INVESTIGATION OF TRAJECTORY TRACKING FRACTIONAL ORDER CONTROLLER FOR ROBOT MANIPULATOR EMPLOYING HIL SIMULATION TECHNIQUE

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Abstract: *A systematic solution for precise trajectory tracking of a three link serial rigid robotic manipulator is presented in this manuscript. A Proportional Integral Derivative (PID) controller based on fractional order fuzzy technique is implemented using Hardware in the Loop (HIL) simulation approach to solve the trajectory tracking problem. Robustness testing of the designed controller for model uncertainties and disturbance rejection is also investigated. To study the performance of fractional order fuzzy based controller, it is compared with different PID variants such as fuzzy, fractional order PID and conventional PID controllers. The performance index of the three link robotic manipulator is observed by computing Integral Absolute Error (IAE) for various percentages of masses and disturbances. The simulink model of the robotic manipulator has been developed using Matlab-simulink environment and its performance analysis are carried out based on HIL via C2000 board to perform the desired task. The HIL simulation for trajectory tracking results clearly indicates the dominance of the fractional order fuzzy controller over the other controllers for various model uncertainties and disturbance rejection.*

Key words: *Robot manipulator, Trajectory tracking, Fractional order controller, Fuzzy logic controller, Hardware-in-the-loop simulation.*

1. Introduction

In this fast growing world, robotic automation in industrial tasks requires very high precision and better stability of in their performance. The main use of autonomous robots is applied to perform hazardous tasks in Industrial Automation. Exciting area of robotics research domains are available with their potential paths that work together with us in our households and workplaces as useful and capable agents based on the long-term vision of robots [1]. Various fuzzy based adaptive controllers were proposed by various researchers for reducing the control effort with negligible chattering in control torque [2], for operating robot manipulators under unknown external disturbances and stochastic perturbations using adaptive neural design [3], parametric uncertainties of robot manipulator were compensated using adaptive control schemes [4]. There are also various other techniques being proposed for the control of robotic manipulator based on feedback linearization or computed torque control, variable structure compensator etc. Also, adaptive control methods are used to track the desired trajectories which are described by arbitrary coordinates that are task oriented [5]. Based on the inverse manipulator dynamics an adaptive jacobian tracking controller is implemented for robots with dynamic uncertainties [6]. To overcome the drawbacks in the velocity measurements, sliding observer design technique can be used to estimate the joint velocities [7]. The robotic manipulator can also be operated in unstructured environment to achieve any reachable position and orientation [8]. Trajectory tracking of end effector and its control is an essential task in a robot manipulator. Although numerous controllers for cooperative manipulators have been proposed in the past few years [9,10], stable execution of tasks with the strange environment using robots is still notorious as one of the main challenges. In the control of robotic manipulators, disturbance and system uncertainties tend to affect the performance of the system. They should be handled properly with effective control algorithms [11]. To further enhance the tracking performance of robotic manipulators in the presence of parametric uncertainties, significant efforts have been made to seek advanced control approaches. Strong and versatile control methods of robot manipulators have been the most active research issues for past few years. Robust control laws can be used for managing external disturbances, formless dynamics, and different sources of uncertainties [12].

The development of fuzzy logic controllers for its application in the field of control engineering is mainly due to their nonlinear linguistic mapping between inputs and outputs. The potential utility of conventional PID controller has improved to a good extent due to its combination with fuzzy logic controllers. Such hybrid fuzzy based PID controllers are implemented to actuate the joints of the robotic manipulators [13,14]. Adaptive variants of the fuzzy PID controllers were also put forward for industrial robotics applications [15,16]. The combination adaptive fuzzy and PID based control also found applications for steering system of armored vehicle [17], in the medical field for employing position control in laparoscopic surgery assisted by robots [18]. Adaptive fuzzy system was implemented as a neuro fuzzy system with complex fuzzy rules for a time domain forecasting used in machine learning problem [19]. By starting with a smaller number of input membership functions in the controller inputs, the fuzzy sets results in a better reference signal tracking in the adaptation process of a nonlinear system [20]. A stable state feedback fuzzy controller was designed based on neuro fuzzy state space model, providing excellent steady state stable performance for a flexible link manipulator [21]. By using iterative feedback tuning for a fuzzy controller combined with a proportional integral controller, an inequality type of convergence condition was derived as a nonlinear dynamical feedback system [22]. They provide a flexible and model-free approach to control engineers for designing an optimal controller for the systems with uncertainties and nonlinearities. Most robot control systems using personal computers are discrete-time systems in nature. On the other hand, the highly coupled nonlinear multiple-input and multiple-output dynamics of robot manipulators limit most control designs and stability analyses to continuous time systems.

By utilizing a suitable HIL technique, in the system design stages of robotic manipulator helps us to increase the system efficiency and reliability as well as the final value of the designed manipulator. Also, it provides a means for proper exploration in the modelling and simulation stages, thereby helps to avoid serious errors in the design of software and hardware as well as their physical interfacing [23]. To test the validity of the algorithm the HIL simulation approach is employed. It includes the actuators of the robotic manipulator as a part of the simulator system and not represented by real hardware. This kind of HIL based simulation modelling aids, to fast prototyping of control algorithms, rather than researchers investing on the actual robotic manipulator [24, 25]. It is used to derive the control law to control the actuators in the joints of the robotic manipulator. In this present work, the control algorithms are implemented in C2000 real time controller.

