Design an Optimal Fuzzy Controller Base on Genetic Algorithm for Chaotic Rikitake System

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Abstract- *In this paper we want to investigate and control the chaos Rikitake system. For this end we should design an optimal controller for chaos Rikitake system. For this end we will divide our work to three steps and will monitor behavior of system for more clarify effect of our approach. At the first step we will briefly introduce and investigate the formulation and behavior of the chaos Rikitake system, after that at the second step we want to control the system by use of fuzzy controller, Takagi–Sugeno(T-S) method that it use for chaos systems, by this method will design a fuzzy controller and apply this controller on chaos Rikitake system, at the third step we will optimize the fuzzy controller, by use of GA(Genetic Algorithm) and new controller will apply on Rikitake system. At the all steps numerical simulations are given to illustrate the effectiveness and validity of the proposed approach.*

Key words - *Chaotic systems, Rikitake system, Takagi–Sugeno model, fuzzy system, GA (Genetic Algorithm).*

1. INTRODUCTION

Many theories have been advanced to explain the origin of the earth's main dipole field, the Rikitake system introduced for describing the irregular polarity switching of the Earth’s magnetic field. But intervals among such geomagnetic polarity reversals are highly irregular. Thus while their average is about 3×10^5 years, there are intervals as long as 3×10^7 years without polarity change. From introduce Rikitake system, till now many control approaches has been presented. At the bellow we will review this works together:

Pecorra and Carroll [1], Ott.E, et.al.[2], Carlos Aguilar-Ibañez, et.al. [3], and Mohammad Ali Khan [4] Synchronization for chaotic system has been investigated. In the last years, some methods to achieve synchronization have been proposed from the control theory perspective such as the famous observer-based approach [5], [6], and the so-called adaptive synchronization method [7]. Two research directions have been already conformed in synchronizing chaos: (i) analysis and (ii) synthesis. Analysis problem comprises: (a) the classification of synchronization phenomena [8], (b) the construction of a general framework for unifying chaotic synchronization [9], and (c) the comprehension of the synchronization properties, for instance, robustness [10] or geometry [11]. Liu Xiao-Jun, et.al. [12] analyzed the dynamics of Rikitake two-disk dynamo to explain the reversals of the Earth’s magnetic field. They concluded that the chaotic behavior of the system can be used to simulate the reversals of the geomagnetic field. The Rikitake chaotic attractor was studied by several authors. T. McMillen [13] and Mohammad Javidi et.al.[14] has studied the shape and dynamics of the Rikitake attractor. J. Llibre .et.al [15] used the Poincare compactification to study the dynamics of the Rikitake system at infinity. Chien-Chih Chen et.al [16] have studied the stochastic resonance in the periodically forced Rikitake dynamo. In the past decade, many researchers start working on controlling the chaotic behaviors. Harb and Harb [17] have designed a nonlinear controller to control the chaotic behavior in the phase-locked loop by means of nonlinear control. Ahmad Harb[18] have designed a controller to control the unstable chaotic oscillations by means of back stepping method. U.E. Vincent, R. Guo [19], Park et.al[20] and Jeong et.al[21] They have presented a controller by use of adaptive method and controlled chaotic Rikitake system.

In this paper we want to control chaotic Rikitake system by use of Takagi–Sugeno (T-S) fuzzy model [22] has attracted a great deal of attention. The main purpose of the T-S fuzzy model is to represent or approximate a complex nonlinear system. The T-S fuzzy model approach will provide a powerful method for analysis of nonlinear systems [23, 24]. After that we will optimize the fuzzy designed controller by use of GA(Genetic Algorithm) technique[25, 26, 27], and monitor operation of final controller on chaos Rikitake system.

