TRANSIENT STATE ANALYSIS OF A PERMANENT MAGNET STEPPER MOTOR USING DISCRETE KALMAN FILTER ALGORITHM

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ABSTRACT:*This paper estimates the micro step performance of an electromechanical stepper motor with permanent magnet rotor when subjected to load torque disturbances. The state of the art is to use sensor-less vector control strategy, that allows measurement and monitoring of un-measurable variables of the system. The Discrete Kalman filter algorithm estimates the mechanical variables of the system based on the initial state for constant motor parameters. The algebraic manipulation of the two phase dynamic system parameters and equations is carried out and the same simulated in MATLAB. The accuracy of the sensor-less control is obtained by determining steady state error between the actual and reference values. Further, the strength of the algorithm for motor parameter variants allows algorithm to check performance.*

Keywords: *Kalman Filter, State Estimation, Sensorless Control, Permanent Magnet Machine.*

1. Introduction

Stepper motors have received considerable attention in computer peripherals, machine tools, aerospace etc., due to its discrete step positioning capability [1-4] and permanet magnet are used as a cost-effective facility and high efficiency [5].

Although PM stepper motor mathematical based exiting work is quite non-linear, considering the consequence of difficulties in the design of control procedures, numerous papers are in existence, making it an optimistic area for research. The control methods usually involve feedback and can be broadly classified under position and/or velocity control [6-9]. By including series resistance i.e. limiting the current or by addition of mechanical gear or by incorporating microstepping for regulating the stator phase winding currents, the position and velocity can be controlled provided the sensed actual rotor position is feedback to the controller [7,10]. As the stepping rate is increased, on account of inertia the rotor takes more time to

reach the steady state [3,9].

Sensor based control strategy as in robotics, aerospace, and high performance applications increase the cost and

size of the system. Further, the characteristic of the sensor deteriorate, under change in temperature, pulsating environment [7,9]. The low cost sensorless control strategy is the state of art [11-13].

Computing the speed and rotor position by means of sensorless method uses reference frame theory. The feedback control signal is obtained by measuring the actual rotor position and estimating the same using observers. The viewer is most commonly used as a sliding mode due to the strong characteristic of motor parameter variants [14-15][23].The Kalman Filter and another version of Kalman Filter are the both universally accepted estimators in sensorless control and are applied till date in numerous fields. For example the Kalman filter is utilized in the new model wind turbine controllers to evaluate the aerodynamic torque quickly and precisely [16]. In an electromechnical actuator the Kalman filter estimates the position and minimizes the angular error [17]. A space teleoperation uses the Kalman filter to estimate the control signals of a delayed feedback state[18]. The Kalman Filter is used tocontrol independently the intensity and power of reactive generated by a non-brushbilaterally supplied electricity generator by controlling the system parameters [19]. A unique aspect of the above applications [16-19] is the use of Kalman filter state estimator to measure the mechanical level variablessuch as torque, angle, velocity by estimating the currents and line voltages. The speed observer is used at high and medium speeds whereas the extended Kalman filter is used for lower speeds [20**].**In addition to accurate evaluation and forecasting capability, the instantaneous response is also one of the main motivating factors for proposing and utilizing the Kalman filter [21].

This study proposes a simple discrete Kalman Filter to obtain the information regarding the position of the shaft by deducing the voltage and currents from the discrete time system equations which act as an auxiliary system. The filter is implemented here for middle and high speed incorporating the advantage of reduced computing time. The dynamic performance is considerably improved by applying this sensorless vector control strategy.

