CLOSED FORM MODELS FOR PULL-IN VOLTAGE OF ELECTROSTATICALLY ACTUATED CANTILEVER BEAMS AND THEIR COMPARATIVE ANALYSIS

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Abstract: *Evaluation of pull-in voltage is significant for the design of electrostatically actuated MEMS devices. This paper presents simple closed form models for fast and accurate computation of pull-in voltage of electrostatically actuated cantilever beams. These models are obtained based on five different capacitance models suitable for wide range of dimensions. Using these models pull-in voltages are computed for different range of dimensions and the results are compared with the experimentally verified 3D finite element analysis results. The results show that, for every given range of dimension, choice of the model changes for the evaluation of the pull-in voltage with a maximum deviation of ±2%. Therefore for a given range of dimension appropriate closed form model is to be chosen for accurate computation of pull-in voltage.*

Keywords: *Capacitance models, cantilever beams, electrostatic actuators, FEM models, MEMS, pull-in voltages.*

1. Introduction

Micro Electro Mechanical Systems (MEMS) capacitive type transducers are used to sense external mechanical excitation such as force, acoustical pressure, acceleration, as a change in capacitance. It requires electrical energy and this energy can be applied as a constant voltage (or) constant charge [1].The voltage controlled parallel plate electrostatic actuator exhibits an important behavior called pull-in. Pull-in voltage is one of the basic parameters of the design of many electrostatic MEMS devices. Accurate evaluation of the pull-in voltages is essential in the design of electrostatically actuated MEMS devices. In particular, in micromirrors, the designer avoids this instability in order to achieve stable motions. But in switching applications, the designer exploits this effect to optimize the performance of the device [3]. The pull-in problem of beams cannot be solved analytically and numerical techniques using Finite Element Analysis (FEA) are computationally expensive as they are time consuming. Closed form expressions are very useful for designers as they provide some basic information regarding pull-in voltage. Many attempts have been made by several authors [10-19] to derive a closed form expression for the pull-in voltage. Chowdhry et al [10] has derived closed form model for pull-in voltage calculation by considering Meji's and Fokkema's capacitance formula [5] as better capacitance model [2]. Here the investigations were done for selective dimensions of cantilever beam. But further investigation for wide range of dimension of cantilever beam shows that the closed form model used in paper [10] alone is not sufficient. Different capacitance models are available in literature [4-9] Chang's model [4] is very accurate [2]. But Chang's model is computationally expensive. Therefore this paper takes into consideration of all other capacitance models available in literature [5-9]. Based on these models pull-in voltage of cantilever beam is computed for wide range of dimensions. Moreover the suitability of each model for the calculation of pull-in voltage has also been investigated in the present paper. A detailed comparative analysis is done, by comparing the pull-in voltages obtained from the closed form models with CoventorWare FEA model results. The results show that, for a given range of dimension one particular model suits better for the evaluation of the pull-in voltage of cantilever beam with a maximum deviation of ±2% as compared with the experimentally verified FEA results.

2. Cantilever Beam Pull-In Voltage Model

The lumped parameter model of the actuator is shown in Figure 1. The actuator is assumed to be in a vacuum environment to ensure zero external mechanical loading of the top electrode. It is also assumed that the movable plate's elastic restoring force (spring force) is linear. Neglecting any damping within the system, the equation of motion of the movable plate due to an electrostatic force FE can be expressed as,

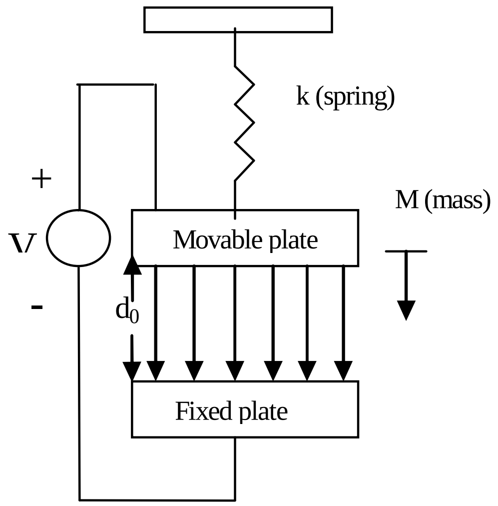

Figure 2. The basic electrostatic actuator - Lumped parameter model of parallel plate electrostatic actuator

$$M\frac{d^2z}{dt^2}+kz=F_E \quad (1)$$

The electrostatic attraction force of the plate can be found by differentiating the stored energy of the capacitor with respect to the position of the movable plate and is given as

$$F_E = -\frac{d\left(\frac{1}{2}CV^2\right)}{dz} = \frac{\varepsilon_0 AV^2}{2\left(d_0 - z\right)^2} \quad (2)$$

where F_E = Electrostatic force

$$C = \frac{\varepsilon_0 A}{d_0 - z} \quad (3)$$

and the spring force (elastic restoring force) is represented as

$$F_M = kz \quad (4)$$

where F_M = Mechanical elastic restoring force

k – spring constant

z - displacement

At static equilibrium $F_M = F_E$

If electrostatic force is increased by increasing the applied voltage and if that force is greater than the elastic restoring force, the equilibrium is lost and the movable plate will collapse on the fixed ground plate. This phenomenon is known as pull-in.

