the minimum and maximum fitness value in a iteration, f(i) indicates the normalized fitness value.

Once the fitness value is calculated, the regional constraints must be checked. Hence a modified fitness value, $mf(i)$, is defined by using a penalty factor. If a particle satisfy the all of constraints of regional level, then $mf(i)$ is equal to $f(i)$, otherwise, the penalty factor is subtracted from $f(i)$. So, the $mf(i)$ and penalty factor (pf) is expressed as:

$$mf(i) = \begin{cases} f'(i) & if \quad constrains \quad are \quad satisfied \\ f'(i) - pf & otherwise \end{cases} \qquad 32$$

$$pf = \frac{f'(i)}{\gamma}$$
33

It is notable that γ is a constant value. It is set to a value that the mf becomes near to zero in an attempt to reduce the effectiveness of particles not satisfying the regional level constraints. With this consideration, the particles that satisfy the constraint will have a significant mf as compared to that of offender particles.

In order to better clarify, the solution of optimization problem in regional level with PSO can be presented by an algorithm in five steps as follows:

Step1: Initialization

In this step $d, n, T, iter_{max}, w, c_1, c_2$ and velocities are assigned.

In this step, the lower and higher bound of regional constraints is specified too. Based on Eq. (29), d initial particles are generated in random in the range of regional constraint. Set iteration=1.

Step2: Objective function calculation

In this step the objective function and fitness value of each particle qi,d is calculated.

Compare fitness value of each particle with its *qbest*. The best fitness value among *qbest* is denoted as *sbest*.

Step3: Velocity modification

In this step the velocity of each particle is modified based on Eq. (32), and then generate the new particles based on Eq. (34).

$$v_{dn}^{t\ r+1} = w.v_{dk}^{r} + c_1 * rand(qbest_{dk} - q_{dn}^{t\ (r)})$$
$$+ c_2 * Rand(sbest_d - x_{dk}^{r})$$
34

$$q_{dn}^{t\ (r+1)} = q_{dn}^{t\ (r)} + v_{dn}^{t\ (r+1)}$$
35

In these equations $q_{dn}^{t\ (r)}$ is a part of (33) in the *rth* iteration.

It should be noted that $q_{dn}^{t\ (r)}$ is defined earlier in (30). In (32), if v_{dn}^{t} reaches to its boundary values, it will be adjust to the extreme values. In other words, If $v_{dn}^{t} > V^{max}$ then

$v_{dn}^{t} = V^{max}$. Similarly, If $v_{dn}^{t} < V^{min}$ then $v_{dn}^{t} = V^{min}$ .Finally the all of regional constraints are checked and the offender particles are penalized with the penalty factor expressed by (35).

Step4: Upgrading of qbest,sbest

If the fitness value of each particle is better than the previous *qbest*,then *qbest* is updated with the current value.

If the best *qbest* is better than *sbest*, then *sbest* will be substituted with the best *qbest*. This is the end of iteration. Set iteration=iteration+1. If *iteration>iter_{max}* then the algorithm is stopped unless it is continued by going to step2. Otherwise step 5.

Step5: Results of PSO

The particle that generates the latest *sbest* is the optimal solution of PSO.

VII. SIMULATION RESULTS

In this research PSO Algorithm is used for optimization procedure. In this paper, photovoltaic and battery is considered for supplying the load. Pool-based power market is considered as constrains equation in optimization procedure in net present value of total system including Distribution Company and photovoltaic owner.

1. Different Cases Analysis

The histogram of power generated from a photovoltaic array for a time step of a year are shown in Fig. 4. Optimization algorithm run and the variables are updated in each time step.
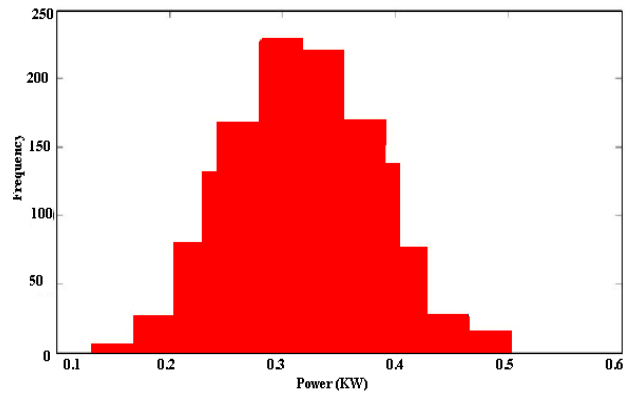

Figure 4.Histogram of a time step photovoltaic array power output

The convergence of PSO algorithm is shown in Fig. 5 and the best combination of photovoltaic and battery units with optimized net present worth for three cases with different values of weighting coefficients are listed in Tables I-VI. The three cases which have been analyzed in this paper are listed as follows:

Case 1: In this case, from point of view of market operator the importance of economical benefits of DISCO and PV owners are same and so the values of $\omega_1 = 0.5$ and $\omega_2 = 0.5$ are considered.

Case 2: In this case, from point of view of market operator the importance of economical benefits of DISCO must be much more than PV owners and so the values of $\omega_1 = 0.6$ and $\omega_2 = 0.4$ are considered.