2. PM stepper motors with mathematical model

The dynamic equations (1) and (2) on the PM stepper motor in state variable form is derived from the Kuo model exploited in state space in 1978 [21].

$$\begin{bmatrix} \dot{i_a} \\ \dot{i_b} \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & 0 \; \frac{K_m}{L}\sin(N_r\,\theta) & 0 \\ 0 & \frac{-R-K_m}{L \quad L}\cos(N_r\theta) & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} \frac{U_a}{L} \\ \frac{U_b}{L} \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix}\dot{\omega}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\frac{-K_m}{J}\sin(N_r\theta) & \frac{K_m}{J}\cos(N_r\theta)\frac{-B}{J} & 0\\ 0 & 0 \quad 1 & 0\end{bmatrix}\begin{bmatrix}\omega\\ \theta\end{bmatrix} + \begin{bmatrix}\frac{-T_L - T_D}{J}\\ 0\end{bmatrix}, \quad (2)$$

The system description after applying D-Q transformation is specified by equations (3) to (6).

$$\frac{di_d}{dt} = \frac{v_d - Ri_d + N_r L\omega i_q}{L}, \quad (3)$$

$$\frac{di_q}{dt} = \frac{v_q - Ri_q - N_r L\omega i_d - K_m\omega}{L}, \quad (4)$$

$$\frac{d\omega}{dt} = \frac{K_m i_q - B\omega - T_L - T_D}{J}, \quad (5)$$

$$\frac{d\theta}{dt} = \omega, \quad (6)$$

The vector control or field oriented control constitute controlling the flux (aligned with d-axis) and torque (aligned with q-axis) independently. The stator currents are transferred to the rotating reference system by Clark transformation. Controlling the motor speed varies the q-axis has a current parameter or by varying the load torque component. This aspect of system can be reduced, a continuous Linear Time Invariant (LTI) system by choosing V_d and V_q after decoupling as in (7) and (8).

$$V_d = \left(Ri_d + L\frac{di_d}{dt}\right) - N_r L\omega i_q, \quad (7)$$

$$V_q = (Ri_q + L\frac{di_q}{dt}) + N_r L\omega i_d + K_m\omega, \quad (8)$$

The continuous LTI system is modeled as in (3) to (6) is discretised as (9) to (12) as in [22].

$$i_d(k+1) = \left(1 - \frac{R\tau}{L}\right)i_d(k) + N_r\tau i_q(k)\omega(k) + \frac{v_d(k)\tau}{L}, \quad (9)$$

$$i_q(k+1) = \left(1 - \frac{R\tau}{L}\right)i_q(k) + N_r\tau i_d(k)\omega(k) + \frac{v_q(k)\tau}{L} - \frac{K_m\tau\omega(k)}{L}, \quad (10)$$

$$\omega(k+1) = \frac{K_m\tau i_q(k)}{J} + \left(1 - \frac{B\tau}{J}\right)\omega(k) - \frac{(T_L + T_D)\tau}{J}, \quad (11)$$

$$\theta(k+1) = \tau\omega(k), \quad (12)$$

3. Discrete kalman filter algorithm

A linear quadratic estimator is a part of the Kalman Filter. It is originally developed to represent a non-linear state space model. However the linear model is just a special case of non-linear model. For a discrete time invariant system the discrete Kalman Filter measures the current state along with the input and predicts the next state of each input sample for every sampling instant. Here the following state vector $X = [\, i_d i_q\ \omega\ \theta\ ]^T$ is selected along with the input and output vector $U = [\, v_d v_q T_L]^T$ and $Y = [\, i_d i_q \omega\ \theta]^T$ respectively. The particular discrete state equation is accounted in equation (13).

$$X(k+1) = f\{X(k), U(k), k\} + W(k),$$
$$Y(k) = h\{X(k), k\} + V(k), \quad (13)$$

then, the measurement and process noise are characterized by,

$$E\{W(k), W(K)^T\} = Q \text{ and } E\{V(k), V(K)^T\} = R$$

The Riccati equations defined in (14) to (17) are applied to the system (13) for a sampling time of $\tau = 100\mu s$ to yield the stationary reference frame has the instantaneous states of the PM stepper motor

$$P_{(k+1)/k} = F_{dk}P_k F_{dk}^T + Q, \quad (14)$$

$$K_{(k+1)/k} = P_{(k+1)/k}H_k^T\left[R + P_{(k+1)/k}H_k^T H_k\right]^{-1}, \quad (15)$$