$$kz = \frac{\varepsilon_0 AV^2}{2(d_0 - z)^2} \quad (5)$$

$$2kz = \frac{\varepsilon_0 AV^2}{d_0{}^2 + z^2 - 2d_0 z} \quad (5a)$$

$$\because\ 2kzd_0{}^2 + 2kz^3 - 4kd_0 z^2 = \varepsilon_0 AV^2 \quad (5b)$$

Differentiating (5b) with respect to z

$$2kd_0{}^2 + 6kz^2 - 8kd_0 z = \varepsilon_0 A2V\frac{\partial v}{\partial z} \quad (5c)$$

At equilibrium (or) at Pull-in $\frac{\partial v}{\partial z} = 0$

$$\because\ 2kd_0{}^2 + 6kz^2 - 8kd_0 z = 0 \quad (5d)$$

simplifying eqn (5d) , eqn (5e) is obtained

$$d_0^2 + 3z^2 - 4d_0 z = 0 \text{ (or) } 3z^2 - 4d_0 z + d_0^2 \quad (5e)$$

$z_{pi} = \dfrac{d_0}{3}$ is obtained by solving eqn (5e),

substituting the z_{pi} in eqn (5) and simplifying, eqn (6) can be arrived

$$V_{PI} = \sqrt{\frac{8kd_0^3}{27\varepsilon_0 A}} \qquad (6)$$

The spring constant of the movable plate is solved from equation (6) as

$$k = \frac{27\varepsilon_0 A V_{PI}^2}{8d_0^3} \qquad (7)$$

If the applied voltage is increased beyond the pull-in voltage, the resulting electrostatic force will overcome the elastic restoring force and will cause the movable plate collapse on the fixed ground plane and the capacitor will be short circuited. By expanding equation (3) using a Taylor series approximation about a distance z_0 as outlined in paper [12], equation (8) can be arrived

$$F_E = \frac{\varepsilon_0 AV^2}{2(d_0 - z)^2} = \frac{\varepsilon_0 AV^2}{2(d_0 - z)^2}\Big|_{z=z_0} + \frac{\varepsilon_0 AV^2(-2)(-1)}{2(d_0 - z)^3}\Big|_{z=z_0}(z - z_0) + \ldots, \quad (8)$$

After simplification and rearrangement of the terms in equation (8), equation (9) has been arrived.

$$F_E = \frac{\varepsilon_0 AV^2}{2(d_0 - z)^2}\left[1 + 2\frac{z - z_0}{d_0 - z_0} + \ldots\right]. \qquad (9)$$

By substituting F_E from equation (9) into equation (1), equation (10) is arrived.

$$M\frac{d^2 z}{dt^2} + kz = \frac{\varepsilon_0 AV^2}{2(d_0 - z_0)^2}\left[1 + 2\frac{z - z_0}{d_0 - z_0} + \ldots\right]. \qquad (10)$$

After rearrangement,

$$M\frac{d^2 z}{dt^2} + \left(k - k_{soft}\right)z = \frac{\varepsilon_0 wlV^2}{2(d_0 - z)^2} \times \left[1 - 2\frac{z_0}{d_0 - z_0} + \ldots\right].$$

That is

$$M\frac{d^2 z}{dt^2} + \left(k - \frac{\varepsilon_0 AV^2}{2(d_0 - z_0)^3}\right)z = \frac{\varepsilon_0 AV^2}{2(d_0 - z_0)^2} \times \left[1 - 2\frac{z_0}{d_0 - z_0} + \ldots\right]. \qquad (11)$$

