ELECTRIC ENERGY POWER IN THE SERIES CIRCUIT WITH A STATIC RESISTANCE ELEMENT AND A DIODE

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Abstract: *Traditional comprehension of the quality indicators of the electric energy's power in the single-phase circuits are reduced to well-known notions (about apparent, active, reactive and distortion powers), which are being criticized by modern researchers in terms of electricity metering. The paper is devoted to the analysis of the power of electric energy in the series circuit with a source of harmonic voltage, a static resistance and a diode in order to use it as an elementary example of the imperfections of mentioned traditional comprehension. The methods of the electrical engineering theory and analysis of instantaneous power's components of polyharmonic periodic current and voltage were used to calculate the power components for each element of the circuit in the analytical and numerical form. The rationality of using these calculations to assess the nature of electric energy's distortion was considered.*

Key words: *diode, converter, polyharmonic periodic current, power of higher harmonics.*

1. Introduction.

Calculations of the energy flows distribution in electric grid is an important task, both in designing and in operation. The indicators which should be determined in order to calculate the energy flows distribution based on the appropriate equivalent circuits are components of active and reactive power [1,2]. They ultimately determine the apparent power circulating at a given point of the circuit. This indicator is used as one of the selection criteria of electric equipment such as, for example, transformers. The presence of consumers with non-linear current-voltage characteristics of loads (arc furnaces, welding machines, semiconductor convertors), which are widely used in electric grids, makes it necessary to take into account indicators that reflect the quality of the processes of electric energy transformation and their quantitative assessment. At present, the quality of the electricity is assessed by the measured voltage in the node of electric grid in according with normative documents [3]. Some scientists research the causes of power quality deterioration [2,4] using well-known methods of circuit theory in order to determine the integral indicators of power. The relationship between electricity distortions and the additional components of electric power which caused the reduced its quality is accounted in papers [4-6]. This approach is useful to analyze the power of electricity, its metering and determining the causes of its deterioration. The correct determination of the electric power's components in according with the processes of generation, transmission, distribution and consumption of electric energy is very important for further developing of this approach.

2. Analysis of previous studies.

Among the considerable number of research devoted to components of the power of electricity there is a separate category which uses the appropriate power of distortion (introduced by Budeanu [7]) for characteristics of electricity distortion. This power is determined for circuits with polyharmonic current and (or) polyharmonic voltage and is calculated as a component that provides equality between the sum of the squares of active and reactive powers of all harmonic components and apparent power, which is determined by the multiplication of the effective voltage and current values. In [7] it is proved that the distortion power is a measure of power fluctuations' change in relation to the active power and reactive power. Similarly, the question of the imbalance of power for elementary circuit with a diode is considered on the base of analysis of integrated indicators in [8]. Versatility of the method and possibility of its wide use are brought in [9] on the basis of the balance of powers in the circuit with a diode, by presenting instantaneous value of power as a polyharmonic function. A detailed study of energy processes in electric elementary circuits with series connected diodes was executed in [10]. The generalized correlation of the distribution between main and higher harmonics of power is presented in the paper. At the same time the questions of forming the harmonic components in the power circuit are left without consideration.

3. The purpose of the work.

Determination of components distribution of electric power in a serial circuit with static resistance element and diode.

4. Setting objectives.

As noted in [10], in contradistinction to the linear loads which form polyharmonic current only under polyharmonic voltage the nonlinear loads distort current even when the input voltage is monoharmonic. That is, if the nonlinear load electric power conversion is accompanied by the formation of additional power components with frequencies which are different from the frequency of the voltage source. Many experts by the term non-linear loads mean such types of equipment as a device with a magnetic core (inductors, transformers and motors), an electric arc devices (electric welders, arc furnace, gas-discharge lamps), a semiconductor converters (DC and AC electric drives, chargers, etc.). In contradistinction to mentioned types of nonlinear loads there is specific group of devices which includes controlled and uncontrolled rectifiers or controlled and uncontrolled semiconductor switches that periodically connect to linear loads in according with certain schemes. It allows to speak about specific character of current in this group. Elementary circuit with a nonlinear element which was considered in [8,10,11] is shown in Fig. 1.a. Source of monoharmonic voltage (u_s) with internal resistance R_s is serially connected to the load R_{ld} and ideal diode VD. Accounting for power is executed on the area marked by gray line. In accordance with the procedure which is described in [13] the diode and load impedance are replaced by equivalent voltage sources as shown in Fig. 1.b. Given the fact that the element loads - static resistance, we will replace only the diode by equivalent sources as shown in Fig. 1.c.