$$X_{\frac{k+1}{k+1}} = K_{\frac{k+1}{k}}\left[Y_{k+1} - H_k X_{\frac{k+1}{k}}\right] + X_{(k+1)/k} \quad (16)$$

$$P_{(k+1)/(k+1)} = P_{(k+1)/k}\left[I - K_{(k+1)/k}H_k\right] \quad (17)$$

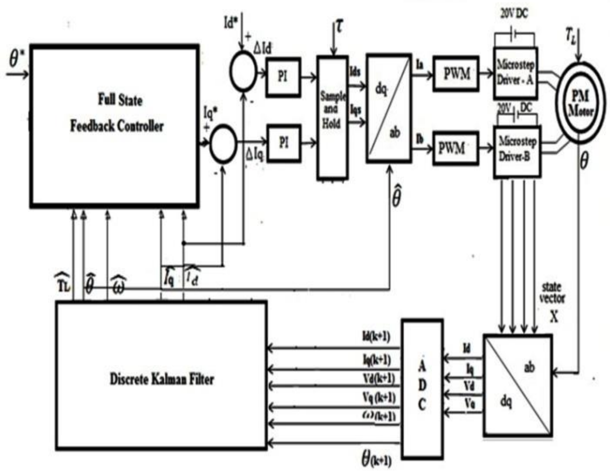

Fig. 1. Control Structure of the Two-phase PM Stepper Motor with Discrete Kalman Filter.

4. Feedback controller of full state

The feedback controller of full state computes the q axis reference current when the gain constants K_ω, K_θ are input. The steps for determining the coefficients of the controller are given below:

STEP 1: Determine the controllability matrix Q_C

STEP 2: Determine the system matrices ofthe open loop system canonical form A_r, B_r

STEP 3: The characteristic of the reference modelequation and compare it with before the step.

STEP 4: Find the controller tuning co-efficient $K_{r\omega}$, $K_{r\theta}$ of the reference model.
STEP 5: Determine $K = [K_{\omega} \quad K_{\theta}]$ from the inverse of Q_C and modal matrix.

5. Simulation results and discussion

The PM stepper motor control structure is shown in Fig. 1. The simulation is conducted for the motor along with the discrete Kalman Filter using SIMULINK in MATLAB as shown in Fig. 2.

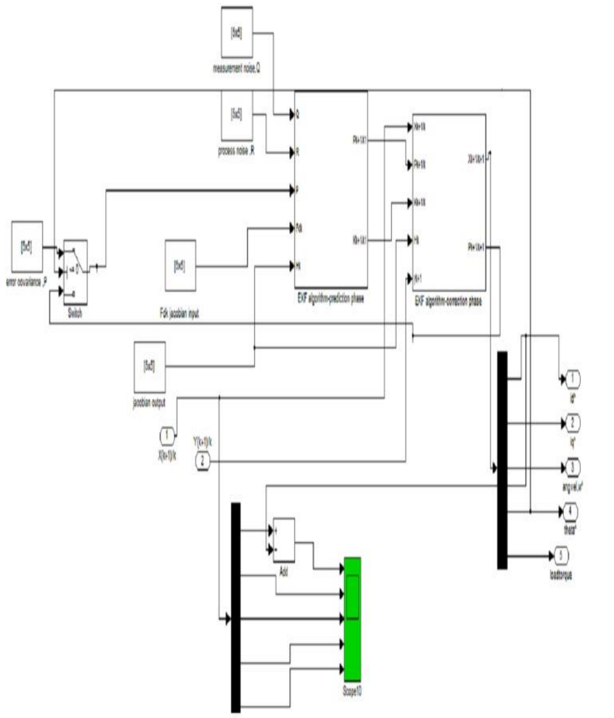

Fig. 2. The Discrete Kalman FilterSimulink Structure with Prediction and Correction Phase.

The motor is driven by a micro-step driver using H bridge configuration for each phase. The micro-stepping currents in the phase windings without feedback controller are given in Fig. 3a and Fig. 3b displaying the orthogonal connection.