From equation (11) it is evident that the electrostatic attraction force effectively modifies the spring constant of the movable plate and the term within the parenthesis on the left-hand side of equation (11) represents the effective spring constant at a specific voltage. The amount of modification is termed as the spring softening and can be expressed as

$$k_{soft} = \frac{\varepsilon_0 AV^2}{2(d_0 - z)^3}. \qquad (12)$$

For parallel plate geometries, the nonlinear electrostatic force is always uniform. But for cantilever beam geometry (figure 2(a)), the electrostatic force becomes increasingly non-uniform as the beam deforms (figure 2(b)). As a result, the tip of the cantilever will experience a higher attractive force comparing to the region closer to the fixed end. Following [11], an expression for a uniform pressure causing a cantilever tip deflection of z can be derived as

$$P = \frac{kz}{wl} = \frac{2}{3}\frac{\tilde{E}h^3}{l^4}z \qquad (13)$$

Where $\tilde{E}$ is the plate modulus E / (1-ν^2) for wide beams (w≥5h). For narrow beams (w <5h), $\tilde{E}$ simply becomes the Young's modulus E [11]. A uniform linearised model of the electrostatic force can be obtained from (11) and (12) by linearising the electrostatic force about zero deflection point ($z_0 = 0$) as shown in figure 3. Since before any deflection the beam surface is assumed to be planar, the parallel-plate approximation can easily be applied without causing any significant error if airgap thickness (d_0) is very small compared to the lateral dimensions of the beam. Linearising (11) about the point z_0=0,

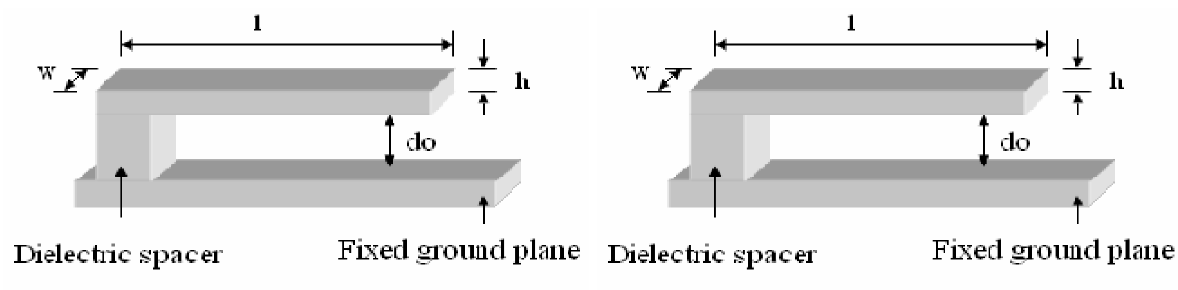

Figure 2. (a). A cantilever beam separated from a fixed ground plane by a dielectric spacer.
(b). Deformation of the beam due to electrostatic force.

$$M\frac{d^2z}{dt^2}+\left(k-k_{soft}\right)z=\frac{\varepsilon_0 wlV^2}{2d_0^{\ 2}} \qquad (14)$$

and

$$k_{soft}=\frac{\varepsilon_0 wlV^2}{2d_0^{\ 3}} \qquad (15)$$

Rearranging (14) and neglecting the time-dependent term for a static case, the force equilibrium relation for any displacement z can be obtained. It is given in equation (16).

$$kz=\frac{\varepsilon_0 wlV^2}{2d_0^{\ 2}}+k_{soft}z \qquad (16)$$

The right-hand side of (16) is equal to approximate, uniform and linear electrostatic force ($F_{E\text{-linear-uniform}}$) and the left-hand side represent the elastic restoring force. The effective linearised uniform electrostatic pressure on the beam can be evaluated from (16) as

$$P_{eff}=\frac{F_{E-linear-uniform}}{wl}=\frac{\varepsilon_0 V^2}{2d_0^2}+\frac{k_{soft}z}{wl} \qquad (17)$$

Substituting k_{soft} from (15) into (17) and replacing z in (17) by the pull-in deflection $z = 1/3d_0$, the pull-in electrostatic pressure $P_{PI\text{-electrostatic}}$ can be evaluated as

$$P_{PI-electrostatic}=\frac{5\varepsilon_0 V_{PI}^2}{6d_0^2} \qquad (18)$$

where V_{PI} represents the pull-in voltage. In order to compensate for the error that arises due to neglecting higher order terms in Taylor series expansion and error due to the linearization, a Compensation Factor (cf) has been determined by a trial and error method, while comparing the results with CoventorWare-FEA model results. The compensation factor is applied to equation (18). Now the compensated pull-in electrostatic pressure $P_{PI\text{-electrostatic}}$ is expressed as

$$P_{PI\text{-electrostatic}} = (cf)\ {}_{0}\ {}^{2}_{PI}\left(\frac{5}{6d_0^2}\right)$$

By substituting the pull-in deflection, $z = 1/3d_0$, into equation (13), the elastic restoring pressure at pull-in is obtained.

$$P_{PI-elastic}=\frac{2}{3}\frac{\tilde{E}h^3}{l^4}\left(\frac{d_0}{3}\right)=\frac{2\tilde{E}h^3d_0}{9l^4} \qquad (20)$$