2. THREE STEPS TO CONTROL AND OPTIMIZE THE CHAOS RIKITAKE SYSTEM

1) The formulation and behavior of the chaos Rikitake system

The Rikitake system consists of two conducting rotating disks (see Fig 1). These disks are connected into two coils. The current in each coil feeds the magnetic field of the other. The self inductance (L) and resistance (R) are the same in each circuit. An external constant mechanical torque (G) for each circuit is applied on the axis to rotate with an angular velocity [28].

The original differential equations derived by Rikitake are:

$$\begin{cases} L_1 \dfrac{dI_1}{dt} + R_1 I_1 = \omega_1 M I_2 \\ L_2 \dfrac{dI_2}{dt} + R_2 I_2 = \omega_2 N I_1 \\ C_1 \dfrac{d\omega_1}{dt} = G_1 - M I_1 I_2 \\ C_2 \dfrac{d\omega_2}{dt} = G_2 - N I_1 I_2 \end{cases} \quad (1)$$

where L, R are the self-inductance and resistance of the coil, the electric currents, I, ω, C, G are the electric currents, the angular velocity , momentum of inertia, and the driving force; M, N are the mutual inductance between the coils and the disks.

Now we consider a further simplification by
L1=L2, R1=R2, M =N, C1=C2, G1=G2
and set:

$$I_1 = \sqrt{\frac{G}{M}}x, \quad I_2 = \sqrt{\frac{G}{M}}y, \quad \omega_1 = \sqrt{\frac{GL}{CM}}z \quad (2)$$

$$\omega_2 = \sqrt{\frac{GM}{CM}}(z-a), \quad t = \sqrt{\frac{CL}{GM}}t', \quad u = R\sqrt{\frac{C}{LGM}}$$

(3)

Where, constant parameter a,u>0.

The system mathematical model can be written as follows[29]:

$$\begin{cases} x_1' = -u x_1 + x_2 x_3 \\ x_2' = -u x_2 + (x_3 - a) x_1 \\ x_3' = 1 - x_1 x_2 \end{cases} \quad (4)$$

Where (x1, x2, x3) ∈ R3 are the state variables and a>0, u > 0 are parameters. Note that system (4) is a quadratic system in R3. The choice of the parameters a > 0 and u > 0 reflects a physical meaning in the Rikitake model. For study physical meaning can see [30]. Here we suppose a=5 & u=2, we know according to ref. [28] this system in some values is unstable and we choose this system in chaotic mode.

Note that x and y are corresponding to the electric currents, while z is corresponding to the angular velocity. For more details read ref. [29, 30]. At the bellow Fig. 1 we see the shape of Rikitake and in Fig. 2 and Fig. 3 behavior of system that it is chaotic behavior.