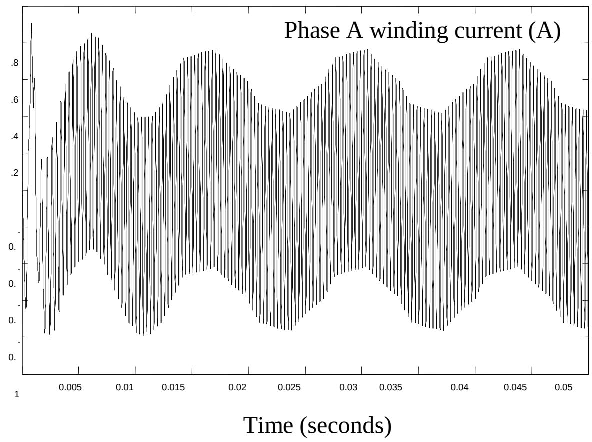

Fig. 3a. PM stepper motor Phase A winding current without feedback controller.

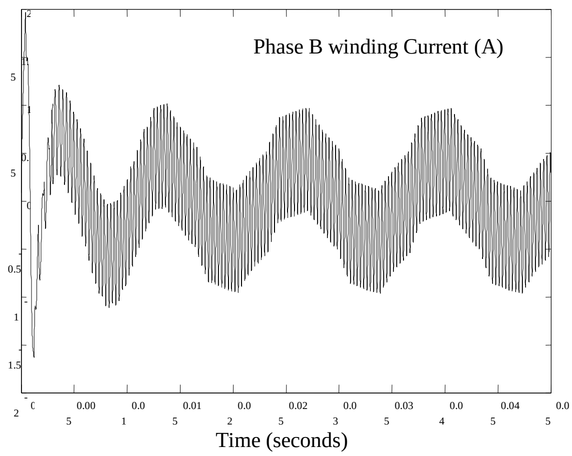

Fig. 3b. PM stepper motor Phase B winding current without feedback controller.

The Fig.4a and Fig.4b display the phase winding currents with feedback controller.

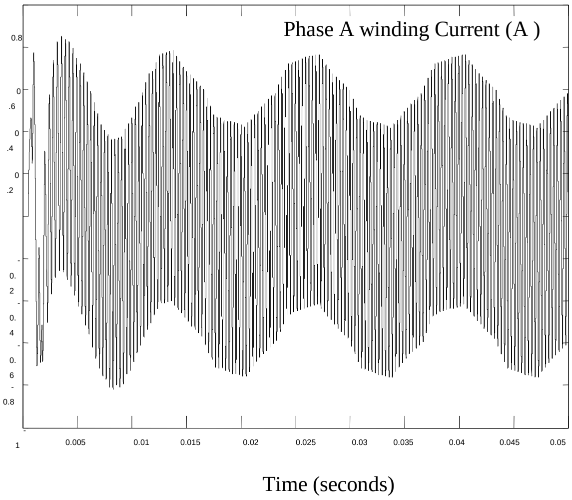

Fig. 4a. PM stepper motor Phase A winding current with feedback controller.

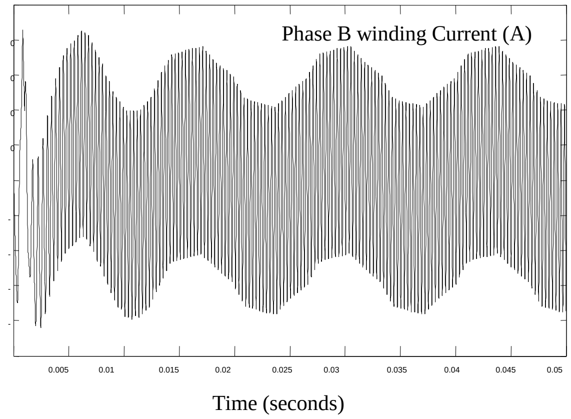

Fig. 4b. PM stepper motor Phase B winding current with feedback controller.

For every sampling instant τ the instantaneous values of state X is fed to the state feedback controller to obtain the reference current $i_q^* = f\{\hat{\omega}, \hat{\theta}\}$. The process and measurement noise of the co-variance matrices Q, R used in the simulation is shown in Table I.