Since at pull-in equilibrium, the electrostatic pressure is just counterbalanced by the elastic restoring pressure ($P_{PI\text{-electrostatic}} = P_{PI\text{-elastic}}$), equation (19) and (20) can now be solved simultaneously to yield the final closed-form expression for the pull-in voltage V_{PI} as

$$V_{PI}=\sqrt{\frac{0.222\tilde{E}\left(h\right)^3 d_0}{\varepsilon_0(l^4)cf\left(\frac{5}{6}d_0^2\right)}} \qquad (21)$$

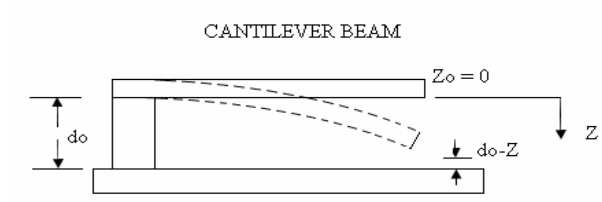

Figure 3. Linearization of the electrostatic force about the zero deflection point (z_0=0)

3. Closed form models of pull-in voltage from capacitance models

This section presents the closed form models of pull-in voltage obtained from various models of capacitances available in literature [5-9], using the procedure explained in the above section. These capacitance models are basically parallel plate models with fringing fields. These capacitance models are found to be suitable for wide range of dimensions of parallel plate capacitors [2]. Based on these capacitance models the closed form models of pull-in voltage are derived and presented in 3(b). Further its suitability for different range of dimensions is extensively investigated in section 4.

A. Meji's and Fokkema's model (Model 1)

Meji and Fokkema [5] improved Sakurai's model [8] by extending the empirical expression. The first term describes the parallel-plate capacitor and the other allows for all side effects:

$$c = \varepsilon_0 l\left[\left(\frac{w}{d_0}\right) + 0.77 + 1.06\left(\frac{w}{d_0}\right)^{0.25} + 1.06\left(\frac{h}{d_0}\right)^{0.25}\right] \qquad (22)$$

With reference to [2, 5], the maximum deviation from Chang's formula [4] is as 2 percent when $w/d_0 \geq 1$, $0.1 \leq h/d_0 \leq 4$ and as 6 percent when $w/d_0 \geq 0.3$, $h/d_0 \leq 10$.

B. Yuan and Trick's model (Model 2)

Yuan and Trick [6] presented simple analytic approximation. They replaced the rectangular line profile with an 'oval' one. The resulting capacitance is given by

$$c = \varepsilon_0 l\left[\frac{w - h/2}{d_0} + \frac{2\pi}{\log\left(1 + \frac{2d_0}{h} + \sqrt{\frac{2d_0}{h}\left(\frac{2d_0}{h} + 2\right)}\right)}\right] \qquad (23)$$

With reference to [2, 6], a maximum error of 10 percent with respect to Chang's formula [4] is stated.

C. Elmasry's model (Model 3)

$$c = \varepsilon_0 l\left[\frac{w}{d_0} + 2\log\left(1 + \frac{h}{d_0}\right) + \frac{2h}{d_0}\log\left(1 + \frac{w/2}{h + d_0}\right)\right] \qquad (24)$$

The first term of (24) represents parallel plate capacitance, second term represents capacitance associated with the side walls, and the third term represents capacitance associated with the top side of the beam.

D. Sakurai and Tamaru's model (Model 4)

$$c = \frac{\varepsilon_0 l}{d_0}\left[1.15\left(\frac{w}{d_0}\right) + 2.80\left(\frac{h}{d_0}\right)^{0.222}\right] \qquad (25)$$

The first term of (25) represents the capacitance of the top and side walls of the beam and the second term represents side wall contribution.

E. Palmer's model (Model 5)

$$c = \frac{\varepsilon_0 l w}{d_0}\left[1 + 2\left(\frac{d_0}{\pi w}\right) + 2\left(\frac{d_0}{\pi w}\right)\log\left(\frac{\pi w}{d_0}\right)\right] \qquad (26)$$

This model includes parallel plate capacitance and includes fringing field capacitance due to the width of the capacitance. But it neglects the capacitance due to the lateral surfaces.

3(a). Pull-in Voltage Models

Based on the above capacitance models the closed form models of pull-in voltages are derived based on the procedure outlined in paper [12], whose final form is presented table 1. It is to be noted that the model 1 shown below has been already discussed in paper [10]. Here as explained before a compensation factor is applied for every model whose value is same irrespective of change in dimensions.

3(b). FEA Based Computation of Pull-in Voltage

The cantilever beam is modeled and analyzed for wide range of dimensions using Finite Element Analysis (FEA) based software platform CoventorWare. Meshing is done based on mesh convergence study. Cosolve (coupled analysis of MemMech and MemElectro) is one of the solvers of CoventorWare that is used to detect the pull-in voltage. The FEA model used in this study is shown figure 4.