2. Dynamic modelling of robotic manipulator

The mathematical model of three link planar serial robot manipulator is used to express the behaviour of robot manipulators. The design of the model based controller has been derived from dynamic equations of robotic manipulator [26] for carrying out the HIL simulation to estimate the tracking joint angle errors and position of end effector is shown in Figure 1.

The dynamic modelling of the robotic manipulator describes the relationship between joint motion, accelerations and torque [7].

Moreover, it is used to describe the particular dynamic effects such as inertia, Coriolis, centrifugal, and the other parameters of the manipulator.

The dynamic mathematical model for a 'n' link, serially connected, direct driven robotic manipulator is given in joint space as follows:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau \qquad (1)$$

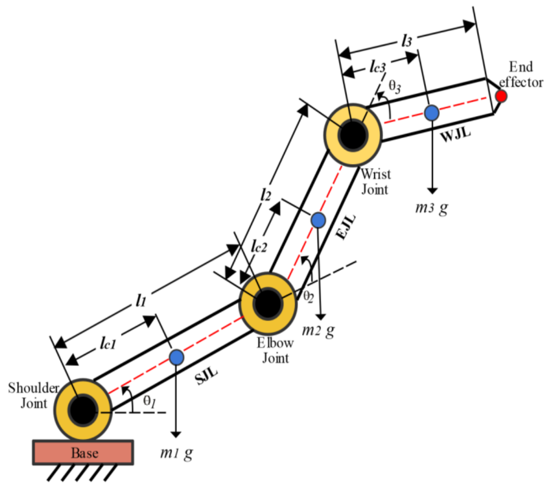


Fig.1. Model of the Robotic Manipulator

where, $M(\theta)$is the n×n inertia matrix of the manipulator, θ is the n×1 joint position vector , $C(\theta,\dot{\theta})$ is the n×1 vector of Centrifugal and Coriolis terms, $G(\theta)$is the n×1 vector of gravity terms, τ is the n×1 vector of joint actuator torques and forces. The manipulator chosen is modelled with three rigid links, the first one connecting the shoulder and elbow joint represented as Shoulder Joint Link (SJL), second link connecting elbow and wrist joint represented as Elbow Joint Link (EJL), and the last link connecting wrist joint and the gripper which is represented as Wrist Joint Link (WJL). The torque vector for those links can be expressed as:

$$\tau = [\tau_1 \tau_2 \tau_3]^T \qquad (2)$$

whereτ_1, τ_2 and τ_3 are torques applied to the shoulder joint, elbow joint and wrist joint actuators respectively.

Table 1: Parameters of the three link planar rigid robotic manipulator

Parameters	**Shoulder Joint Link(SJL)**	**Elbow Joint Link(EJL)**	**Wrist Joint Link(WJL)**
Mass	m_1=0.15kg	m_2=0.1kg	m_3 =0.28kg
Length	l_1=0.09m	l_2=0.063m	l_3=0.115m
Acceleration due to gravity (g)	9.81 m/s^2	9.81 m/s^2	9.81 m/s^2

The equation (1) and equation (2) show the controller output torque and the link positions $\theta_1, \theta_2, \theta_3$ of three links. The Table 1 lists the relevant parameters used in the model of robotic manipulator, where $m_1, m_2$and m_3 are the mass of SJL, EJL and WJL respectively, $l_1, l_2$and l_3 are the length of those each links.

3. Design of PID based fractional order fuzzy controller using HIL

This part presents the comprehensive description about the design and realization of PID based fractional order fuzzy controller using HIL. A brief illustration about the design steps carried out in PID based fuzzy, fractional order and the conventional PID controller is also discussed.

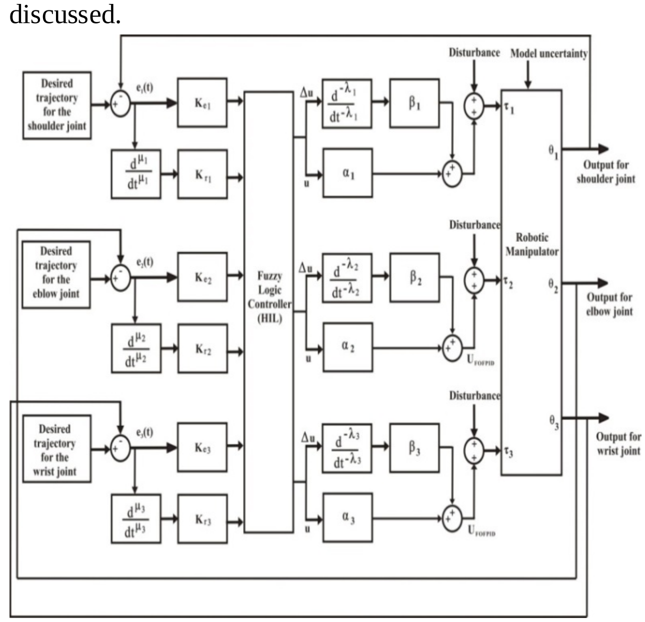


Fig.2.Block diagram of PID based fractional order fuzzy controller using HIL for a three link robotic manipulator

