

CORRELATION OF THERMAL-WIND-HYDRO UNITS FOR SOLVING UNIT COMMITMENT PROBLEM USING FIREFLY ALGORITHM

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Abstract: Owing to the development of renewable energy resources, paves the way for overcoming the difficulties of electric power generation. On combining renewable energies together brings more advantages than conventional methods. For this many more models have been proposed for scheduling the generating units according to the need of energy demand. In order to supply balanced power to the utility and the proper management of power system operation, allocation of units is the foremost concern. Usually, economic dispatch and unit commitment are the problems which may arise due to scheduling of operating units. Unit commitment is a mixed integer programming problem, linear or nonlinear and NP- hard problem involving all the operational constraints. This paper recommends a new approach for resolving unit commitment problem based on firefly algorithm. This proposed meta-heuristic optimization has a preference over earlier methods to determine the best possible solutions. The ultimate aim is to trim down the operational tariff and computational time of existing system. The proposed technique has been investigated on various case studies taking account of 38 and 100 thermal units, 2 wind and 2 hydro units respectively. The results attained from these studies satisfies all the basic constraints and on/off status of distinct generators over a time period of 24 hours. The acquired outcome is to be compared with other conventional approaches. It is apparent that the numerical results elucidate that proposed algorithm is more eminent and efficient tool for settling large scale unit commitment problems within a promising execution time.

Keywords: Unit commitment, Renewable energy, Firefly algorithm, Generation cost, Operational time.

1. Introduction

In modern environment, the evolving electricity demand arouses the necessity to study different operational substitutes for planning electric power generation by coordinating conventional resources with renewable energy resources while ensuring better operational cost [1,2] and enhances the profitability in comprehensive market [3], together with efficiency of energy can be improved in power systems by using various ways [4,5]. Furthermore, multi-period and multi-paradigm models have also been proposed so as to scheme and elevate the energy system and their components for a long time planning horizon [6-8]. In recent times, a general idea of state of ability and process of current system opportunities in power

generation, transmission and distribution have been studied [9]. The generation scheduling for electric power distribution is based on three distinct classifications defined according to the duration of the scheduling time horizon:

- Long-term planning - capacity, type and number of power generators.
- Medium term planning- scheduling of the existing units.
- Short-term planning- calibration of the power that each committed unit must produce to confront the real-time electricity demand.

UC problem has been more extensively considered because of its practical prominence [10,11]. Besides, this problem has varied applications across chemical engineering area, for example the constraints of unit commitment were applied to air separation plants to wrap up when to turn on and off compressors and liquefiers [12]. On account of its goal, the Unit Commitment can be expressed as a mathematical programming problem with constraints using various alternative models. The planning mainly focuses on the most advantageous solutions of these models to be executed, may result in noteworthy cost-effectiveness savings. Nevertheless, undertaking the UC problem is very challenging in nature and it is a mixed integer programming problem, linear or nonlinear, large scale that is commonly be a NP-hard problem due to the exponential computational time that may be vital in the worst case [13].

This paper has a faith on the thermal UC problem. The solution techniques proposed for solving this may be either deterministic or heuristic. Methodologies based on deterministic approaches include: priority list method [14], integer mathematical programming (linear and nonlinear) [15-20], dynamic programming [21] and other decomposition techniques [22-24]. Thus far, few of these proposed approaches guarantee total optimality. As for heuristic approaches, the most widely used are: artificial neural networks [27], genetic algorithms [28,29] evolutionary programming [30,31], simulated annealing [32], fuzzy systems [33],

particle swarm optimization [34,35], Tabu search [36], and hybrid methods [37-38]. The complete analysis about contributions on deterministic and heuristic methodologies for solving the UC problem is provided [10,11].