4. Model validation

Pull-in voltages of cantilever beam have been computed using the models in section 3 over wide range of dimensions. The specific ranges used for the calculation of pull-in voltage are based on the paper [2].

Table 1. Pull-in voltage formulas based on various models.

Pull-in voltage based on model 1	$$\mathrm{V}_{PI,1} = \sqrt{\frac{0.222\tilde{E}(h)^3 d_0}{\varepsilon_0 (l)^4 (cf)[A_1]}} \qquad (27)$$ $$where\ A_1 = \frac{5}{6(d_0)^2} + \frac{0.189}{(d_0)^{1.25}(w)^{0.75}} + \frac{0.398(h)^{0.5}}{(d_0)^{1.5} w}$$ $and\ \ cf = 0.76$
Pull-in voltage based on model 2	$$\mathrm{V}_{PI,2} = \sqrt{\frac{0.222\tilde{E}h^3 d_0}{\varepsilon_0 l^4 (cf)[B_1]}} \qquad (28)$$ $$where\ B_1 = \left[\frac{5(2w-h)}{12 d_0^2 w}\right] + \left[\frac{16\pi d_0\left(\frac{2d_0}{h}+2\right) + 4\pi \log(sigma)(2d_0+h)\left(\frac{2d_0\left(\frac{2d_0}{h}+2\right)}{h}\right)^{\frac{1}{2}}}{12 d_0 h(w)\log(sigma)^3\left(\frac{2d_0}{h}+2\right)^2}\right] + \pi\frac{\frac{\left(\frac{\frac{2d_0}{h}+2}{h}\right)+\frac{2d_0}{h^2}}{\left(\frac{2d_0\left(\frac{2d_0}{h}+2\right)}{h}\right)^{\frac{1}{2}}}+\frac{2}{h}}{(sigma)(w)\log(sigma)^2}$$ $$where\ sigma = \frac{2d_0}{h} + \left(\frac{2d_0\left(\frac{2d_0}{h}+2\right)}{h}\right)^{\frac{1}{2}} + 1$$ $where\ \ cf = 0.76$
Pull-in voltage based on model 3	$$\mathrm{V}_{PI,3} = \sqrt{\frac{0.222\tilde{E}(h)^3 d_0}{\varepsilon_0 l^4 (cf)[C_1]}} \qquad (29)$$ $$where\ C_1 = \frac{5}{6d_0^2} + \frac{h(5d_0+4h)}{3d_0} + \frac{2h}{3(d_0+h)(2d_0+2h+w)^2} + \frac{(5h)\log\left(\frac{2d_0+2h+w}{2d_0+2h}\right)}{3d_0^2 w} + \frac{h(6d_0+5h)}{3d_0(d_0+h)^2(2d_0+2h+w)}$$ $where\ \ cf = 0.5$
Pull-in voltage based on model 4	$$\mathrm{V}_{PI,4} = \sqrt{\frac{0.222\tilde{E}(h)^3 d_0}{\varepsilon_0 (l)^4 (cf)[D_1]}} \qquad (30)$$ $$Where\ D_1 = \frac{0.958}{(d_0)^2} + \frac{0.438(h)^{0.2222}}{w(d_0)^{1.222}}$$ $where\ \ cf = 0.8$

Pull-in voltage based on model 5	$$V_{PI,5} = \sqrt{\frac{0.222\tilde{E}(h)^3 d_0}{\varepsilon_0 (l)^4 (cf)[E_1]}} \qquad (31)$$ $$Where\ E_1 = \frac{5}{(6d_0)^2} + \frac{4}{3\pi d_0 w}$$ $$where\ cf = 0.89$$

Here the values of d_o are selected as very low to suit the parallel plate approximation as considered in the derivation of closed form models. The pull-in voltage for different ranges is computed and presented from Table 1 through Table 4. The values of pull-in voltage are validated using Cosolve FEA results; where in Cosolve FEA results have been already verified with the experimental results [6, 10]. It was reported that the difference of the experimentally measured values and Cosolve FEA results as 0.83%. Therefore the authors of paper [10] have used Cosolve FEA results as a bench mark. Though it is claimed that the accuracy of pull-in voltage obtained from FEA based models are best compared with closed form models [2, 10], the former is time consuming compared to the later.

Here it is to be noted that for each closed form model the compensation factor applied is unique across all dimensions and it has taken care of the reduction of error substantially. If the closed form model's error within ±2% is considered as less error [10], then that model can be considered as a better model. The better suited model amongst them is in bold form as shown in Table 1 through 4.