3.1 Design of fractional order fuzzy PID controller (FOFPID)

With the aid of three inputs, namely, error, integration of error and rate of change of error are required to be fuzzified in addition to a single output to implement the fuzzy PID controller. The PID controller output expression in the time domain can be described as

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \quad (5)$$

where, K_p, K_i and K_d are the proportional, integral and derivative gains respectively. The error $e(t)$ is the variation between the desired and actual trajectory followed by the links. From equation (3), to implement the HIL based fuzzy PID controller, bigger rule base is required for the three inputs, with significant number of Membership functions. With the assistance of just two inputs, the error and the rate of change of error fuzzy PID controllers can be implemented simply as fuzzy PI plus fuzzy PD controllers. The acquired single output is aggregated single action of both the controllers. The fuzzy PD and fuzzy PI controllers are in position and velocity form respectively.

The position form of fuzzy PD, in discrete notation can be expressed as,

$$u(k) = e(k)K_p + r(k)K_d \quad (4)$$

The velocity form of fuzzy PI, in discrete notation can be expressed as,

$$\Delta u(k) = r(k)K_p + e(k)K_i \quad (5)$$

where, $r(k)$ represents rate of change of error.

The control action of aggregate PID can be expressed as,

$$U_{PID}(k) = \frac{T}{1-z^{-1}} \Delta U_{PI}(k) K_p + U_{PD}(k) \quad (6)$$

This can be achieved with a single rule base, which can reduce the complexity and improve the efficiency of the system. With additional gain parameters α and β are introduced in order to increase the degrees of freedom, the input and the output relation can be represented as.

$$\frac{T}{1-z^{-1}} \beta \Delta u(k) + \alpha u(k) = e(k)K_e + \frac{1-z^{-1}}{T} e(k)K_r \quad (7)$$

Furthermore, the fractional order fuzzy based PID controller is implemented as,

$$U_{FOFPID}(k) = \left[\frac{T}{1-z^{-1}}\right]^{\lambda} \beta \Delta u(k) + \alpha u(k) = e(k)K_e + \left[\frac{1-z^{-1}}{T}\right]^{\mu} e(k)K_r \quad (8)$$

To enhance the flexibility of the controller design, two additional tuning parameters λ and μ were introduced represented as order of integrator and differentiator respectively in the equation (8). The basic block diagram of fractional order fuzzy based PID controllers implemented for each link of the three link planar robotic manipulator is shown in Fig. 2 where K_{e1}, K_{r1}, a_1, b_1, K_{e2}, K_{r2}, a_2, b_2, K_{e3}, K_{r3}, a_3 and b_3 are the scaling factors and k_1, l_1, k_2, l_2, k_3 and l_3 are the fractional order parameters.

3.2 Design of fuzzy PID controller (FPID)

By adjusting the values of fractional order integrator and derivative operators in the above illustrated design of fractional order fuzzy PID to 1, the design of fuzzy PID can be obtained. Active parameters are the four scaling factors of a fuzzy PID controller.

3.3 Design of fractional order PID controller (FOPID)

The design illustration of the fractional order PID controller is shown in Fig. 3. The fractional order parameters γ and δ along with controller gains K supports the implementation of the design. In the time domain PID controller along with its fractional order operators λ and μ can be expressed as,

$$U_{FOPID}(t) = e(t)K_p + \frac{d^{-\lambda}e(t)}{dt^{-\lambda}} K_i + \frac{d^{\mu}e(t)}{dt^{\mu}} K_d \quad (9)$$

The above equation (9) can be expressed in s-domain as,

$$U_{FOPID}(s) = \left(K_p + \frac{1}{s^{\lambda}} K_i + s^{\mu} K_d\right) E(s) \quad (10)$$

3.4 Design of PID controller

The PID controller is a straightforward version of the fractional order PID controller. By fixing the values of fractional order integrator and derivative operator to 1 in the fractional order PID as shown in Fig. 3, the conventional PID controller can be designed, thus leaving only three parameters to be tuned.

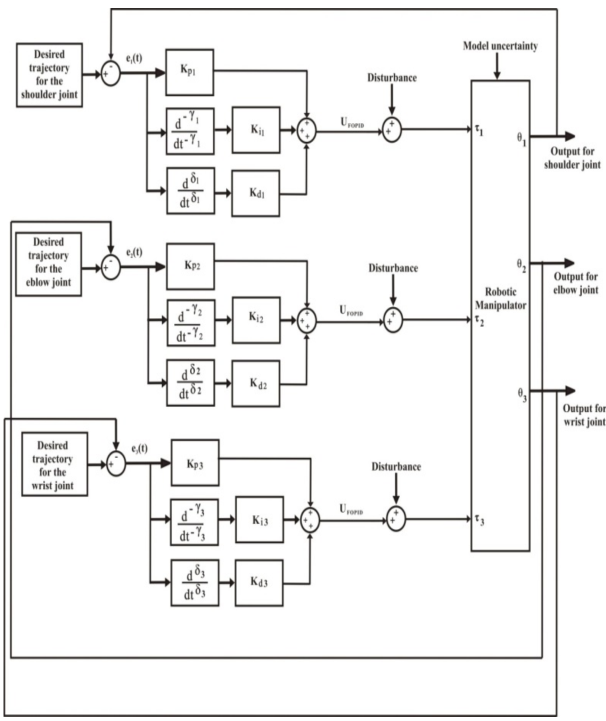


Fig.3. Block diagram of PID based fractional order controller using HIL for a three link robotic manipulator