5. Electric calculation.

By analogy with [10], the power supply voltage:

$$u_s = U_{sm}\cos(\omega t),$$

where U_{sm} - voltage amplitude; ω - angular frequency.

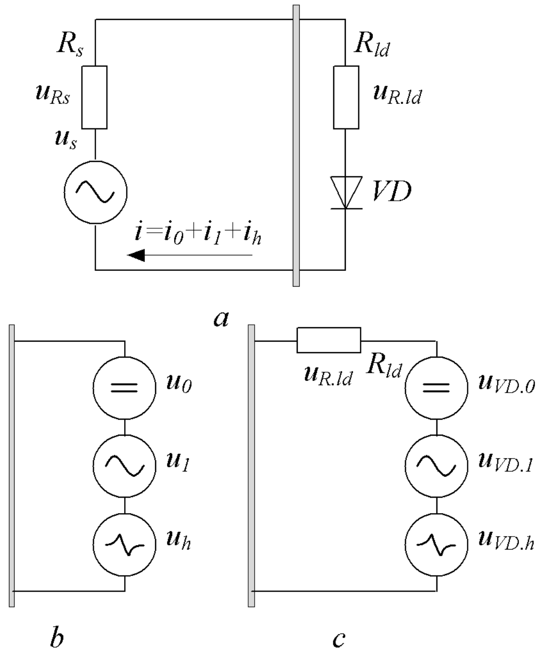

Fig. 1

The current is distorted and it will consist of three components:

$$i = i_1 + i_0 + i_h;$$

where $i_1 = \frac{U_{sm}}{2(R_s + R_{ld})}\cos(\omega t)$ - basic current harmonic; $i_0 = \frac{U_{sm}}{\pi(R_s + R_{ld})}$ - direct current component; $i_h = -\frac{2}{\pi}\frac{U_{sm}}{(R_{gr} + R_{ld})}\sum_{h=2,4,6,..}\frac{\cos(h\omega t)}{h^2 - 1}$ - higher current harmonics.

Distorted current causes a voltage drop on the resistance $R = R_s + R_{ld}$ that has the same harmonic sequence, as well as the current and consists of three components $u = u_1 + u_0 + u_h$.

This observation allows us to replace non-linear element by fictitious equivalent voltage source which has the same spectrum as the actual voltage. That is, the diode acts as a generator and generates in its closed state interval the voltage which is equal in level and opposite in sign of voltage source, thereby providing zero current. Similarly as the current, the voltage can be decomposed $u_{VD} = u_{VD.1} + u_{VD.0} + u_{VD.h}$.

Based on Kirchhoff's voltage law:

$$u_{VD} = u_s - u_R \rightarrow$$

$$\rightarrow u_{VD.1} + u_{VD.0} + u_{VD.h} = u_{s.1} - u_{R.1} - u_{R.0} - u_{R.h}$$

Let us write balance for each component of the elements' voltage

$$u_{VD.1} = u_{s.1} - u_{R.1} = \frac{U_{sm}}{2}\cos(\omega t);$$

$$u_{VD.0} = -u_{R.0} = -\frac{U_{sm}}{\pi};$$

$$u_{VD.h} = -u_{R.h} = -\frac{2U_{sm}}{\pi}\sum_{h=2,4,..}\frac{\cos(h\omega t)}{h^2-1}.$$

The current in all elements of the circuit is the same, so the voltage on the resistive components R_s, R_{ld} of the circuit is proportional to the current

$$u_R = Ri_1 + Ri_0 + Ri_h.$$

Figure 2 shows diagrams of current, voltage and instantaneous power for source i_s, u_s, p_s (fig. 2.a), diode i_{VD}, u_{VD}, p_{VD} (fig. 2.b), source resistance i_{Rs}, u_{Rs}, p_{Rs} (fig. 2.c), load resistance $i_{Rld}, u_{Rld}, p_{Rld}$ (рис.2.d).