Yet, the approaches proposed so far are not always able to solve real world problems to optimality in acceptable computational times. In this paper firefly method is proposed for solving the unit commitment problem coordinating thermal-wind-hydro units. The problems adopted for these units with constraints are explained in [41]. This can be identified as given a number of thermal power generators and a specified time-variant demand over the planning time horizon, in order to reduce the operational costs and computational time while meeting out demand. This paper is outlined as follows. Section 2 affords a detailed description of the mathematical formulation for the proposed approach which includes all the constraints of hybrid systems. Section 3 defines the meta-heuristic optimization approach called firefly algorithm. Section 4 presents computational tests with the proposed optimization approach. In Section 4.1, the performance of the firefly algorithm is demonstrated. In Section 4.2 two application examples are presented with the proposed technique, compared with other methods. Finally, Section 5 contributes the general conclusions.

2. Problem Formulation

The foremost goal of resolving the hybrid units scheduling problem is to govern when to startup and shutdown units so that the total operating cost can be diminished, while simultaneously determine substantially their system and generator constraints. This section explicitly frameworks the mathematical problem formulation for solving Unit Commitment problem. It also comprises binary and continuous variables significantly. Consider a set of thermal generating units I and a time-varying demand over T time periods convey the planned time horizon over twenty-four hours however the units be numbered as $i = 1, \dots, I$ and the time intervals as $t = 1, \dots, T$.

The main objective function is formulated and it subject to different constraints.

$$\text{Min Cost} = \sum_{i=1}^I \sum_{t=1}^T F_i + SU_{i,t} + SD_{i,t} \quad (1)$$

Where $F_i = a_i * y_{i,t} + b_i * P_{i,t} + c_i * P_{i,t}^2$

2.1 Thermal constraints

The constraints associated with thermal units are discussed below and these to be satisfied. The power demand for each time period is

$$PD_t \leq \sum_{i=1}^I P_{i,t}; t = 1, \dots, T \quad (2)$$

The spinning reserve requirement is guaranteed by the available capacity of active units:

$$PD_t + Res_t \leq \sum_{i=1}^I P_i y_{i,t}; t = 1, \dots, T \quad (3)$$

The power generation limits for each unit at each time period are given by

$$y_{i,t} P_i^{\min} \leq P_{i,t} \leq P_i^{\max} y_{i,t} \quad (4)$$

The minimum uptime and downtime constraints are given below

$$y_{i,t} = 0 \forall i : T_i^{\text{initial}} < 0; j = 1, \dots, (TD_i + T_i^{\text{initial}}) \quad (5)$$

$$y_{i,t} = 1 \forall i : T_i^{\text{initial}} > 0; j = 1, \dots, (TU_i - T_i^{\text{initial}}) \quad (6)$$

where T_i^{initial} is the number of periods that unit i has been initially switched off ($T_i^{\text{initial}} < 0$) or turned on ($T_i^{\text{initial}} > 0$).

The constraints (7) and (8) model the unit minimum uptime for the general case and for the time period respectively.

$$y_{i,t} - y_{i,t-1} \leq y_{i,t+j}; \quad i = 1, \dots, I; t = 2, \dots, T; j = 1, \dots, (TU_i - 1) \quad (7)$$

$$y_{i,1} \leq y_{i,1+j} \forall i : T_i^{\text{initial}} < 0; j = 1, \dots, (TU_i - 1) \quad (8)$$

Similarly, constraints (9) and (10) model the minimum down time for the units

$$y_{i,t+j} \leq y_{i,t-1} - y_{i,t-1} + 1; i = 1, \dots, I; \\ t = 2, \dots, T; j = 1, \dots, (TD_i - 1) \quad (9)$$

$$y_{i,1+j} \leq y_{i,1} + j \forall i: T_i^{initial} > 0; \\ j = 1, \dots, (TD_i - 1) \quad (10)$$

The start-up cost function defined as hot start cost if down time is less than $(TD_i + T_i^{cold})$ and cold start costs, the cost function of various constraints are explained from equation (11)-(20).