4. Fractional order Fuzzy Logic Controller

Pre-processing is performed in the implementation of the core part of the fractional order fuzzy logic control as show in Fig 4. First stage of the pre-processing is to scale the input variables and output variables with the aid of the scaling factors that are normalized by suitable multiplication and mapped to the range between [-1,1].The second stage involves the fuzzification phase in which the crisp values of the normalized variables are mapped to the linguistic variables or fuzzy values. Due to the smooth nature of Gaussian membership function, its nonlinear characteristics and non-zero at all points, they are used in this preprocessing stage. The third stage is the rule base which acts as core part in the design of fractional order fuzzy logic control with totally 49 rules mapped with errors and fractional order errors in the rows and columns of the matrix respectively. The fourth stage holds the inference engine that acts as a central processing unit and provides the overall control action. In the proposed work, Mamdani min–max inference is chosen as inference method that acts on the fuzzy data and makes it ready for defuzzification. The fifth and final stage is the defuzzification phase that takes responsibility of converting the linguistic values of the fractional order fuzzy logic controller into a crisp value. Finally, using the center of gravity method defuzzification is done to convert the linguistic values of a fuzzy logic controller output into a crisp value.

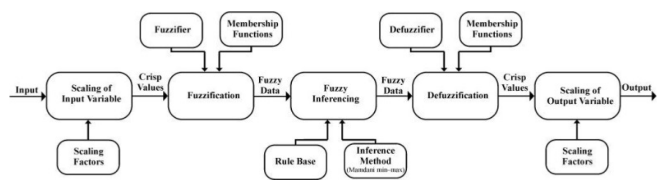


Fig.4. Block diagram of implementation of the core part of fractional order fuzzy logic controller

In this study, the input and output variables are represented with seven membership functions namely NL (Negative Large), NM (Negative Medium), NS (Negative Small), ZR (Zero), PS (Positive Small), PM (Positive Medium) and PL (Positive Large). The 50% overlapping between adjacent fuzzy sets of both inputs and output variable is also used. The membership functions of inputs as well as the output are shown in Fig. 5.

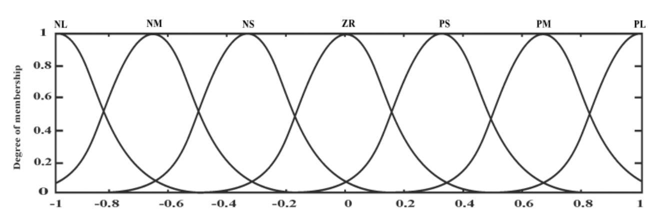


Fig.5. Membership functions of inputs and outputs

The rule base part of a fuzzy logic controller design is based on the process dynamics, expert's knowledge and experience. The rule base listed in Table 2 looks like a two dimensional matrix of the linguistic values of the control action to be taken which is formulated between the seven linguistic values of error and fractional order change of error as the row and column respectively.

Table 2. Rule Base between the 7 linguistic values of error

Fractional Rate of Change of Error		Error						
		NL	**NM**	**NS**	**ZR**	**PS**	**PM**	**PL**
	NL	NL	NL	NL	NL	NM	NS	ZR
	NM	NL	NL	NL	NM	NS	ZR	PS
	NS	NL	NL	NM	NS	ZR	PS	PM
	ZR	NL	NM	NS	ZR	PS	PM	PL
	PS	N	NS	ZR	PS	PM	PL	PL
	PM	NS	ZR	PS	PM	PL	PL	PL
	PL	ZR	PS	PM	PL	PL	PL	PL

4.1 Fractional order operator implementation

The fractional calculus has been utilized in accumulation with the control theory nowadays by the researchers. The unique feature of fractional calculus is its ability of utilizing the real number order of integral as well as differential operators in lieu of fine-tuned integer order. The generalized differential-integrator definition is given by equation (11) is as follows

$$a_0 D_t^{\alpha_0} = \begin{cases} \frac{d^{\alpha_0}}{dt^{\alpha_0}} R(\alpha_0) > 0 \\ 1 R(\alpha_0) = 0 \\ \int_{a_0}^{t} (d\tau)^{-\alpha_0} R(\alpha_0) < 0 \end{cases} \quad (11)$$

where, a_0 is concerned with initial conditions and α_0 is the fractional order of operation. Since, the implementation part of the algorithm is carried out using the HIL simulation a particular choice of approximation technique is carried out. Here we use an approximation technique for the fractional order controller implementation because of its capability for real hardware implementation. Here the fractional order controller is preferred because it involves the recursive distribution of poles and zeroes. The transfer function for the chosen approximation is represented as shown in equation (12)