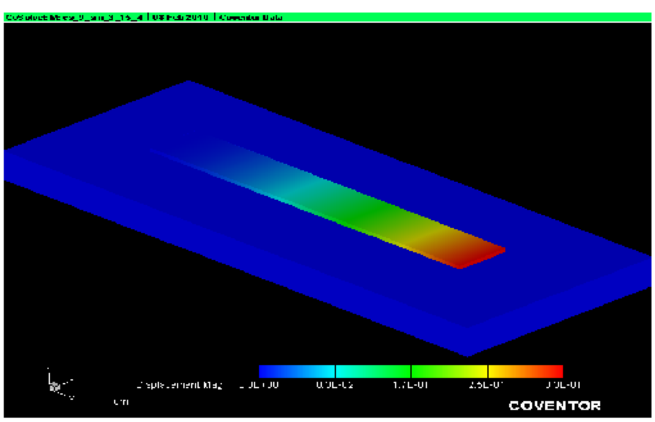

Figure 4. FEA simulation model in CoventorWare

Table.2 Pull-in voltage and %Error comparison for $w/d_0 \leq 2$

Common parameters: h = 1.3µm, d_0 = 0.75 µm, Poisson's ratio v = 0.06, Young's modulus E=169GPa, l=100 µm.				
w/d_0	**0.667**	**1.2**	**1.333**	**2**
$V_{PI,1}$	5.2554	6.0076	6.1286	6.5473
%Error	**0.4856**	**1.4185**	**0.6871**	**0.1937**
$V_{PI,2}$	5.1518	5.9663	6.0977	6.5512
%Error	1.4955	2.0957	1.1872	0.1339
$V_{PI,3}$	5.1956	5.8984	6.0132	6.4249
%Error	0.6568	3.2098	2.5568	2.0597
$V_{PI,4}$	5.3769	5.9885	6.0806	6.3840
%Error	2.808	1.7308	1.4648	2.6774
$V_{PI,5}$	5.4816	6.1	6.1930	6.4996
%Error	4.8103	0.0985	0.3559	0.9206
Cosolve FEA	5.23	6.094	6.171	6.56

Table 2 shows the Pull-in voltage and its %Error for $w/d_0 \leq 2$. In this range $V_{PI,1}$'s error is within 2%. Therefore for the range of $w/d_0 \leq 2$, $V_{PI,1}$ is a better suited model.

Table 3. Pull-in voltage and %Error comparison for $w/d_0 \geq 10$

Common parameters h = 1.3µm, d_0 = 0.75 µm, Poisson's ratio v = 0.06, Young's modulus E=169GPa, l=100 µm.						
w/d_0	**10**	**15**	**28**	**30**	**40**	**50**
$V_{PI,1}$	7.5155	7.6125	7.7358	7.7449	7.773	7.7975
%Error	8.3321	9.663	10.0249	9.6621	10.1214	10.4070
$V_{PI,2}$	7.5624	7.6678	7.7697	7.7778	7.806	7.8231
%Error	9.0071	10.2778	10.5069	10.1275	10.5273	10.7693
$V_{PI,3}$	7.7872	8.0902	8.5180	8.5615	8.733	8.8552
%Error	13.3881	17.5357	21.148	22.4550	24.9121	26.6558

			6			
$V_{PI,4}$	6.9954	7.0553	7.1084	7.1126	7.1277	7.1368
%Error	**0.8348**	**1.4411**	**1.1003**	**0.7101**	**0.9235**	**1.0522**
$V_{PI,5}$	7.1145	7.1727	7.2280	7.2323	7.2474	7.2566
%Error	2.5514	3.1579	2.8016	2.4041	2.6185	2.7477
Cosolve FEA	6.94	6.95	7.031	7.06	7.06	7.06

Table 3 shows the pull-in voltage and the %Error for the range of $w/d_0 \geq 10$.In this range $V_{PI,4}$'s error is less compared to other models. Therefore in the range of $w/d_0 \geq 10$, it is concluded that $V_{PI,4}$ is a better suited model

Table 4. Pull-in voltage and %Error comparison for w ≥ h/2 and h ≈ d_0

Common parameters h = 0.75µm, d_0 = 0.75 µm, Poisson's ratio v = 0.06, Young's modulus E=169GPa, l=100 µm. (h ≈ d_0)				
w	**0.375**	**1.5**	**2.5**	**3.5**
$V_{PI,1}$	2.2603	2.9469	3.1073	3.1874
%Error	**0.2325**	**0.7339**	**0.5662**	**1.9965**
$V_{PI,2}$	2.1396	2.9168	3.0996	3.1892
%Error	5.5628	1.7475	0.8123	2.0549
$V_{PI,3}$	2.2637	2.9728	3.1803	3.3071
%Error	0.0836	0.1386	1.777	5.8261
$V_{PI,4}$	2.2688	2.8313	2.9428	2.9948
%Error	0.1393	4.6274	5.8305	4.1673
$V_{PI,5}$	2.2454	2.8482	2.9713	3.0292
%Error	0.89	4.0604	4.9176	3.0645
Cosolve FEA	2.2656	2.9687	3.125	3.125

Table 4 shows that $V_{PI,1}$'s percentage error is less compared with other models for the range of w ≥ h/2 and h ≈ d_0. If the thickness of the beam is equal to the gap between the substrate and the beam, and w ≥ h/2, the model $V_{PI,1}$ is a better suited model.