$$(y_{i,t} - y_{i,t-1})H_{sci} \leq SU_{i,t}; i = 1, \dots, I; t = 2, \dots, T \quad (11)$$

$$(y_{i,1})H_{sci} \leq SU_{i,1}; \forall i: T_i^{initial} < 0 \quad (12)$$

If unit i is turned on at the time period t and downtime at that moment is greater than $(TD_i + T_i^{cold})$, equations (15) and (16) impose $CU_{i,t}$ to be greater or equal than the cold start cost C_{sci} ,

$$\left(y_{i,t} - \sum_{j=TD_i+T_i^{cold}+1}^{t-1} y_{i,t-j} \right) C_{sci} \leq SU_{i,t}; \\ i = 1, \dots, I; (TD_i + T_i^{cold}) < t \leq T \quad (13)$$

$$\left(y_{i,t} - \sum_{j=TD_i+T_i^{cold}+1}^{t-1} y_{i,t-j} \right) C_{sci} \leq SU_{i,t}; \\ (TD_i + T_i^{cold} + 1) < t \leq (TD_i + T_i^{cold}) \quad (14)$$

Equation (15) ensures that variables takes value zero when the unit i is not turned on at the time period t :

$$0 \leq SU_{i,t}; i = 1, \dots, I; t = 1, \dots, T \quad (15)$$

Since, variables $SU_{i,t}$ are only involved in (11) to (15), the optimization procedure will ensure that the cost expected will be exactly zero for each case. Some units can incur in a shut-down cost when cost when they are turned off. This is modeled from equations (16) and (17). Constraint (18) prevents variable $SD_{i,t}$ taking negative values if the unit is not shut down at that time period t ,

$$(y_{i,t-1} - y_{i,t})D_{ci} < SD_{i,t}; t = 2, \dots, T \quad (16)$$

$$(1 - y_{i,t})D_{ci} < SD_{i,1} \forall i: T_i^{initial} > 0 \quad (17)$$

$$0 \leq SD_{i,t}; i = 1, \dots, I; t = 1, \dots, T \quad (18)$$

Similar to the start-up cost, after optimization $SD_{i,t}$ exactly takes either of the values 0 or D_{ci} . Finally, the specification on the variables is as follows,

$$0 \leq P_{i,t}; i = 1, \dots, I; t = 1, \dots, T \quad (19)$$

$$y_{i,t} \in \{0,1\}; i = 1, \dots, I; t = 1, \dots, T \quad (20)$$

The mathematical formulation for elucidating thermal unit commitment problem is provided from (1) -(20).

2.2 Hydro constraints

The constraints for Hydraulic balance is

$$V_{hk}(t+1) = V_{hk}(t) + Ah_k(t) - Q_{hk}(t) - Kh_k(t) \quad (21)$$

The constraints for initial and final reservoir,

$$V_{hk}(1) = V_{hk}^{start} \quad (22)$$

$$V_{hk}(T) = V_{hk}^{end} \quad (23)$$

The generation of hydroelectric power

$$P_{hk}(t) = \rho k \times Q_{hk}(t) \quad (24)$$

Storage and turbine volume limit:

$$V_{hk,L} \leq V_{hk}(t) \leq V_{hk,U} \quad (25)$$

$$Q_{hk,L} \leq Q_{hk}(t) \leq Q_{hk,U} \quad (26)$$

2.3 Wind Generator constraints

The wind power curve constraints:

$$P_{wi}^*(t) = \begin{cases} 0 & v(t) \leq v_{inl} \text{ or } v(t) > v_{outl} \\ \phi(v(t)) & v_{inl} \leq v(t) \leq v_{ratedl} \\ P_{wi}^{max} & v_{ratedl} \leq v(t) \leq v_{outl} \end{cases} \quad (27)$$

Total available power generations:

$$P_{wi}^*(t) = \sum_{j=1}^{NW} P_{wi}^*(t) \quad (28)$$

Total actual wind generation limits:

$$0 \leq P_w(t) \leq P_w^*(t) \quad (29)$$

3. Firefly algorithm

Firefly algorithm is one of the meta-heuristic nature inspired approach among the most powerful

algorithms for solving optimization complications. The Firefly algorithm is a novel technique motivated by the behavioral activities of fireflies. This was developed by Xin-She Yang at Cambridge university which is stochastic optimization approach. He particularized the flashing nature of fireflies attracted towards each other on their searching area. It is very affluent to implement and find optimum solution precisely. This proposed nature inspired algorithm was advanced by using three significant guidelines. They are categorized as follows.