$$s^a = k_0 \prod_{k_0=-N}^{N} \frac{s+W_{k_z}}{s+W_{k_p}} \quad (12)$$

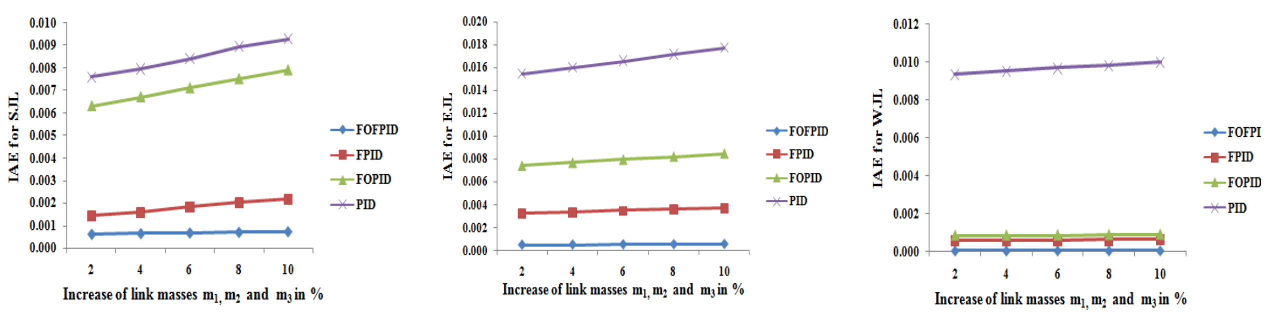
where, a is the real number, which is orders of fractional differentiator-integrator; $[W_h, W_b]$ is the expected fitting range; 2N + 1 represents the order of approximation. k_0 is the gain parameter, W_{k_p} and W_{k_z} are the poles and zeros of the filter, which are calculated recursively as follows:

$$W_{k_p} = W_b \left(\frac{W_h}{W_b}\right)^{\frac{k+N+\frac{a}{2}+\frac{1}{2}}{2N+1}} \quad (13)$$

$$W_{k_z} = W_b \left(\frac{W_h}{W_b}\right)^{\frac{k+N-\frac{a}{2}+\frac{1}{2}}{2N+1}} \quad (14)$$

$$k_0 = W_h^a \quad (15)$$

Smaller values of N produces ripple both in gain and phase behaviors. It can be eliminated by choosing higher value of N which leads to complexity. In the present study, the value of N is chosen as 3 and the frequency range as $[10^{-3}, 10^{2}]$.

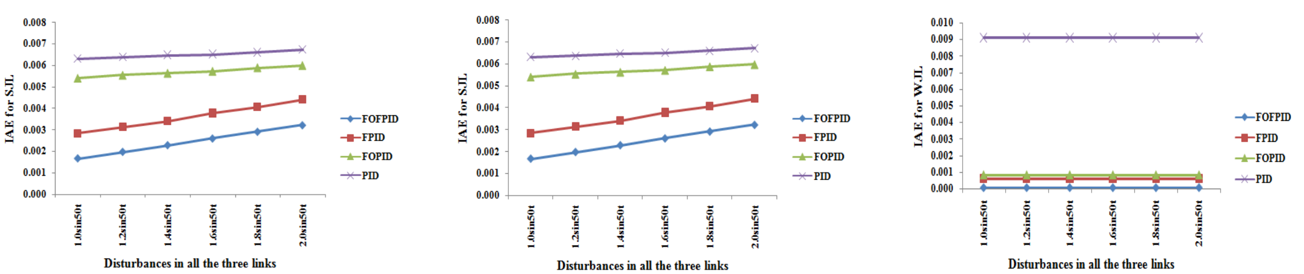


(a) (b) (c)

Fig. 6. Variation in IAE for increase in link mass m_1, m_2 and m_3 in % (a) for SJL (b) for EJL and (c) for WJL.

Table 3: IAE of PID based fractional order fuzzy, PID based fuzzy, PID based fractional order and PID controllers for increasing in masses of the links