Table1
The Discrete Kalman FilterNoise Covariance Matrices.

Q =

0.0010	0	0	0	0
0	0.0010	0	0	0
0	0	0.0100	0	0
0	0	0	0.0000	0
0	0	0	0	0.0000

R =

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

A standard step signal is used to obtain the load disturbance for the discrete time system in transient state analysis. The mathematical calculations obtained from the full state feedback controller computed offline as $K_\omega = 0.3901$, $K_\theta = 39.01$ is fed subsequently to the simulator show that the controller is fast and accurate in reference tracking capability as observed in Fig. 5.

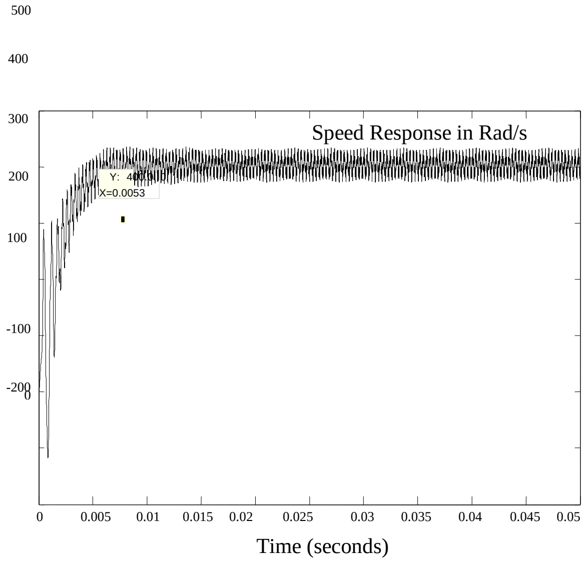

Fig. 5. Speed Response in Rad/s.

The voltages of d and q axis undergo inverse d-q transformation before being feedback to the microstep driver for commutation of the conducting devices. The estimated speed converges to the true value in 0.0053s.

5.1. Case A: Variations in Load Torque

The load torque variation is carried out from 0 to 100% by providing a disturbance input to the motor. The corresponding motor torque and speed response is observed as in Fig. 5 and Fig. 6 corresponding. Fig. 7a shows the variation of motor torque for load variation of 0 to 50%.

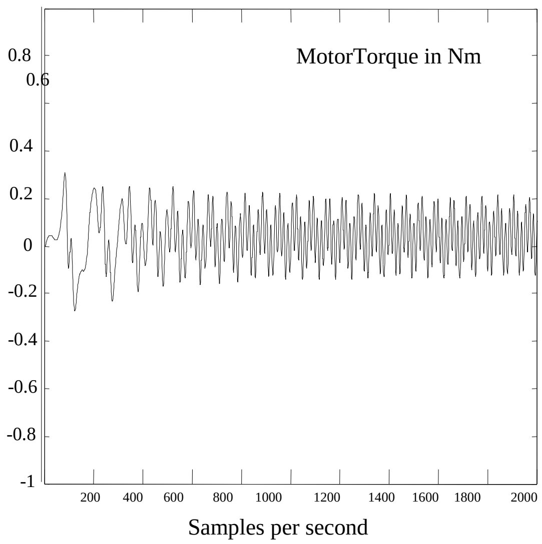

Fig. 6. Motor Torque Response for load variation from 0 to 100%.