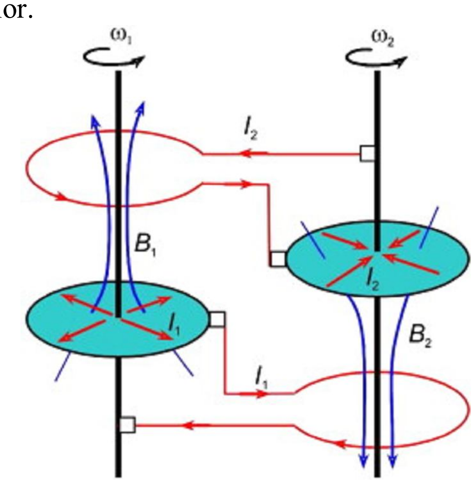

Fig.1 The Rikitake dynamo is composed of two disk dynamos coupled to another

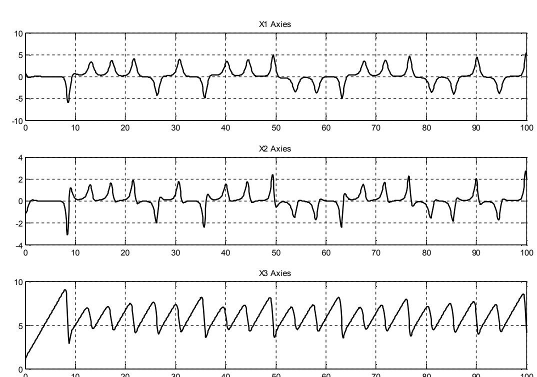

Fig.2. Behavior of Rikitake system without controller

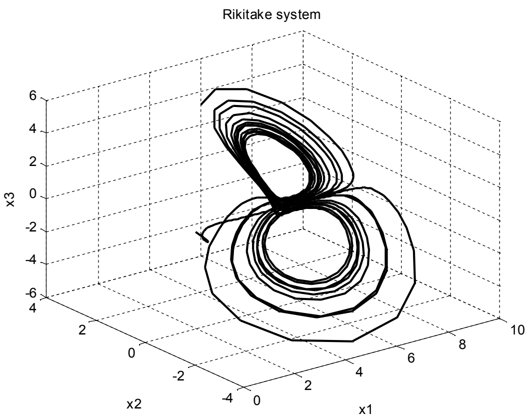

Fig.3. Behavior of Rikitake system in 3D plot

2) Modeling and control by use of fuzzy rules

For more convenience, the equations of Rikitake system can be written in the state-space matrix form as:

$$\dot{x}(t) = Ax(t) + B \qquad (5)$$

From the system mathematical model, calculate matrix A & B and will have:

$$A = \begin{bmatrix} -u & x_3(t) & 0 \\ x_3(t) - a & -u & 0 \\ 0 & -x_1(t) & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

For the nonlinear terms, define z1(t) = x3(t) and z2(t) = x1(t), then we can write:

$$A = \begin{bmatrix} -u & z_1(t) & 0 \\ z_1(t) - a & -u & 0 \\ 0 & -z_2(t) & 0 \end{bmatrix}$$

Now we want to calculate the values of Z1(t), Z2(t) when X1 ∈ [-1 1] and X3 ∈ [-1 1] then we will get following values:

$$\underset{x_1, x_3}{Z_1} = \begin{cases} 1 & \max \\ -1 & \min \end{cases}$$

$$\underset{x_1, x_3}{Z_2} = \begin{cases} 1 & \max \\ -1 & \min \end{cases}$$

From min and max values of Z1(t), Z2(t) we can write:

$$z_1(t) = x_3(t) = M_{11}(z_1(t)) \cdot 1 + M_{12}(z_1(t)) \cdot (-1)$$

$$z_2(t) = x_1(t) = M_{21}(z_2(t)) \cdot 1 + M_{22}(z_2(t)) \cdot (-1)$$

Where:

$$M_{11}(z_1(t)) + M_{12}(z_1(t)) = 1$$

$$M_{21}(z_2(t)) + M_{22}(z_2(t)) = 1$$

Therefore, we can calculate the membership function as bellow, the membership function has been drawn in fig.4:

$$M_{11}(z_1(t)) = \frac{1}{2}(1 + x_3(t)) \quad M_{12}(z_1(t)) = \frac{1}{2}(1 - x_3(t))$$

$$M_{21}(z_2(t)) = \frac{1}{2}(1 + x_1(t)) \quad M_{22}(z_2(t)) = \frac{1}{2}(1 - x_1(t))$$

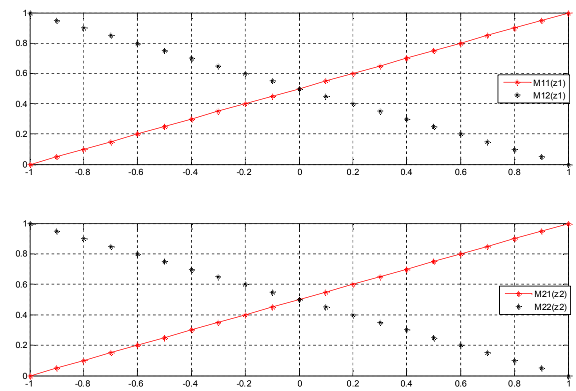

Fig.4. Membership functions M11, M12, M21 and M22

From M11, M12, M21 and M22 we want to write rules of T-S fuzzy models:

Rule 1: If Z1(t) is M11 and Z2(t) is M21 then:

$$\dot{x}(t) = A_1 x(t) + B + Cu(t)$$

Rule 2: If Z1(t) is M11 and Z2(t) is M22 then:

$$\dot{x}(t) = A_2 x(t) + B + Cu(t)$$

Rule 3: If Z1(t) is M12 and Z2(t) is M21 then:

$$\dot{x}(t) = A_3 x(t) + B + Cu(t)$$

Rule 4: If Z1(t) is M12 and Z2(t) is M22 then:

$$\dot{x}(t) = A_4 x(t) + B + Cu(t)$$

From these rules will obtain:

$$A_1 = \begin{bmatrix} -u & 1 & 0 \\ 1-a & -u & 0 \\ 0 & -1 & 0 \end{bmatrix} \qquad A_2 = \begin{bmatrix} -u & 1 & 0 \\ 1-a & -u & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -u & -1 & 0 \\ -1-a & -u & 0 \\ 0 & -1 & 0 \end{bmatrix} \qquad A_4 = \begin{bmatrix} -u & -1 & 0 \\ -1-a & -u & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

We suppose that:

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D = [0]$$

By use of defuzzifier method we can reformulate the Rikitake system:

$$\dot{x}(t) = \sum_{i=1}^{4} h_i(z(t))\{A_i x(t) + B\} \qquad (6)$$

And

$$h_1(z(t)) = M_{11}(z_1(t)) \times M_{21}(z_2(t))$$
$$h_2(z(t)) = M_{11}(z_1(t)) \times M_{22}(z_2(t))$$
$$h_3(z(t)) = M_{12}(z_1(t)) \times M_{21}(z_2(t))$$
$$h_4(z(t)) = M_{12}(z_1(t)) \times M_{22}(z_2(t))$$

We can simplify and write the controlled system bellow:

$$\dot{y}(t) = \sum_{i=1}^{4} h_i(z(t))\{A_i y(t) + B \ + C\ u(t)\} + D$$

(7)

Where

$e(t) = [y_1 - x_1 \quad y_2 - x_2 \quad y_3 - x_3]^{\mathrm{T}} \in R^3$ and C is the constant matrix.

We have:

$u(t) = [u_1 \quad u_2 \quad u_3]^{\mathrm{T}}$ That it is the control input, and it's equals to $u = -k.x$. At the here –k=C and x=e , then $u = C \times e$.

For plotting the behavior of system and the initial conditions of the drive and response system are chosen to be $(x_1, x_2, x_3) = (1, -1, 1)$ and $(y_1, y_2, y_3) = (1, -1, 1)$ respectively.