- Every member in firefly family is assumed to be as unisexual and their attraction is irrespective of their sex.
- The amount of the attractiveness of each firefly towards other is analogous to brightness, and accordingly for any two flashing fireflies, the less bright one will step towards the brighter one. Thus, for more illumination the distance between two fireflies to be lesser. Yet, if any two fireflies hold the identical intensity, then they move randomly.
- The light intensity of a firefly is determined by the assessment of the objective function. For a maximization problem, the brightness is proportional to the consequence of the objective function and vice versa.

3.1 Attractiveness and intensity of light:

The foremost parameters related with this proposed algorithm are peculiarity in light intensity and the initiation of the attractiveness of individual firefly. Here, the intensity of light is represented as $I(r)$ differs with distance r of each firefly monotonically and exponentially.

$$I(r) = I_0 e^{-\gamma r^2} \quad (30)$$

Where I_0 is the initial intensity and γ is the absorption coefficient of light. Meanwhile attractiveness of firefly is relative to its brightness grasped by adjacent fireflies. At this time, the attractiveness ' β ' of a firefly can be stated below.

$$\beta(r) = \beta_0 e^{-\gamma r^2} \quad (31)$$

3.2 Distant between the fireflies

The distant between any two fireflies to be u and v at X_u and respectively, can be categorized as Cartesian distance. Here, d is the number of dimensions, $X_{u,n}$ is the n th component of the spatial coordinate of X_u and $X_{v,n}$ is the n th component of X_v of v^{th} firefly.

$$r_{uv} = \sqrt{\sum_{n=1}^d (X_{u,n} - X_{v,n})^2} \quad (32)$$

3.3 Movement of firefly

If one of the firefly u is get attracted by another brighter firefly v and its movement will be determined by using below equation.

$$X_u' = X_u + \beta(r)(X_u - X_v) + \alpha(rand - 0.5) \quad (33)$$

Here, the second term states an attraction of fireflies where the third term specifies randomization process having a step length factor α and r is a random number which is uniformly distributed in $[0, 1]$.

3.4 Algorithm of proposed technique

The main steps of the proposed algorithm are as follows:

- Step1: Start the program.
- Step 2: Initialize the population of fireflies randomly and set the control parameters of fireflies(γ, β_0), maximum number of generation, size of fireflies.
- Step 3: Check the individuals for feasibility of solution in which generation is greater than the load demand. If it is infeasible, such individuals are excluded and generate a new random population.
- Step 4: If the solution is feasible, then examine the minimum up time/down time constraints to be satisfied.
- Step 4: Compute the status of attractiveness of each individual using the equation (30).
- Step 5: Adjust the position of individuals and evaluate the fitness function
- Step 6: Select the brighter firefly and obtain the minimum cost function.
- Step 7: Obtain the best commitment of units and go for scheduling of next generation.
- Step 8: If the maximum number of iteration is attained, stop the process otherwise go to step (4).
- Step 9: Stop the program.

The flowchart of the proposed algorithm is shown in Figure 1.