It should be noted that power of diode VD in the considered case is absent (fig. 2), i.e. $p_{VD} = 0$. Thus, the analysis of this power as a signal is not possible.

6. Calculation of power.

Integral components of the power are determined using indicators [2], apparent power, active power and inactive power

$$S = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} u^2 dt}\sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2 dt};$$

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} uidt;$$

$$N = \sqrt{S^2 - P^2}.$$

for each circuit element, on the condition that $U_{sm} = 220V$, $R_s = 0.5Ohm$, $R_{ld} = 2Ohm$ $\omega = 314s^{-1}$. The results of calculations are shown in the table 1. Reactive power calculation was performed with an additional integral expression [7]

$$Q_H = \frac{1}{T}\int_{t_0}^{t_0+T} uH(i)dt,$$

where $H(i)$ - Hilbert transform for the current.

Analysis of the results (Table 1) shows the distribution of active power P between the source, source resistance and load resistance. Apparent power S is observed at the all elements besides S is equal to the active power at the active resistances. Inactive power N in the circuit with power supply, diode and without any reactive elements cannot be classified as a reactive. Definition of power Q_H by [7] is some interest. The diode VD is a generator of this power (Table 1).

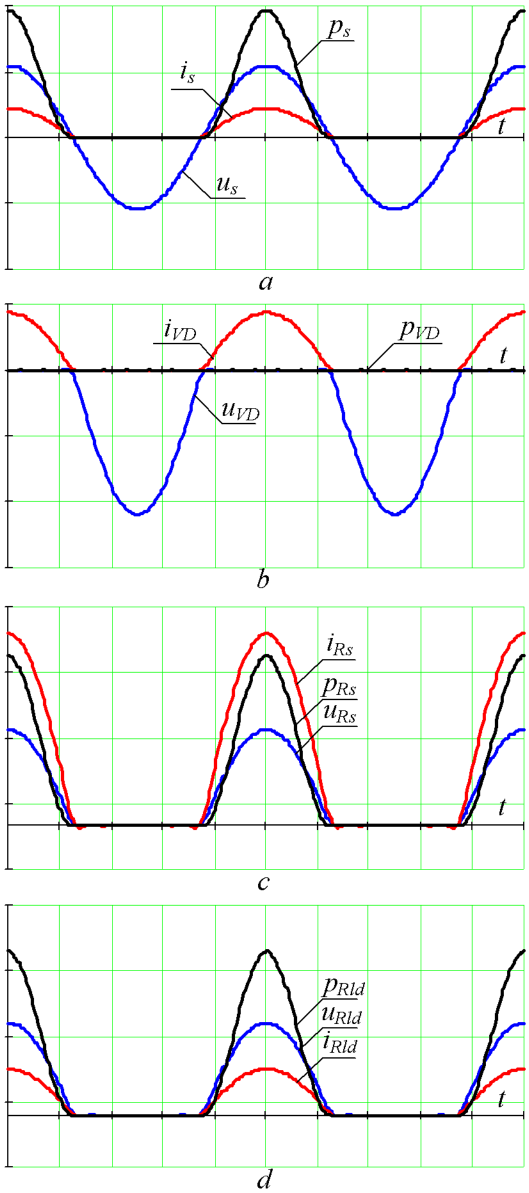

Fig. 2.

Table 1.

	P, W	S, VA	N, var	Q_H, var
Source	4833	6837	4837	-1,807
R_s	966,419	966,419	0	395,924
R_{ld}	3866	3866	0	1584
VD	0	4837	4837	-1981

It disseminates the power between the resistances in this circuit (the power of source in this case is on the level of calculation error). So using of known integral indicators doesn't afford full understanding concerning of the energetic processes in the circuit.