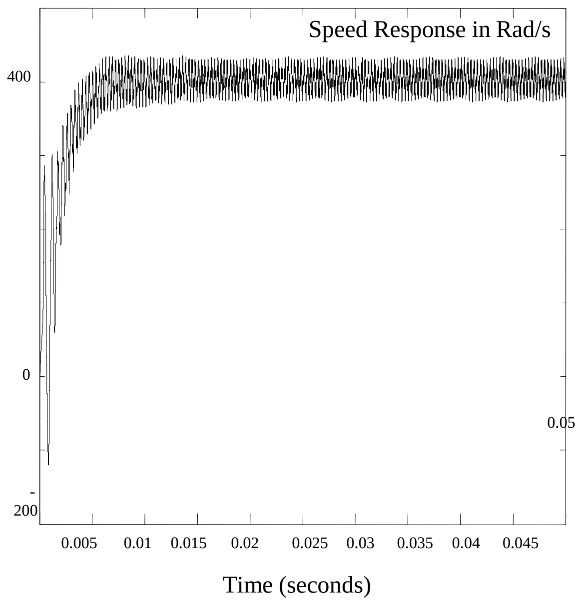

Fig. 7b. Speed Response for load variation from 0 to 50%.

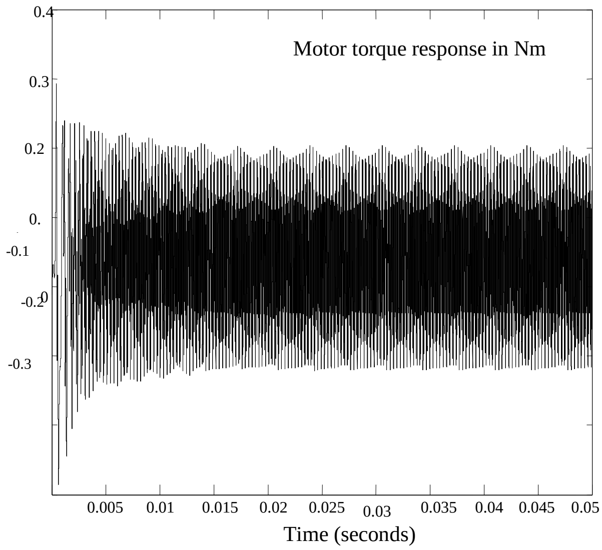

Fig. 7a. Motor Torque Response for load variation from 0 to 50%.

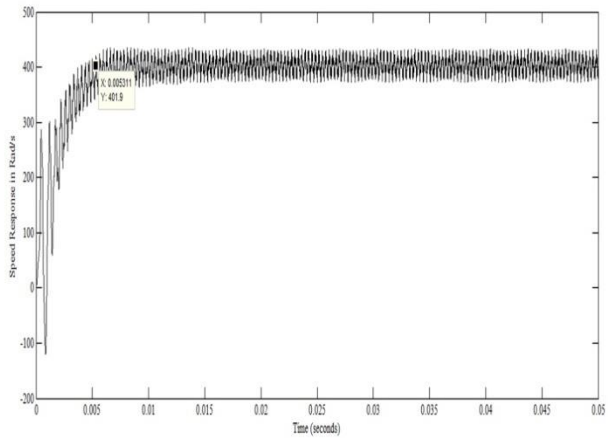

Fig. 7b. Speed Response for load variation from 0 to 75% and I_d^*=0.

Fig. 7b and Fig. 7c show the speed response for load variations from 0 to 50% and 0 to 75% respectively.

The observations of Case A demonstrate that the full state feedback controller is robust to variations in load and the motor torque remains at 0.2 Nm. Further, the motor seep response is regulated from no load on the whole load.

5.1. Case B: Variations in Reference Current

The motor speed and torque response for the reference current I_d^* has the variations are shown in Fig.8 and Fig.9. Table II gives the settling time and the Kalman filter steady state error for variations in the reference current.

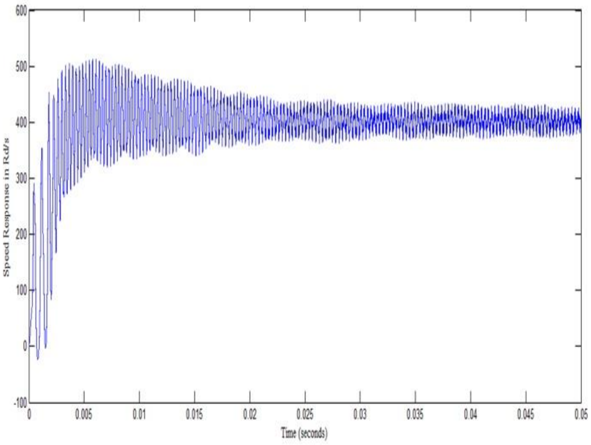

Fig. 8a. Speed response in Rad/s when I_d^* is positive constant.