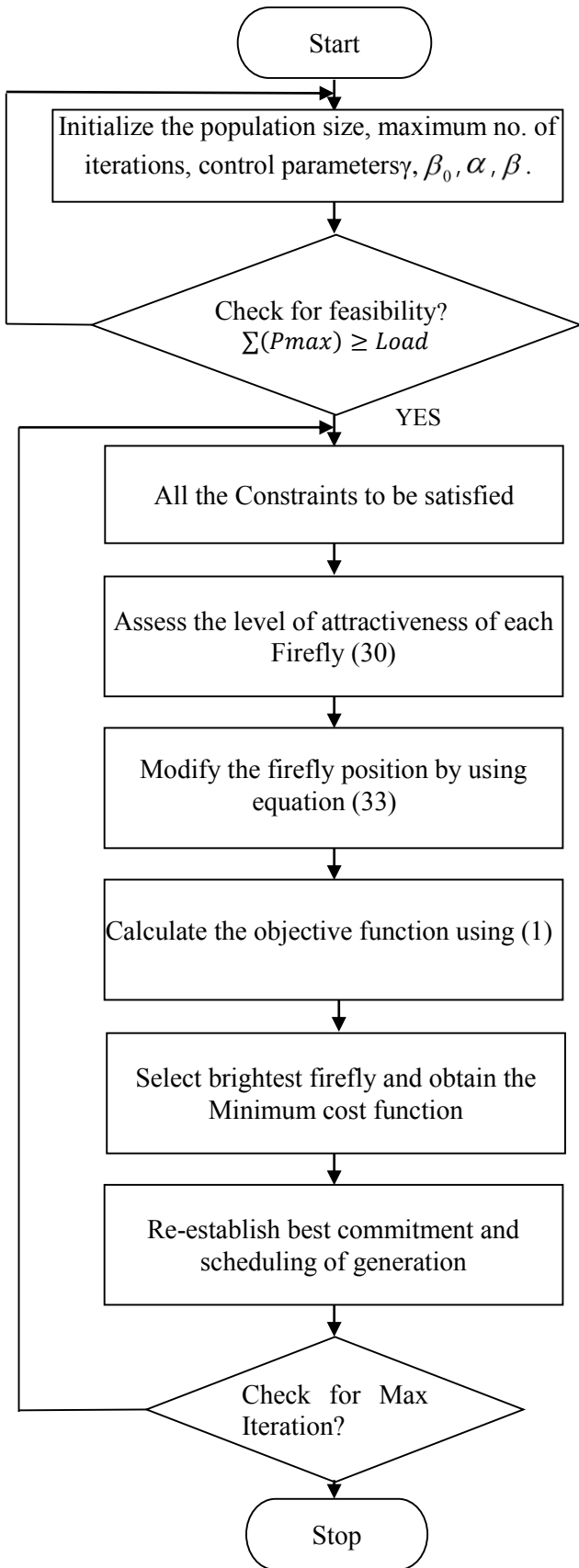


Figure 1: Flowchart for proposed algorithm

4. Results and Discussion:

The proposed firefly algorithm is verified using MATLAB 2013a 2.5GHz, 8GB, i7 intel core processor, in order to validate the feasibility of available generating units. In preference to this purpose, various case studies are instigated and their solutions are simultaneously related with other deterministic and non-deterministic approaches.

4.1 Performance of Firefly

In order to demonstrate the execution of the proposed firefly approach, primarily implemented to trim down the operating cost on evident instances of a UC problem. This compresses generating units which includes thermal, wind and hydro units over a scheduling time horizon of 24hours in a day. The foresighted power demand for the 38-unit system over a 24 hours are known in [39]. Furthermore, renewable energy resources such as wind, hydro units are assimilated with thermal units provides better economical savings. The optimal scheduling for the 42-units and 122 unit's system are listed on Tables A.1 and A.2 of Appendix A, respectively.

4.2 Test study 1:

The Proposed optimization technique FA is applied to solve the huge scale unit commitment problem and it is performed using MATLAB. In this test system, adopted from [39] is consists of 38 thermal units, 2 wind farms each consists of 10 units and 2 hydro units. The hourly load demand distribution on the system is given in [39]. During the hours 10,11,12,14,15 the load demand is at peak for about 7800,8000,8100,8150,8250 MW respectively. At this time periods, prior aim to mitigate these demands and provide service at ease as possible. For this, FA technique is employed to reduce total operating costs and computational time of each unit. The results gathered from firefly algorithm method is to compare by other methods.