Let us determine analytically the signal of power for each element using known analytical equations of current and voltage on the circuit elements and the multiplication algorithm for Fourier series. Thus, let us limit a series that reflects the current signal and a series that reflects the voltage signal, for reduction of calculations volume, by tenth harmonic. As shown by previous studies the mean square deviation of current and voltage signals which were created by a limited series of harmonics does not exceed in this case 3% if compare them with original signals. Figure 3 shows (for specific considered example) the harmonics distribution of: current amplitudes (fig. 3.a); voltage amplitudes (fig. 3.b); current phase angle (fig. 3.c); voltage phase angle (fig.3.d).

Source voltage power

$$p_s = u_s i = u_s\left(i_0 + i_1 + i_h\right) =$$

$$= U_{s1}\cos(\omega t)\left[I_0 + I_{m1}\cos(\omega t) + \sum_{h=2,4,6,8,10} I_{mh}\cos(h\omega t)\right] =$$

$$= \frac{U_{sm}^2}{R_s + R_{ld}}\cos(\omega t)\times$$

$$\times\left[\frac{1}{\pi} + \frac{1}{2}\cos(\omega t) + \frac{2\cos(2\omega t)}{3\pi} - \frac{2\cos(4\omega t)}{15\pi} + \frac{2\cos(6\omega t)}{35\pi} - \frac{2\cos(8\omega t)}{63\pi} + \frac{2\cos(10\omega t)}{99\pi}\right],$$

$$p_s = P_{s.0} + \sum_{k=1}^{11} P_{s.k}\cos(k\omega t),$$

where $P_{s.0}$ - constant power of voltage source (average); $P_{s.k}$ - amplitude of k-harmonic components a voltage source power.

Power of static resistance

$$p_R = u_R i = \left(u_{R0} + u_{R1} + u_{Rh}\right)\left(i_0 + i_1 + i_h\right) =$$

$$= R\left(i_0 + i_1 + i_h\right)^2 = \frac{RU_{sm}^2}{\left(R_s + R_{ld}\right)^2}\left[\frac{1}{\pi} + \frac{1}{2}\cos(\omega t) + \frac{2\cos(2\omega t)}{3\pi} - \frac{2\cos(4\omega t)}{15\pi} + \frac{2\cos(6\omega t)}{35\pi} - \frac{2\cos(8\omega t)}{63\pi} + \frac{2\cos(10\omega t)}{99\pi}\right]^2,$$

$$p_R = P_{R.0} + \sum_{k=1}^{20} P_{R.k}\cos(k\omega t),$$

where $P_{R.0}$ - constant power of resistance (average); $P_{R.k}$ - amplitude of k-harmonic power components of resistance.

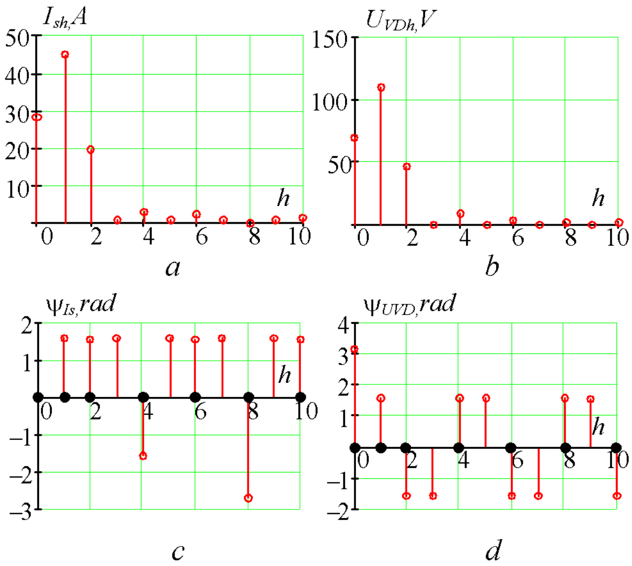

Fig. 3.