Increase in link mass (%)		**IAE of FOFPID controller**			**IAE of FPID controller**			**IAE of FOPID controller**			**IAE of PID controller**		
mass	**%**	**SJL ($x10^{-4}$)**	**EJL ($x10^{-4}$)**	**WJL ($x10^{-5}$)**	**SJL ($x10^{-3}$)**	**EJL ($x10^{-3}$)**	**WJL ($x10^{-4}$)**	**SJL ($x10^{-3}$)**	**EJL ($x10^{-3}$)**	**WJL ($x10^{-4}$)**	**SJL ($x10^{-2}$)**	**EJL ($x10^{-2}$)**	**WJL ($x10^{-3}$)**
m_1	2	4.834	4.632	4.430	0.921	2.946	5.910	5.162	6.738	8.314	0.634	1.407	9.151
	4	4.832	4.631	4.430	0.934	2.946	5.950	5.099	6.738	8.377	0.641	1.407	9.220
	6	4.832	4.631	4.430	1.011	2.946	6.000	5.035	6.738	8.441	0.648	1.407	9.290
	8	4.832	4.631	4.430	1.023	2.946	6.050	4.971	6.738	8.505	0.655	1.407	9.359
	10	4.832	4.631	4.430	1.034	2.946	6.090	4.908	6.738	8.568	0.662	1.407	9.429
m_2	2	5.210	4.720	4.230	1.191	3.005	5.930	5.392	6.872	8.352	0.632	1.435	9.194
	4	5.360	4.820	4.280	1.193	3.064	6.001	5.561	7.007	8.453	0.638	1.463	9.306
	6	5.490	4.910	4.330	1.210	3.123	6.072	5.729	7.142	8.555	0.644	1.491	9.419
	8	5.620	5.000	4.380	1.223	3.182	6.143	5.898	7.277	8.656	0.649	1.519	9.531
	10	5.750	5.090	4.430	1.234	3.241	6.214	6.064	7.411	8.758	0.655	1.548	9.644
m_3	2	5.586	4.808	4.030	1.319	3.064	5.95	5.622	7.006	8.39	0.631	1.463	9.237
	4	5.888	5.009	4.130	1.397	3.182	6.052	6.023	7.276	8.529	0.635	1.519	9.392
	6	6.148	5.189	4.230	1.411	3.300	6.144	6.423	7.546	8.669	0.640	1.575	9.548
	8	6.408	5.369	4.330	1.422	3.418	6.236	6.825	7.816	8.807	0.644	1.631	9.703
	10	6.668	5.549	4.430	1.437	3.536	6.338	7.220	8.084	8.948	0.648	1.689	9.859
m_1, m_2	2	5.188	4.724	4.260	1.097	3.005	5.976	5.329	6.872	8.415	0.639	1.435	9.263
	4	5.292	4.816	4.340	1.114	3.064	6.093	5.433	7.007	8.581	0.652	1.463	9.445
	6	5.396	4.908	4.420	1.223	3.123	6.210	5.538	7.142	8.746	0.665	1.491	9.627
	8	5.500	5.000	4.500	1.251	3.182	6.327	5.643	7.277	8.911	0.677	1.519	9.809
	10	5.604	5.092	4.580	1.276	3.241	6.444	5.746	7.411	9.076	0.689	1.548	9.990
m_1, m_2 m_3	2	6.240	5.085	3.930	1.438	3.241	6.022	6.316	7.410	8.504	0.627	1.547	9.366
	4	6.542	5.286	4.030	1.589	3.359	6.124	6.717	7.680	8.643	0.631	1.603	9.521
	6	6.802	5.466	4.130	1.842	3.477	6.216	7.117	7.950	8.783	0.636	1.659	9.677
	8	7.062	5.646	4.230	2.014	3.595	6.308	7.519	8.220	8.921	0.640	1.715	9.832
	10	7.322	5.826	4.330	2.173	3.713	6.410	7.914	8.488	9.062	0.644	1.773	9.988



(a) (b) (c)

Fig. 7. Variation in IAE for (a) SJL with disturbance in all three links (b) EJL with disturbance in all three links and (c) WJL with disturbance in all three links.

Table 4: IAE of PID based fractional order fuzzy, PID based fuzzy, PID based fractional order and PID controllers due to disturbances