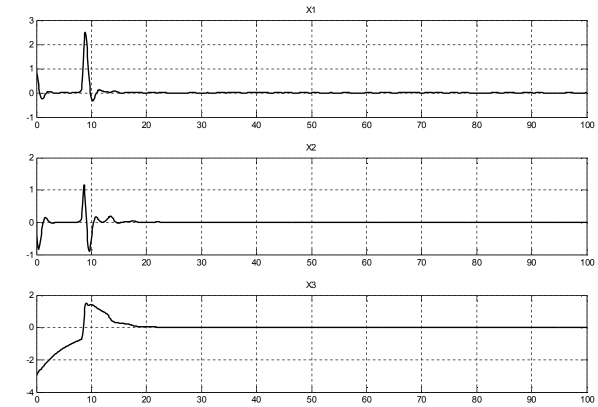

Fig.5. Behavior of Rikitake system after control

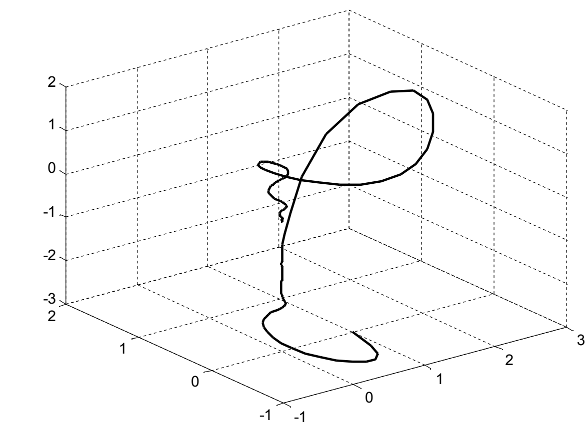

Fig.6. Behavior of Rikitake system after control in 3D plot

When applied controller on system we observed, the system be stable very fast and before twenty seconds we have some little fluctuations. We see the results in Fig. 5 and Fig.6.The block diagram of this controlled system has been presented at the below in Fig. 7.

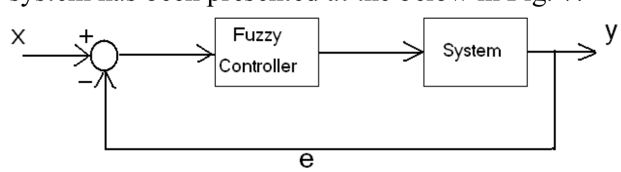

Fig. 7 Block diagram of fuzzy controller on Rikitake system

3) Optimization the fuzzy controller, by use of GA

Optimization is the process of making something better. An engineer or scientist conjures up a new idea and optimization improves on that idea. Optimization consists in trying variations on an initial concept and using the information gained to improve on the idea [27]. Genetic Algorithm (GA) is a search technique that mimics the mechanisms of natural selection.

Recently, GA has been recognized as an effective and powerful technique to solve optimization and search problems which represents an intelligent utilization of a random search within a defined search space to solve a problem. During GA computing, a population of artificial individuals is modified repeatedly based on biological evolution rules that converge towards better solution of the problem being solved. At each step individuals are selected in random from the current population to be parents. These individuals are used to produce children for the next generation. Based on biological basics, the fittest individuals survive and the least fit die. Through successful generations, the population evolves towards an optimal solution. Compared with other optimization techniques, particularly gradient search methods, GA is superior in avoiding local minima which is a common aspect of nonlinear systems. GA is considered as a part of evolutionary computing technique which is a rapidly emergent area of artificial intelligence. The Genetic Algorithm is a collection of functions that extend the capabilities of the Optimization Toolbox and the MATLAB numeric computing environment [25, 31].

After this prologue we want by use of numerical values that obtained in past section (fuzzy controller) optimize the controller and monitor the effect of optimized fuzzy controller on chaos Rikitake system.

For optimization of controller there are many option for optimize, but we choice member function.

At the section of fuzzy controller we suppose the length of member function [-1, 1] now we want to get the best point for start and end of member function, the general equation of this choice is:

$$f(t)=\alpha L+x_0 \qquad (8)$$

We suppose that coefficient of L is one, α=1 and the length of member function, L is the same length, L=2, we want to gain new x0 by use of GA and again tune fuzzy controller. For this goal at the first write cost function and use from gatool in MATLAB, the optimum point will gain and will write the new member function and repetition of stages of fuzzy.

We set the values of system for gatool like bellow[32]:

Population size: 100
Scaling function: Rank
Selection function: Elitist
Crossover fraction: 0.65
Mutation function: 0.01

And other parameters are default that the system set on it.