Power of diode

$$p_{VD} = u_{VD} i = \left(u_{VD0} + u_{VD1} + u_{VDh}\right)\left(i_0 + i_1 + i_h\right) =$$

$$= \left[U_{VD0} + U_{VDm1}\cos(\omega t) + \sum_{h=2,4,6,8,10} U_{VDmh}\cos(h\omega t)\right]\times$$

$$\times\left[I_0 + I_{m1}\cos(\omega t) + \sum_{h=2,4,6,8,10} I_{mh}\cos(h\omega t)\right] =$$

$$= \frac{U_{sm}^2}{\left(R_s + R_{ld}\right)}\left[-\frac{1}{\pi} + \frac{1}{2}\cos(\omega t) - \frac{2\cos(2\omega t)}{3\pi} + \frac{2\cos(4\omega t)}{15\pi} - \frac{\cos 2(6\omega t)}{35\pi} + \frac{2\cos(8\omega t)}{63\pi} - \frac{2\cos(10\omega t)}{99\pi}\right]\times$$

$$\times\left[\frac{1}{\pi} + \frac{1}{2}\cos(\omega t) + \frac{2\cos(2\omega t)}{3\pi} - \frac{2\cos(4\omega t)}{15\pi} + \frac{2\cos(6\omega t)}{35\pi} - \frac{2\cos(8\omega t)}{63\pi} + \frac{2\cos(10\omega t)}{99\pi}\right],$$

$$p_{VD} = P_{VD.0} + \sum_{k=1}^{20} P_{VD.k}\cos(k\omega t),$$

where $P_{VD.0}$ - constant power of diode (average); $P_{VD.k}$ - amplitude of k-harmonic power components of diode.

Given the fact that the variable component of power when calculating the average value will be zero result, consider the constant power component which is formed from equal-frequency components of current and voltage.

Therefore, the average value for the period (t_0, t_0+T), where $T = 2\pi/\omega$, of voltage source power

$$P_s = \frac{1}{T}\int_{t_0}^{t_0+T} p_s dt = P_{s.0} = \frac{2}{\pi}\frac{U_{sm}^2}{(R_s + R_{ld})}.$$

The power average value of static resistance

$$P_R = \frac{1}{T}\int_{t_0}^{t_0+T} p_R dt = P_{R.0} = R\left[\frac{U_{sm}^2}{\pi^2(R_s+R_{ld})^2} + \frac{U_{sm}^2}{8(R_s+R_{ld})^2} + \right.$$

$$+\frac{2U_{sm}^2}{9\pi^2(R_s+R_{ld})^2} + \frac{2U_{sm}^2}{225\pi^2(R_s+R_{ld})^2} + \frac{2U_{sm}^2}{1225\pi^2(R_s+R_{ld})^2} +$$

$$\left. + \frac{2U_{sm}^2}{3969\pi^2(R_s+R_{ld})^2} + \frac{2U_{sm}^2}{6400\pi^2(R_s+R_{ld})^2}\right].$$

The power average value of diode

$$P_{VD} = \frac{1}{T}\int_{t_0}^{t_0+T} p_{VD} dt = P_{VD.0} =$$

$$= -\frac{U_{sm}^2}{\pi^2(R_s + R_{ld})} + \frac{U_{sm}^2}{8(R_s + R_{ld})} -$$

$$-\frac{2U_{sm}^2}{9\pi^2(R_s + R_{ld})} - \frac{2U_{sm}^2}{225\pi^2(R_s + R_{ld})} -$$

$$-\frac{2U_{sm}^2}{1225\pi^2(R_s + R_{ld})} - \frac{2U_{sm}^2}{3969\pi^2(R_s + R_{ld})} -$$

$$-\frac{2U_{sm}^2}{6400\pi^2(R_s + R_{ld})} = 0.$$

It should be noted that the power average value of diode is zero. Omitting the multiplication process of currents and voltages of series components for each of the circuit elements (fig. 1.a.) we note in the present case there are the cosine components only and the amplitude of each harmonic power element is defined as $P_k = \sum_{|h\pm g|=k} 0.5U_{mh}I_{mg}$, or for resistance $P_k = \sum_{|h\pm g|=k} 0.5RI_{mh}I_{mg}$. Calculation results are tabulated in table 2 taking into account that the number of harmonics is limited to twelve. The constant component of power for each circuit element defined by this way, as shown in Table 2, differs from active power in Table 1 not more than 1%. In this case, find a certain physical sense in the higher harmonic power, which according to [9, 10], formed by all combinations of amplitude harmonic current and voltage, assuming that $|g \pm h| = k$ is impossible.