4.2.1 Convergence characteristics:

The operating characteristics of 42 units (38T+2W+2H) system are accomplished and is shown in figure 2. This explains total generation cost versus maximum number of iterations. The convergence rate is fast on smaller dimensions. On conclusion, FA technique is more adequate than other conventional approaches.

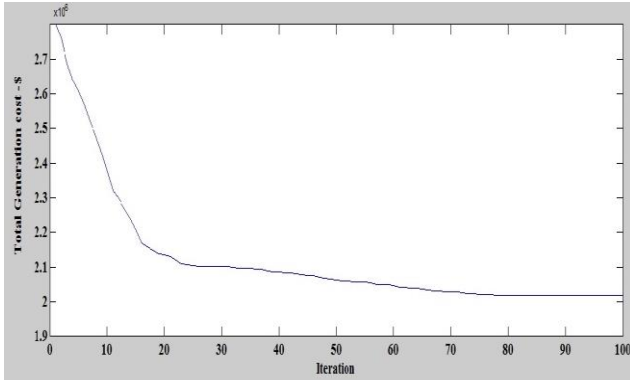


Figure 2: Total generation cost Vs Iteration for 42 units

4.2.2 System optimality:

The proposed approach is based on metaheuristic method which predominantly involves population size and maximum number of iterations. Here, we are considering *pop size* up to 50 and *max iter* as 100. Thus different population sizes such as 10, 20, 30, 40, 50 is implemented for different trails to obtain an optimal solution which is shown in Figure 3. From this analysis, pop 50 gets better optimal solution with reducing operational price and execution time.

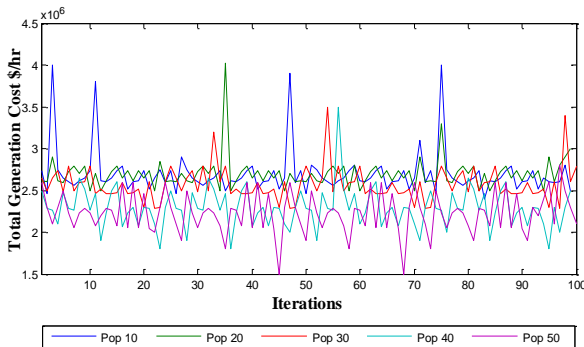


Figure 3: Population profile for 42 units

4.3 Test study 2:

In this sample system, holding of 100 thermal units correlated with 2 wind farms possessing 10 units each, 2 hydro units for ensuring the optimality of feasible solution. In order to implement this case study, 10 thermal units are scheduled as each unit are multiplied by 10 according to the load variations over a peak time period. To simulate this sample, FA is predominantly involved to reduce the generating tariff and estimating time. On distinguishing with other methods shown in Table 1, explains the proposed technique curtails the generating expenditure (\$) and executing duration of operating units.

4.3.1 Convergence characteristics:

The convergence characteristics of 122 units (100T+20W+2H) are obtained and it is showcased in figure 4.

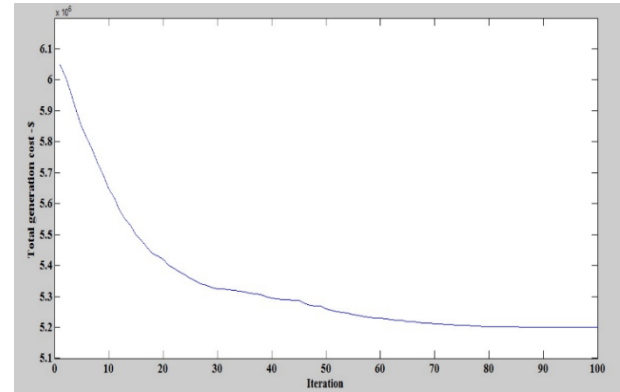


Figure 4: Total generation cost Vs Iteration for 104 units

4.3.2 System optimality:

In this test study, considering *pop size* 10, 20, 30, 40, 50 for different iterations for the best fittest solution. Here, pop size 50 implicates an optimal solution for various trials. Furthermore, if increase in population size reduces the performance and its convergence rate is too slow.