After optimization we gained x0=2 . With this new point draw new member function the new length of member function is [2 4], the new membership function has been drawn in fig.8:

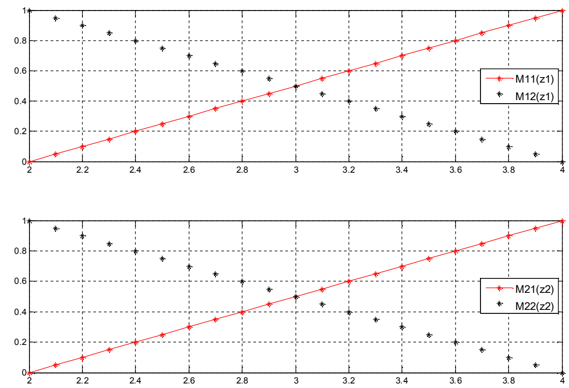

Fig. 8 New membership functions M11, M12, M21 and M22

Now we want to calculate the values of Z1(t), Z2(t) when X1 ∈ [2 4] and X3 ∈ [2 4] then we will get following values:

$$\underset{x_1,x_3}{z_1}=\begin{cases}4 & \max\\ 2 & \min\end{cases}$$

$$\underset{x_1,x_3}{z_2}=\begin{cases}4 & \max\\ 2 & \min\end{cases}$$

From min and max values of Z1(t), Z2(t) we can write:

$$z_1(t)=x_3(t)=M_{11}(z_1(t))\cdot(4)+M_{12}(z_1(t))\cdot(2)$$
$$z_2(t)=x_1(t)=M_{21}(z_2(t))\cdot(4)+M_{22}(z_2(t))\cdot(2)$$

Where:

$$M_{11}\left(z_1(t)\right)+M_{12}\left(z_1(t)\right)=4$$
$$M_{21}\left(z_2(t)\right)+M_{22}\left(z_2(t)\right)=4$$

Therefore, we can calculate the membership function as bellow:

$$M_{11}\left(z_1(t)\right)=\frac{1}{2}(x_3(t)-8)$$

$$M_{12}\left(z_1(t)\right)=\frac{1}{2}(16-x_3(t))$$

$$M_{21}\left(z_2(t)\right)=\frac{1}{2}(x_1(t)-4)$$

$$M_{22}\left(z_2(t)\right)=\frac{1}{2}(8-x_1(t))$$

From M11, M12, M21 and M22 we want to write rules of T-S fuzzy models:

Rule 1: If Z1(t) is M11 and Z2(t) is M21 then:

$$\dot{x}(t) = A_1 x(t) + B + Cu(t)$$

Rule 2: If Z1(t) is M11 and Z2(t) is M21 then:

$$\dot{x}(t) = A_2 x(t) + B + Cu(t)$$

Rule 3: If Z1(t) is M12 and Z2(t) is M21 then:

$$\dot{x}(t) = A_3 x(t) + B + Cu(t)$$

Rule 4: If Z1(t) is M12 and Z2(t) is M22 then:

$$\dot{x}(t) = A_4 x(t) + B + Cu(t)$$

From these rules will obtain:

$$A_1 = \begin{bmatrix} -u & -8 & 0 \\ -8-a & -u & 0 \\ 0 & 4 & 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} -u & -8 & 0 \\ -8-a & -u & 0 \\ 0 & \frac{-8}{3} & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -u & \frac{16}{3} & 0 \\ \frac{16}{3}-a & -u & 0 \\ 0 & 4 & 0 \end{bmatrix} \qquad A_4 = \begin{bmatrix} -u & \frac{16}{3} & 0 \\ \frac{16}{3}-a & -u & 0 \\ 0 & \frac{-8}{3} & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

By use of defuzzifier method we can reformulate the Rikitake system:

$$\dot{x}(t) = \sum_{i=1}^{4} h_i(z(t))\{A_i x(t) + B\} \tag{9}$$

And

$$h_1(z(t)) = M_{11}(z_1(t)) \times M_{21}(z_2(t))$$
$$h_2(z(t)) = M_{11}(z_1(t)) \times M_{22}(z_2(t))$$
$$h_3(z(t)) = M_{12}(z_1(t)) \times M_{21}(z_2(t))$$
$$h_4(z(t)) = M_{12}(z_1(t)) \times M_{22}(z_2(t))$$