Additionally, consider the components of power considering equal-frequency components of current and voltage only, i.e. $P_k = \sum_{|h\pm g|=k, h=g} 0.5U_{mh}I_{mg} = P_{hh}$, or for resistance $P_k = \sum_{|h\pm g|=k, h=g} 0.5RI_{mh}I_{mg} = P_{hh}$.

The results of determine the power component on this principle are shown in Table 3. The total power of each circuit element defined in this way according to Table 3 differs from active power in Table 1 not more than 1%. In the present case such interpretation of the components of power allows to form a view of its distribution. Whole circuit is provided by power supply (total power 4840 *VA*). The voltage and current harmonics of the main frequency are shared between all elements of the scheme, and half of the power is transmitted to diode.

Components of the diode's power *VD* caused by constant components and higher harmonics of current and voltage have the opposite sign in relation to the power caused by main harmonic. As a result, the sum of power harmonic components of main and higher harmonics gives zero, i.e. $P_{11} + \sum_{h=0,2,4,6,8,10} P_{hh} = 0$

Analysis of tables 2 and 3 lets us to sum up. With the energy position all elements of the circuit are provided with electricity power by source (fig. 4). But in this case the diode acts as a power converter and it is the reason for distribution of high harmonics power for all circuit elements except the source. So, electricity metering by indicators of power caused by actions of equal-frequency harmonic currents and voltages leads to an undercount of the power which used by the semiconductor converter for generation in the grid.

Table 2.

	P_0, *W*	P_1, *VA*	P_2, *VA*	P_3, *VA*	P_4, *VA*	P_5, *VA*	P_6, *VA*	P_7, *VA*	P_8, *VA*	P_9, *VA*	P_{10}, *VA*	P_{11}, *VA*	P_{12}, *VA*
Source	4840	8217	4840	1643	0	-234,7	0	78,25	0	-35,57	0	62,247	0
R_s	968	1643,5	968,5	328,65	-0,3775	-82,165	35,475	35,215	-18,64	-7,115	4,56	12,45	7,585
R_{ld}	3872	6573	3873	1315	-1,51	-328,67	141,9	140,9	-64,55	-28,46	18,24	49,8	30,34
VD	0,49	0	-1,3	0	1,89	0	-177,38	0	80,69	0	-22,8	0	-37,9

Table 3.

	P_{00}, W	P_{11}, VA	P_{22}, VA	P_{44}, VA	P_{66}, VA	P_{88}, VA	P_{1010}, VA	P_{sum}, VA
Source	0	4840	0	0	0	0	0	4840
R_s	392,25	484	87,175	3,4875	0,64	0,19775	0,08	967,8303
R_{ld}	1569	1936	348,7	13,95	2,56	0,791	0,32	3871,321
VD	-1962	2420	-435,9	-17,44	-3,2	-0,99	-0,4	0,07

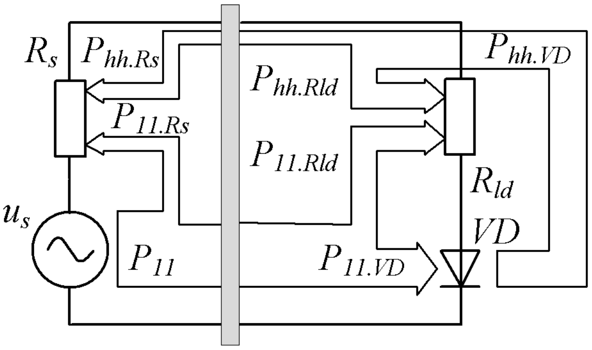

Fig. 4.

7. Conclusions.

The existing approach for calculating the apparent power for polyharmonic currents or voltages loses its informativeness and does not account the completeness of the electricity distributing process.

In the considered case (from the standpoint converting of electricity power), the diode with zero instantaneous power and its corresponding average value generates higher harmonic components that are distributed in the circuit and that are carriers of distorted energy flow. The procedure for determining a power of diode as product of voltage and current does not give understanding of its participation in the conversion of electric energy because an ideal diode instantaneous power is zero.

Description of the diode power by multiplying voltage and current component in the form of field harmonic signals allows to determine the components of the power and detail analyze the processes of the distribution of energy in a given scheme.

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