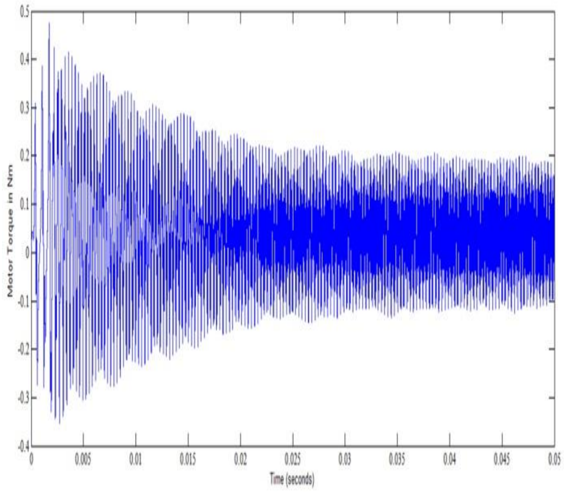

Fig. 8b. Motor Torque in Nm when I_d^* is positive constant.

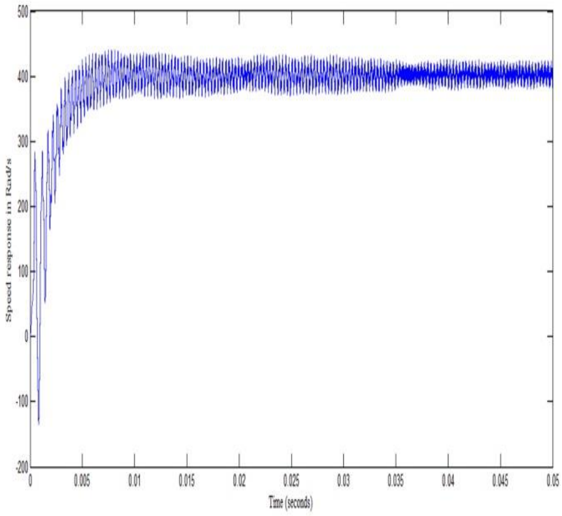

Fig. 9a. Speed response in Rad/s when I_d^* is negative constant.

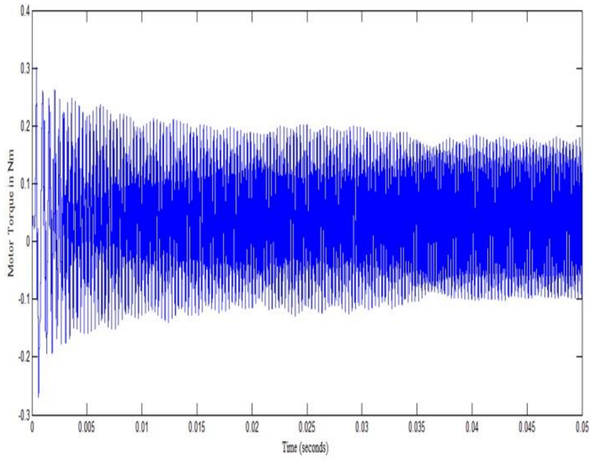

Fig. 9b. Motor Torque in Nm when I_d^* is negative constant.

Table 2

Variation in Reference Current

Reference Current	**Actual Rotor Speed in Rad/s**	**Kalman Filter Speed output Rad/s**	**Settling Time (s)**
I_d*=0	401.5	399.1	0.00531
I_d* >0	401.5	394.6	0.032
I_d*<0	401.5	383.1	0.035

The dynamic process is speedy and reliable, converging to the steady-state within a short span of .0053s as seen in Table 2. An extended portion of this work can further be the implementation of the same in FPGA.