Table 1. Pull-in voltage and %Error comparison for $h/d_0 \leq$ 15

Common parameters w = 1.5µm, d_0 = 0.75 µm, Poisson's ratio v = 0.06, Young's modulus E=169GPa, l=100 µm.				
h/d_0	**2**	**2.667**	**6.667**	**13.333**
$V_{PI,1}$	8.0504	12.1803	44.9252	118.7092
%Error	0.045	2.5574	7.6981	12.9238
$V_{PI,2}$	8.1083	12.5188	56.0039	355.332
%Error	0.7640	0.1505	15.0140	160.6458
	7.8712	11.8764	45.0835	125.6792
%Error	2.1821	4.9889	7.3728	7.8112
$V_{PI,4}$	7.8872	12.0599	46.5252	128.8569
%Error	1.983	3.5209	4.4108	5.4802
$V_{PI,5}$	8.0558	12.4027	49.0261	138.6667
%Error	**0.1119**	**0.7782**	**0.7275**	**1.7155**
Cosolve FEA	8.0468	12.5	48.67	136.3

In Table 5 pull-in voltages and %Error for $h/d_0 \leq$ 15 is given. In this range $V_{PI,5}$ is with less error compared to other models. Therefore for the beams with higher thickness (h), the model $V_{PI,5}$ is a better suited model.

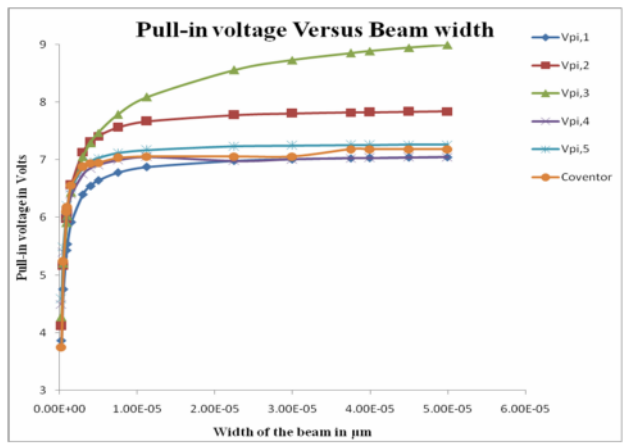

Figure 5. Pull-in voltage Versus width of the beam

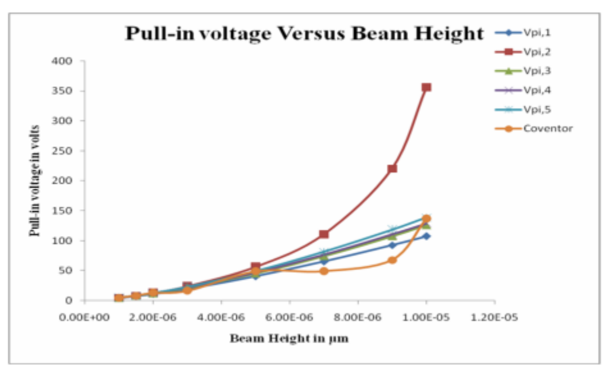

Figure 6. Pull-in voltage Versus height of the beam

Moreover, figure 5 and figure 6 shows the pull-in voltage versus width of the beam and the pull-in voltage versus height of the beam respectively with respect to both the closed form model results and the CoventorWare results. Here the results are plotted for further a wide range; width varying from 0.25 µm to

50µm and height varying from 1 µm to 10 µm. It could be seen that the models 1, 4 and 5 are found to be closer to CoventorWare results. This corroborates with the results already presented from Tables 1 through 4. Therefore it could be concluded that a particular model is better suited for a particular dimension but indeed not in the entire range.

Paper [10] presented a closed form model for the computation of pull-in voltage based on Meji's and Fokkema's capacitance model. Investigation shows that this model is suitable only for particular dimensions but not for wide range of variation in dimension of cantilever beam. Therefore appropriate model has to be chosen for the appropriate dimension of cantilever beam for the calculation of pull-in voltage.

It is observed that the pull-in voltage values can be computed in a few micro-seconds with regard to every model as against the FEA model, which takes relatively a much larger time duration. It is clear that the closed form models can be used for specific range of dimensions as against the FEA model for speedier computation of pull-in voltage without sacrificing the accuracy.