Disturbances (N-m)		IAE of FOFPID controller			IAE of FPID controller			IAE of FOPID controller			IAE of PID controller		
		SJL ($x10^{-4}$)	EJL ($x10^{-4}$)	WJL ($x10^{-5}$)	SJL ($x10^{-3}$)	EJL ($x10^{-3}$)	WJL ($x10^{-4}$)	SJL ($x10^{-3}$)	EJL ($x10^{-3}$)	WJL ($x10^{-4}$)	SJL ($x10^{-2}$)	EJL ($x10^{-2}$)	WJL ($x10^{-3}$)
SJL	1.0sin50t	14.680	9.430	4.180	2.001	4.002	5.860	5.401	6.846	8.251	0.560	1.470	9.082
	1.2sin50t	17.880	11.030	4.180	2.28	4.348	5.860	5.535	6.943	8.251	0.575	1.504	9.083
	1.4sin50t	21.160	12.670	4.180	2.551	4.725	5.860	5.620	7.086	8.252	0.599	1.548	9.083
	1.6sin50t	24.480	14.330	4.180	2.912	5.134	5.860	5.712	7.270	8.252	0.620	1.601	9.083
	1.8sin50t	27.860	16.020	4.180	3.19	5.578	5.860	5.875	7.497	8.252	0.641	1.667	9.084
	2.0sin50t	31.240	17.710	4.180	3.51	6.048	5.860	5.942	7.795	8.252	0.657	1.744	9.084
EJL	1.0sin50t	4.920	4.630	4.340	0.846	2.946	5.905	5.244	6.737	8.230	0.619	1.407	9.040
	1.2sin50t	4.830	4.630	4.430	0.858	2.946	5.919	5.248	6.737	8.226	0.619	1.407	9.032
	1.4sin50t	4.700	4.630	4.560	0.864	2.946	5.935	5.252	6.737	8.222	0.620	1.407	9.024
	1.6sin50t	4.540	4.630	4.720	0.876	2.946	5.952	5.256	6.737	8.218	0.621	1.407	9.016
	1.8sin50t	4.320	4.630	4.940	0.892	2.946	5.970	5.259	6.737	8.215	0.622	1.407	9.008
	2.0sin50t	4.060	4.630	5.200	0.896	2.946	5.990	5.262	6.737	8.212	0.623	1.407	9.001
WJL	1.0sin50t	5.080	7.030	4.260	0.732	3.474	5.883	5.223	6.792	8.241	0.627	1.439	9.061
	1.2sin50t	5.080	7.830	4.305	0.744	3.647	5.890	5.223	6.840	8.239	0.627	1.456	9.058
	1.4sin50t	5.080	8.650	4.370	0.749	3.836	5.898	5.223	6.912	8.237	0.627	1.478	9.054
	1.6sin50t	5.080	9.480	4.450	0.756	4.040	5.906	5.223	7.004	8.235	0.627	1.504	9.050
	1.8sin50t	5.080	10.325	4.560	0.767	4.262	5.915	5.223	7.117	8.234	0.627	1.537	9.046
	2.0sin50t	5.080	11.170	4.690	0.787	4.497	5.925	5.223	7.266	8.232	0.627	1.576	9.043
SJL & EJL	1.0sin50t	14.681	9.440	4.350	2.102	4.005	5.908	5.402	6.849	8.231	0.628	1.470	9.041
	1.2sin50t	17.881	11.040	4.450	2.381	4.352	5.923	5.537	6.947	8.227	0.629	1.504	9.033
	1.4sin50t	21.161	12.680	4.580	2.652	4.730	5.940	5.622	7.092	8.223	0.631	1.548	9.026
	1.6sin50t	24.481	14.340	4.760	3.022	5.140	5.958	5.713	7.277	8.222	0.634	1.601	9.018
	1.8sin50t	27.862	16.030	4.990	3.292	5.585	5.978	5.876	7.651	8.217	0.636	1.667	9.011
	2.0sin50t	31.242	17.730	5.270	3.616	6.057	5.998	5.944	7.809	8.215	0.642	1.744	9.003
SJL, EJL & WJL	1.0sin50t	16.610	12.470	4.762	2.834	5.501	5.987	5.412	6.851	8.272	0.631	1.621	9.124
	1.2sin50t	19.710	14.870	4.807	3.125	5.674	5.994	5.546	6.977	8.270	0.637	1.638	9.121
	1.4sin50t	22.860	17.330	4.872	3.401	5.862	6.002	5.634	7.194	8.268	0.647	1.660	9.117
	1.6sin50t	26.000	19.820	4.952	3.778	6.067	6.011	5.723	7.281	8.266	0.651	1.686	9.113
	1.8sin50t	29.150	22.355	5.062	4.059	6.289	6.020	5.886	7.758	8.265	0.659	1.719	9.109
	2.0sin50t	32.270	24.900	5.192	4.403	6.524	6.030	5.984	8.065	8.263	0.672	1.758	9.106

5. Experimental Results and Discussion

In this section, the results obtained for trajectory tracking, disturbance rejection and model uncertainties for the PID based fractional order fuzzy, fuzzy, fractional order and conventional PID controllers are discussed. The HIL simulation was done with Matlab Simulink and C2000 real time controller. A simulink model of robotic manipulator was developed using equation (1) and equation (2) and was solved using fourth order Runge–Kutta method. The sampling time was taken as 1 ms during the simulation. The torque constraints for all the three links were taken as [-20,20] N-m during the simulation. The third order approximation was used with N = 3 and range of frequency $[10^{-3},10^{2}]$ radian/s for the implementation of the fractional order operator.

The fitness function chosen includes IAE and Integral of Absolute Change in Controller Output (IACCO) of all the three links as shown in equation (16) and equation (17) respectively. It can be noticed that aggregate fitness function F is a weighted sum of IAE and IACCO of all the three links. The main motivation of employing these fitness functions is to minimize the effects of error between actual and desired trajectories and also, to minimize the effects of change in the controller signal for better trajectory tracking.