6. ROBUSTNESS OF THE CONTROLLER

The proposed discrete Kalman filter algorithm effectiveness is tested through varying the electrical constants of the motor. The increase in R does not affect the rotor speed response as much as the increase in L as observed in Fig. 10 and Fig. 11.

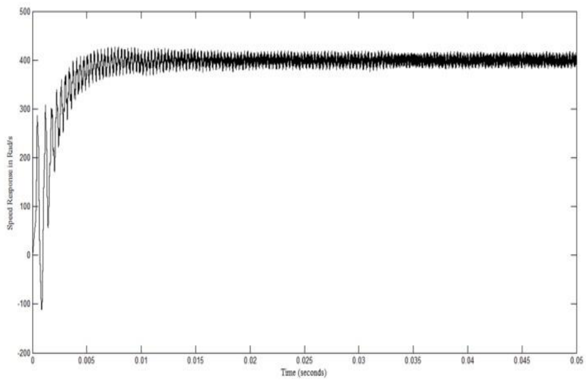

Fig. 10. Speed response in Rad/s when R is increased by 50%.

The increase in L leads to a large amount of overshoot at the onset of tracking and hence more time to settle at the steady-state as in Figure 11.

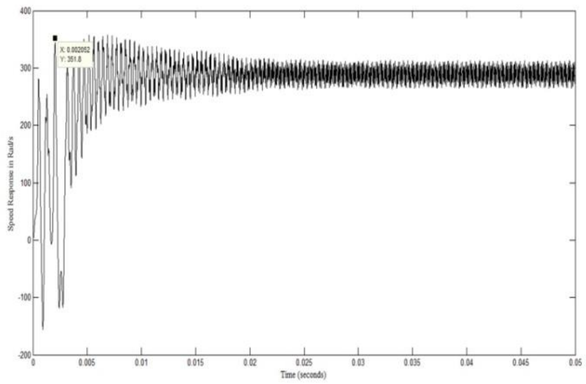

Fig. 11. Speed response in Rad/s when L is increased by 50%

On the contrary if the L/R ratio is maintained constant then the speed response and settling time remain the same as in Fig. 7c. The performance in measures is specified in Table 3.

Table3
Variation of Winding Constants of the PM stepper motor.

Phase winding	**Max. Speed Rad/s**	**Min . Speed Rad/s**	**Mean Speed Rad/s**	**Std. Deviation Rad/s**	**Speed at t=0.025s in Rad/s**
R increased by 25%	426.8	-112	383.8	61.41	396.9
R increased by 50%	432.8	-102	381.2	61.02	393.9
L increased by 25%	357.4	-156	272.5	66.82	287.5
L increased by 50%	392.2	-284	233.0	74.79	255.0
L increased by 75%	342.0	-188	190.4	80.31	244.4

The Table 4 is shown the parameters of motor utilized in the simulation.

Table4
PM Stepper motor Parameters and driver

Phase Winding Resistance	R=0.7 Ω
Phase Winding Inductance	L=7.5 mH
Step Angle	θ = 1.8 °
Max. Flux Linkage	Φ_M = 0.005 Vs
Detent Torque	T_d = 0.12 Nm
Moment of Inertia Of motor	J=1.2 x 10^{-7} Kg m^2
Frictional Constant	B=1 x 10^{-4} Kg m/s
Initial Speed of the rotor	0 Rd/sec
Initial Step angle of the rotor	0 degree
Microstep Driver Input Voltage	20V DC
Shunted Resistance per Phase	1 Ω , 5 mH

7. Conclusion

A discrete mathematical model of the 2 phase PM stepper motor is developed. The linearized case of the non-linear model neglecting the detent torque is fed to the discrete Kalman filterand tested for effectiveness. The simulation results obtained show that the discrete Kalman filter when applied to a physical system filters the thermic noise in the resistors and quantization noise through complex mathematical routines. This algorithm has the advantage of low cost, feasibility and accurate estimation of the speed and position of the rotor during load transients without encoder.The controller gain constants $\boldsymbol{K_\omega}$ and $\boldsymbol{K_\theta}$ are determined offline through MATLAB programming.

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