5. Conclusion

In this paper five closed form models for the calculation of pull-in voltages have been derived from different capacitance models. These models are validated by comparing the results with Cosolve FEA results for wide range of dimensions. Comparison of the pull-in voltages determined from these models shows that particular models are better suited for particular given ranges. The error is within ±2% deviation as compared to the experimentally verified results for those particular models. It is observed that to get accurate value of pull-in voltage, appropriate models have to be chosen for the respective range of dimensions. These models are relatively simple and less time consuming against the FEA models. Such analysis can also be extended for other ranges, if necessary.

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References

1. S. D. Senturia, *Microsystems Design*, Boston, MA: Kluwer Academic, 2000, pp. 249-259.
2. Erich Barke, "Line to ground Capacitance calculation for VLSI: A comparison". IEEE Transactions on Computer – Aided Design, Vol.7, no. 2, Feb. 1988.
3. R. C. Batra, M. Porfiri, and D. Spinello, "Review of modeling of electrostatically actuated micromechancial system", *J Smart Mater Struct* Vol. 16, no. 6, pp.R23-R21, Dec. 2007, DOI: 10.1088/0964-1726/16/6/R01
4. W. H. Chang, "Analytical IC Metal Line Capacitance formulas", *IEEE Transactions in Microwave Theory and Techniques*, Sep. 1976.
5. N. P. Van Der Meijs and J. T. Fokkema, "VLSI circuit reconstruction from mask topology", *Integration*, Vol 2, no. 2, pp. 85-119, Mar. 1984.
6. C. P. Yuan and T. N. Trick, "Simple Formulas for the two and three dimensional capacitances", *IEEE Conf.Comput-Aided Design*, 1984, pp. 263-265
7. M. I. Elmasry., "Capacitance Calculations in MOSFET VLSI", *IEEE Electron Device Letts*. Vol. EDL-3, no. 1, pp. 6-7, Jan. 1982, DOI: 10.1109/EDL.1982.25454.
8. T. Sakurai and K. Tamaru, "Simple formulas for two- and three-dimensional capacitances", *IEEE Trans. Electron Devices*, vol. ED 30, no. 2, pp. 183-185, Feb. 1983, DOI: 10.1109/T –ED.1983.21093.
9. H. B. Palmer, "The capacitance of a Parallel-plate by the Schwartz – Christoffel transformation", *Trans. AIEE*, vol.56, pp. 363, Mar. 1937.
10. S. Chowdhury, M. Ahmadi and W. C. Miller, "A closed-form model for the pull-in voltage of electrostatically actuated cantilever beams", *J Micromech Microeng*, vol. 15, no. 4, pp. 756-763, Apr. 2005.
11. P. M. Osterberg, "Electrostatically actuated micromechanical test structures for material property measurements" *PhD Dissertation*, Massachusetts Institute of Technology, MA, 1995.
12. I. Ladabaum, X. Jin, H. T. Soh, and B. T. Khuri – Yakub, "Surface Micromachined Capacitive Ultrasonic Transducers", *IEEE transactions on Ultrasonic and Ferro electronics, Freq. control.* Vol. 45 pp. 679-90, 1999.
13. P. M. Osterberg and S. D. Senturia, "M test: a test chip for MEMS material property measurement using electrostatically actuated test structures", *Journal of Micro Electro Mechanical Systems*, Vol.6, no. 2, pp. 107-118, Jun. 1997.
14. H. A. C. Tilmans and R. Legtenberg, "Electrostatically driven vacuum-encapsulated polysilicon resonators Part II. Theory and Performance", *Sensors and Actuators A Physical*, Vol. 45, no.1, pp.67-84, Oct. 1994, DOI: 10.1016/0924-4247(94)00813-2.
15. Y. Nemirovrsky and O. Bochosza-Degani, "A Methodology and model for the pull-in parameters of electrostatic actuators", *Journal of Micro Electro*

Mechanical Systems, Vol.10, N0.4, pp. 600-615, Dec. 2001.

16. Y. C. Hu, "Closed form solutions for the pull-in voltage of microcurled beam subjected to electrostatic loads", *J Micromech. Microeng*, Vol.16, pp. 648-655, Feb.2006.
17. Y. C. Hu and G. D. Lee, "A Closed form solution for the pull-in voltage of the micro bridge", *Tamkang journal of Science and Engineering*, Vol.10, no.2, pp. 14-150, Apr. 2007.
18. A. Ramezan, A. Alasty, and J. Akbari, "Closed form solutions of the pull-in instability in nano cantilevers under electrostatic and intermolecular surface stress", *International Journal of Solids and Structures*, Vol.44, pp. 4925-4941, July. 2007.
19. S.Chaterjee and G.Pohit, "A large deflection model for pull-in analysis of electrostatically actuated microcantilever beams", *Journal of Sound and vibration*", Vol. 322, pp. 964-986, Jan. 2009.