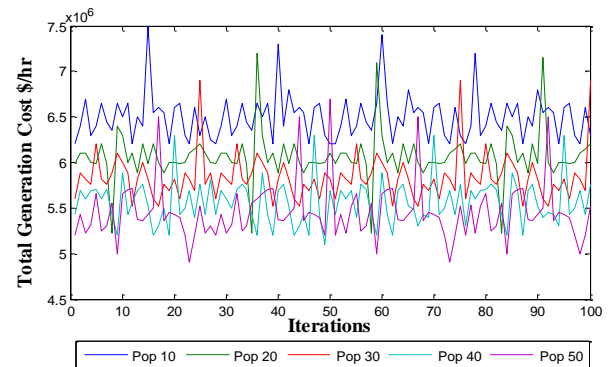


Figure 5: Population profile for 122 units

No.of Units	Methods	Computational time s	Operating cost \$
10	BC	0.4	563990
	FA	0.38	494134
20	BC	4.0	1124858
	FA	3.75	1035200
30	BC	14.0	1683532
	FA	12.5	1398457
40	B&C	19.7	2243688
	FA	18.0	1938010
50	BC	25.8	2801238
	FA	23.3	2350000
60	BC	37.9	3361951
	FA	35.8	2960554
70	BC	52.1	3921228
	FA	49.8	3156082
80	BC	22.4	4480798
	FA	21.1	3843000
90	BC	23.4	5040234
	FA	21.5	4534000
100	BC	28.6	5597993
	FA	26	4867980

Table 1: Comparison of B&C and FA approach

5. Conclusions

In this work, a heuristic optimization approach consisting of FA has been proposed to satisfy all basic constraints of unit commitment problem and to curtail the operating cost and response time of generating units. This imposes a flashing behavior of firefly under specified manner. The suggested approach is proved to highly efficient, since it could achieve global optimality in all cases tested. The performance of firefly is predominant using binary and continuous variables. Its implementation provides minimal generating cost and a reasonable computation time.

Nomenclature

Indexes

i	Unit index
t	Time index

Constants

I	total number of thermal generating units
T	length of the planning time horizon

a_i, b_i, c_i	coefficients of fuel cost function of unit i
D_t	power load demand for time period t
R_t	spinning reserve required at time period t
p_{Li}	minimum power generation of unit i
p_{Ui}	maximum power generation of unit i
T_{Ui}	minimum uptime of unit i
T_{Di}	minimum downtime of unit i
$T_{i\text{ ini}}$	initial status of unit i
$T_{i\text{ cold}}$	cold start hours of unit i
D_{ci}	shut-down cost of unit i