Case 3: In this case, from point of view of market operator the importance of economical benefits of PV owners must be much more than DISCO and so the values of $\omega_1 = 0.4$ and $\omega_2 = 0.6$ are considered.
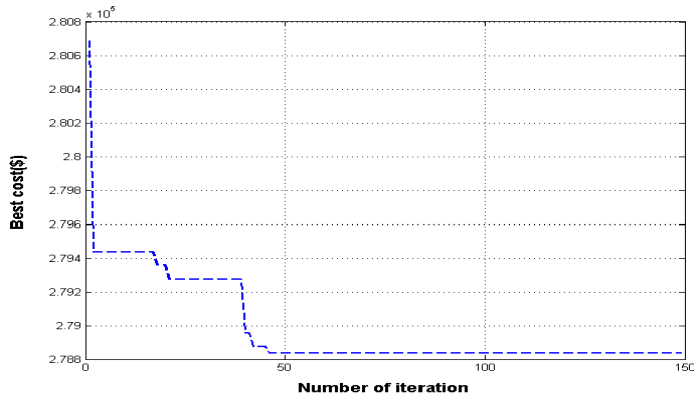

Figure 5.Convergence of PSO algorithm

Table I
Optimal prices for PV owner and DISCO under pool electricity market for case 1

Case	NPW PV-Owner	NPW DISCO	NPW Total
Uniform Pricing	54201$	551546$	598603$
Pay As Bid Pricing	59188$	592206$	589403$

Table II
Optimal number for PV units and battery under pool electricity market for case 1

Case	Costs [$]	PV units	Batt. Num.
Uniform Pricing	203430	219	16
Pay As Bid Pricing	201130	210	11

Table III
Optimal prices for PV owner and DISCO under pool electricity market for case 2

Case	NPW PV-Owner	NPW DISCO	NPW Total
Uniform Pricing	43249$	190546$	301203$
Pay As Bid Pricing	43138$	190206$	304412$

Table IV
Optimal number for PV units and battery under pool electricity market for case 2

Case	Costs [$]	PV units	Batt. Num.
Uniform Pricing	202612	210	18
Pay As Bid Pricing	201140	210	12

Table V
Optimal prices for PV owner and DISCO under pool electricity market for case 3

Case	NPW PV-Owner	NPW DISCO	NPW Total
Uniform Pricing	34201$	211546$	328603$
Pay As Bid Pricing	33188$	222206$	319403$

Table VI
Optimal number for PV units and battery under pool electricity market for case 3

Case	Costs [$]	PV units	Batt. Num.
Uniform Pricing	202510	220	17
Pay As Bid Pricing	202080	212	13

2. Sensitivity analysis

The sensitivity analysis is performed in this study too. A set of sensitivity parameters, the average of annual solar radiation, to interest rates for photovoltaic arrays, annual

inflation rate are investigated.

A. *Sensitivity to the average of annual solar radiation*: In this study, two different cases have been considered. $P_{request}$ *[kW]*,$E_{emergency}$ *[kWh]* and *Average of annual radiation [kW/m2]* are 800,1500,0.35 for case 1 and 800,1500,0.25 for case 2 respectively. The results of this sensitivity analysis are listed in Table VII.

Table VII. Comparison of system combinations using different average of annual solar radiation

Case	Cost [$]	PV Units	Batt. Num.	NPW		
				PV	DISCO	Total
1	202310	216	15	35698	255426	315403
2	203290	216	14	34638	235726	305123

B. *Sensitivity to interest rates for Photovoltaic arrays*: In this state, two interest rates for Photovoltaic arrays are considered as i_{pv}=0.0790 for case 1 and i_{pv}=0.0815 for case 2. The results of interest rates sensitivity analysis are listed in Table VIII.

Table VIII. Comparison of system combinations using different interest rates

Interest rates	Cost [$]	PV Units	Batt. Num.	NPW		
				PV	DISCO	Total
i_{pv}=0.0790	282140	215	16	36638	243826	315403
i_{pv} =0.0815	291610	214	14	35438	234126	306222

C. *Sensitivity to annual inflation rate:* In this state, two inflation rate are considered as f_{pv}=0.0345for case1and f_{pv} =0.0424for case 2. Results of inflation rate sensitivity analysis are listed in Table IX.

Table IX. Comparison of system combinations using different interest rates

Interest rates	Cost [$]	PV Units	Batt. Num	NPW		
				PV	DISCO	Total
f_{pv}=0.0345	297070	215	12	35698	244826	312313
f_{pv} =0.0424	289020	214	13	34638	235126	301222

VIII. CONCLUSION

This paper presents a multi-objective formulation with the aim of optimal sizing of photo-voltaic resources in distribution systems in order to maximize net present worth of system. The proposed system in this paper includes two sources such as upstream network i.e. 63Kv/20Kv substation and photovoltaic arrays as private sector for meeting load. Total net present worth considered in this paper as objective function consists of net present worth of distribution company (DISCO) and of photovoltaic owners (PV-Owner). In order to obtain accurate results, in this paper the uncertainty of solar radiation is considered. The implemented technique is based on PSO and weight method that employed to obtain the best compromise between these costs. Simulation results on 33-bus distribution test system under pool power market operation are presented to demonstrate the effectiveness of the proposed procedure. In order to clarify the best probability density functions (PDF) of solar radiation, 15 years data are used. The data has been used as every four-hour period, i.e. one year is separated to 2190 samples of solar radiation and each four-hour sample have 15 samples of all 15 years, so the time step is four-hour. The purpose is getting the PDF model for every four-hour of solar radiation. In order to compute the PDF for solar radiation, various representation of solar radiation for a year, month and day is investigated. For a year, month and day Weibull distribution is known as suitable distribution for solar radiation. In this research, a distribution style for every time step solar radiation data is considered, that for each time step the parameters of distribution are different from other hours. It will be found that by decreasing the annual average radiation, PV array using rate decreased too. It is noticeable that power production of PV sources depend on the average of annual solar radiation (e.g. for 0.25 kW/m2 annual radiation a 2 kW PV array can generate 1.1562 kW and for 0.35 kW/m2, 1.3562kW).

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