We can simplify and write the controlled system bellow:

$$\dot{y}(t) = \sum_{i=1}^{4} h_i(z(t))\{A_i y(t) + B + C\, u(t)\} + D$$

(10)

Where

$e(t) = [y_1 - x_1 \quad y_2 - x_2 \quad y_3 - x_3]^{\mathrm{T}} \in R^3$

The initial conditions of the drive and response system are chosen to be $(x_1, x_2, x_3) = (1, -1, 1)$ and $(y_1, y_2, y_3) = (1, -1, 1)$ respectively.

At the below illustrate our results, Fig. 9 and Fig. 10:

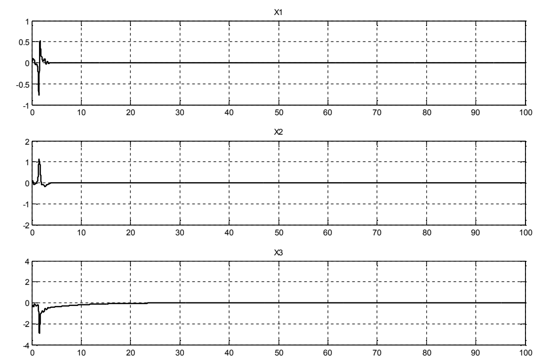

Fig.9. Behavior of Rikitake system after optimization

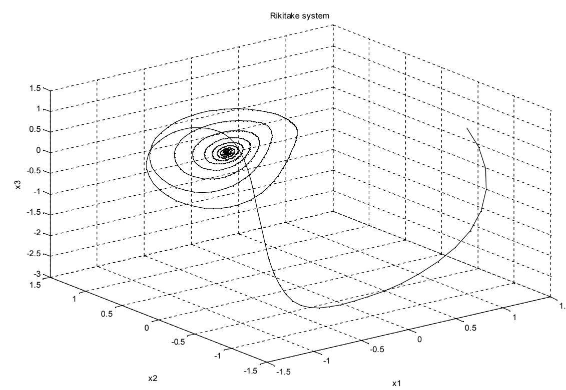

Fig.10. Behavior of Rikitake system after optimization in 3D plot

When applied optimized controller on system we observed, the system after a few seconds became stable also the fluctuations were small. This controller is more effective and powerful than pure fuzzy controller. The block diagram of optimized fuzzy controller has been drawn in Fig. 11.

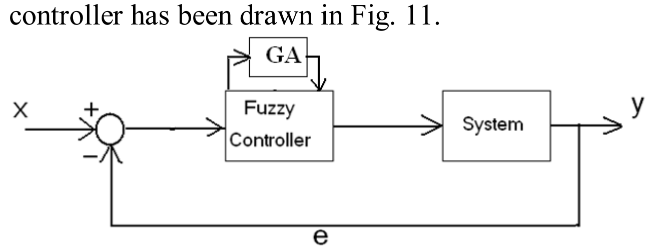

Fig. 11 Block diagram of optimized fuzzy controller

3. CONCLUSION

In this paper we investigated chaotic Rikitake system and monitored behavior of system. We suggested and designed a fuzzy controller (with Takagi–Sugeno techniques), and applied this fuzzy controller on chaotic system and monitored behavior of system. We observed the speed and effect of fuzzy controller on this system is good but in some cases it is not acceptable, the over shoot and the time should be optimized, thus optimized fuzzy controller, by use of

GA(Genetic Algorithm) and again applied optimized controller on chaos system and observed after a few secods the system became stabled. In this paper we calculated, designed and optimized fuzzy controller by use of GA. After that we illustrated effectiveness and validity of our proposed approach.

Acknowledgment

We acknowledge our friend, Mehdi fatemi, and the Associate Editor and anonymous reviewers for their valuable comments and suggestions that have helped us to improving the paper.

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