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APPENDIX

	Hours																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
THERMAL UNITS																								
1	500.0	450.0	450.0	450.0	450.0	450.0	500.0	475.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0	550.0
2	500.0	450.0	450.0	435.0	439.0	400.0	400.0	475.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
3	499.0	450.0	450.0	435.0	439.0	392.0	450.0	439.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
4	499.0	450.0	450.0	435.0	439.0	392.0	450.0	439.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0	500.0
5	499.0	450.0	450.0	400.0	400.0	392.0	392.0	439.0	500.0	500.0	500.0	500.0	495.0	494.0	495.0	498.0	498.0	500.0	450.0	500.0	500.0	440.0	400.0	400.0
6	499.0	450.0	450.0	400.0	400.0	392.0	392.0	439.0	500.0	500.0	500.0	500.0	495.0	494.0	495.0	498.0	498.0	500.0	450.0	500.0	500.0	440.0	400.0	400.0
7	499.0	500.0	500.0	400.0	450.0	450.0	392.0	500.0	500.0	500.0	500.0	500.0	495.0	494.0	495.0	498.0	498.0	500.0	450.0	500.0	500.0	440.0	400.0	400.0
8	499.0	500.0	450.0	400.0	450.0	450.0	392.0	500.0	500.0	500.0	500.0	500.0	495.0	494.0	495.0	498.0	498.0	500.0	450.0	500.0	500.0	440.0	400.0	392.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	490.0	487.0	496.0	0.0	500.0	500.0	500.0	481.0	483.0	433.0	483.0	477.0	440.0	400.0	350.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	490.0	487.0	496.0	0.0	500.0	500.0	500.0	481.0	483.0	433.0	483.0	477.0	448.0	398.0	350.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	150.0	160.0	160.0	160.0	160.0	150.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	405.0	405.0	405.0	435.0	421.0	435.0	435.0	405.0	405.0	205.0	205.0	405.0	205.0	205.0	205.0	205.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	317.0	475.0	475.0	495.0	495.0	495.0	495.0	475.0	475.0	275.0	275.0	275.0	275.0	275.0	275.0	275.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	351.0	351.0	351.0	350.0	350.0	350.0	350.0	351.0	301.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	82.0	82.0	82.0	82.0	82.0	82.0	82.0	82.0	176.0	176.0	176.0	186.0	0.0	236.0	336.0	176.0	176.0	176.0	176.0	176.0	176.0	176.0	176.0	176.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	184.0	184.0	184.0	184.0	184.0	184.0	184.0	184.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	266.0	266.0	166.0	166.0	166.0	166.0	166.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0
21	266.0	266.0	166.0	166.0	166.0	166.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0	266.0
22	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
23	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0	190.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0
26	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0	110.0
27	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0	58.0
28	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
29	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
31	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
32	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
33	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
34	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
35	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
37	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	31.0	31.0	31.0	31.0	31.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
38	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0	27.0
WIND UNITS																								
39	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0	33.0
40	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0	9.0
HYDRO UNITS																								
41	60.0	54.0	48.0	42.0	36.0	42	48	54	60	66.0	72.0	60.0	66.0	72.0	66.0	60.0	54.0	48.0	42.0	36.0	42.0	48.0	54.0	60.0
42	48.0	48.0	54.0	54.0	48.0	42	36	42	48.0	54.0	54.0	48.0	54.0	54.0	54.0	48.0	42.0	36.0	42.0	48.0	54.0	48.0	48.0	48.0

A.1: Optimal scheduling of 42 units

	Hours																							
Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
THERMAL UNITS																								
1	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
2	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
3	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
4	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
5	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
6	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
7	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
8	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
9	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
10	432.6	433.2	432.9	433.8	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	443.4	433.2	432.6
11	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
12	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
13	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
14	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
15	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
16	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
17	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
18	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
19	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
20	215.0	265.0	365.3	365.0	356.6	406.6	426.6	425.4	424.2	423.0	422.4	424.2	423.0	435.0	435.0	415.0	365.0	415.0	435.0	435.0	435.0	415.0	365.0	315.0
21	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
22	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
23	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
24	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
25	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
26	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
27	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
28	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
29	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
30	0.0	0.0	0.0	100.0	100.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	102.4	103.0	89.2	90.4	91.6	115.0	115.0	115.0	100.0	50.0	0.0
31	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
32	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
33	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
34	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
35	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
36	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
37	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
38	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
39	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
40	0.0	0.0	0.0	0.0	50.0	100.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	115.0	50.0	50.0	100.0	106.6	106.6	105.4	59.8	0.0	0.0
41	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
42	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
43	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
44	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
45	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
46	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
47	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
48	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
49	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	150.0	150.0	150.0	150.0	150.0	150.0	50.0	0.0	0.0	0.0	50.0	150.0	150.0	0.0	0.0	0.0
51	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	75.0	75.0	50.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	50.0	0.0	0.0	0.0