$$f_1 = \int|e_1(t)|dt + \int|e_2(t)|dt + \int|e_3(t)|dt \quad (16)$$

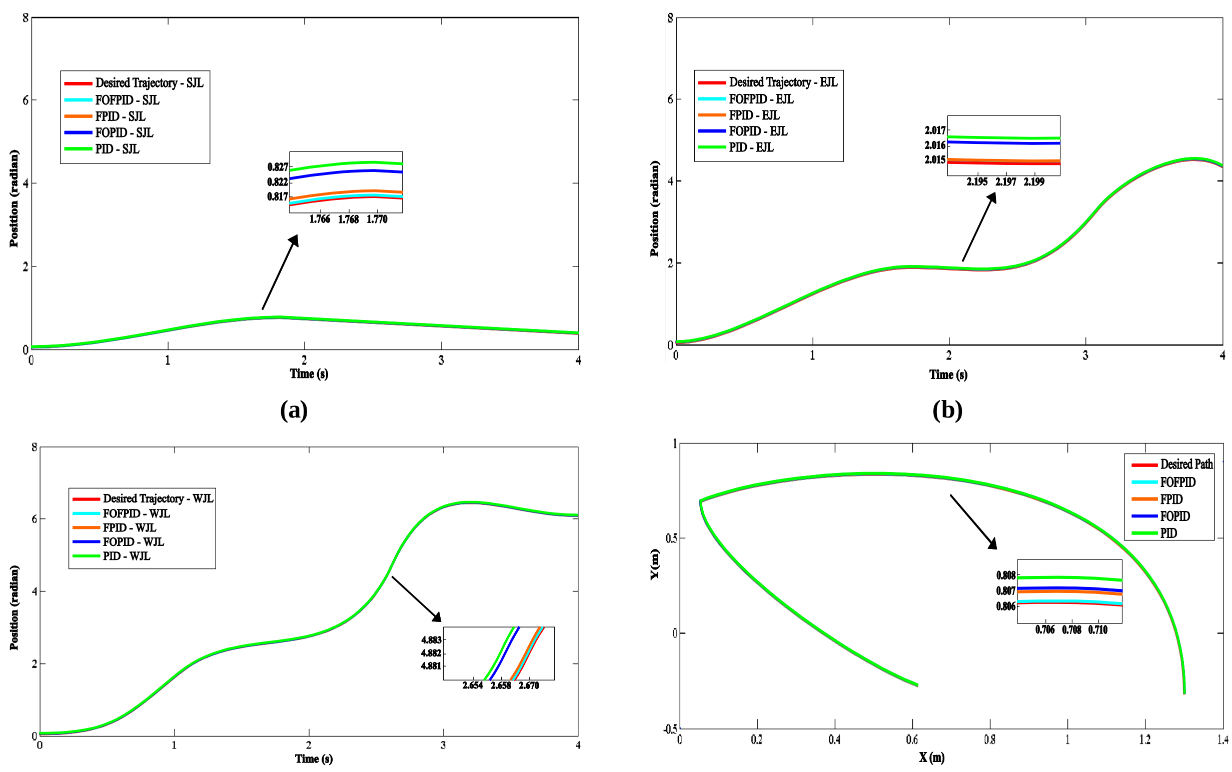
$$f_2 = \int|\Delta\tau_1(t)|dt + \int|\Delta\tau_2|dt + \int|\Delta\tau_3|dt \quad (17)$$

$$F = w_1 f_1 + w_2 f_2 \quad (18)$$

where $\Delta\tau_1, \Delta\tau_2$ and $\Delta\tau_3$ are changes in the control output for SJL, EJL and WJL respectively. The errors for the SJL, EJL and WJL are $e_1(t), e_2(t)$ and $e_3(t)$ respectively. Also w_1 and w_2 are the weights assigned to f_1 and f_2 respectively.

In the present work, the desired trajectory chosen was a cubic polynomial nature. The robustness of the proposed controllers for handling the model uncertainties includes the mass changes from the nominal 0.1 kg value. For robustness testing, the mass of each link of the robotic manipulator was varied from 2% up to 10% of the actual value in m_1, m_2 and m_3 each, combination of m_1 and m_2 with all three masses simultaneously. The values of IAE of PID based fractional order fuzzy, PID based fuzzy, PID based fractional order and PID controllers for increasing in masses of the links SJL, EJL and WJL are summarized in the Table 3. The equivalent graphical analysis of the variation in IAE for increase in link masses m_1, m_2 and m_3 in percentage for SJL, EJL and WJL of the robotic manipulator is illustrated in Fig.6.

The deviation in IAE for fuzzy based PID controllers implemented in the C2000 launch pad, with disturbances applied to the PID based fractional order fuzzy, PID based fuzzy, PID based fractional order and PID controllers output of SJL, EJL and WJL and all three links for examining the robustness of controller is presented in Table 4. The equivalent graphical analysis of the variation in IAE for SJL, EJL and WJL with disturbance in all three links of the robotic manipulator is given in Fig. 7. The graphs for the trajectory tracking of the path tracked by the SJL, EJL, WJL and the end effector of robotic manipulator for the PID based fractional order fuzzy, PID based fuzzy, PID based fractional order and PID controllers for disturbance of 1.0sin50t N-m are presented in Fig. 8. From the experimental results, it is evident that the disturbance rejection and robustness in the robotic manipulator is superior for PID based fractional order fuzzy controller when compared with other controllers.



(c) (d)

Fig. 8. Trajectory tracking of (a) SJL (b) EJL (c) WJL (d) End effector of robotic manipulator

6. Conclusion

The lack of precision in trajectory tracking of robotic manipulators, one of the serious issues was solved to reasonable extent using PID based fractional order fuzzy controller. It was implemented for a three link planar rigid robotic manipulator. The fractional operator has decreased the stiffness of implementation in the fuzzy PID controller as it requires two additional variables to be tuned. Alternatively, tuning of the large number of controller parameters in the resulting PID based fractional order fuzzy controller becomes the tricky task. A relative study of the controller proposed in this work has also done with three more controllers such as PID based fuzzy, PID based fractional order and conventional PID controllers. Besides that, for those mentioned controllers robustness testing of model uncertainties and disturbance rejection are also being investigated. From the experimental results it is obvious that, the designed fractional order fuzzy PID controller possesses superior trajectory tracking and it is robust among all studied controllers.

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