# Input-Output Linearizing Control Based on a Newly Extended MVT Observer Design: an Induction Motor Application

**Okba ZEGHIB [1], Abdelkrim ALLAG [1], Meriem ALLAG [2], Bilal HAMIDANI [1]**

1 LABTHOP Laboratory, Department of Electrical Engineering, El Oued University, BP 789, El Oued, Algeria

2 LMSE Laboratory, Department of Electrical Engineering, Biskra University, BP 145, Biskra, Algeria

Tel: +213669888256

E-mail:okba92.ce@gmail.com

***Abstract*** *In this paper, a nonlinear control strategy of the Induction Motor (IM) which associate an input-output linearizing control conception under a PWM inverter with a new nonlinear extended observer design. At first, a nonlinear control performed using a nonlinear feedback linearization technique that controls separately the flux and the speed. Secondarily, a new extended observer approach relies on the Mean Value Theorem (MVT) using the sector nonlinearity to exhibit the error dynamics as a convex theory association of known matrices with time-varying parameters after representing the nonlinear system as a Takagi-Sugeno (TS) model. Using the Lyapunov theory, the stability terms are obtained and expressed in form of linear matrix inequalities (LMIs), the observer gain is gotten by solving the LMIs. With two line currents and the rotor speed are measured, the extended observer estimates with all IM machine states, the load torque, and the rotor position. Finally, the suggested approach is applied to IM machine through an illustrative simulation to affirm the effectiveness of the concept.*

***Key words****: Induction Motor (IM), input-output linearizing control, extended observer design, Mean Value Theorem (MVT), Linear Matrix Inequalities (LMIs).*

## 1. Introduction

The induction motor (IM) has the most uses of electrical machines in industrial applications due to its simple, their high efficiency, high power density, and higher torque to weight ratio. However, IMs are multivariable nonlinear, strongly coupled time-varying systems and the rotor variables are not measurable. Owing to these and more IM control is so hard [1, 2].

Due to these disadvantages of nonlinear systems, many authors are tried to transfer the nonlinear and coupling systems as specific forms of systems, such as Lipschitz systems. different approaches have been after studied, among them, the high gain observer in [3] and the use of the Lie Algebraic Transformation [4]. To reach a more precise, robust and fast states estimation performance of the IM for control of IM, many strategies were suggested in literature among them: extended sliding mode observer, extended Kalman filter, MRAS (system adaptive reference model) observer, nonlinear Luenberger observer and others. In detail, An extended Sliding mode observer to estimate rotor flux for nonlinear IM control was submitted in[5]. A nonlinear control using an extended Kalman filter method applied to IM was illustrated in [6]. In [7] the authors proposed an MRAS observer for IM control that estimates the speed and the rotor flux. A Nonlinear Luenberger observers were proposed for IM speed servo drive [8]. A nonlinear observer was used to estimate the IM flux which proved to be satisfactory[9]. A technique relied on the changing of the original system into a linear was proposed [10]. In all mentioned observers methods above it was so difficult to reach them owing to the strong conditions under which these transformations existed.

The TS fuzzy approach was extensively used to nonlinear systems [11, 12]. The basic idea was to transfer the nonlinear system model into a series of linear subsystems with associated nonlinear weighting functions [13, 14]. The Lipschitz nonlinear systems' class was exhibited [15, 16]. An adaptive resilient observer for a Lipschitz nonlinear system was designed [15], while in [16] the authors proposed an observer based on differential mean value theorem for a nonlinear system. An observer design based LMIs was submitted in[16] for a class of Lipschitz nonlinear dynamical systems.

In this paper, a state feedback linearizing controller of the induction motor is used in combination with a state extended observer designed via the MVT approach combined with a transformation via the sector nonlinearity. The observer gain matrices are determined as a solution of linear matrix inequalities (LMIs) (was obtained from the Lyapunov theory) to ensure that the observer error dynamics converge toward zero. The master idea of this paper is to find the extended observer gain so as that the nonlinear and coupling model of the IM machine makes as in linear model feedback control theory so as for the nonlinear control of the IM. The main advantage of the MVT theory is to find an observer gain which is calculated offline and doesn't depend on the states machine contrarily as in other technics for the nonlinear systems.

This paper is arranged as followings: the mathematical model of IM is performed in Section 2, Sections 3 provides the nonlinear controller design and show the newly extended observer design respectively. In Section 4, simulation results are submitted to prove the effectiveness of suggested approach. At last, conclusions are noted in Section 5.

## 2. Mathematical model of induction motor

The choice of the IM model is related to the objective of the used approach. The extended model chosen contains the usual dynamic model of the IM in a $d$, $q$ synchronous reference [2] with the dynamics of the load torque and the rotor position:

$$\dot{x}_e = f_e(x_e) + BU \tag{1}$$

Where:

$$f_e(x_e) = \begin{bmatrix} -\gamma i_{sd} + w_s i_{sq} + \frac{k_s}{\tau_r}\Psi_{rd} + k_s n_p w_r \Psi_{rq} \\ -w_s i_{sd} - \gamma i_{sq} - k_s n_p w_r \Psi_{rd} + \frac{k_s}{\tau_r}\Psi_{rq} \\ \frac{M}{\tau_r} i_{sd} - \frac{1}{\tau_r}\Psi_{rd} + (w_s - n_p w_r)\Psi_{rq} \\ \frac{M}{\tau_r} i_{sq} - (w_s - n_p w_r)\Psi_{rd} - \frac{1}{\tau_r}\Psi_{rq} \\ \frac{n_p M}{J L_r}(\Psi_{rd}\, i_{sq} - \Psi_{rq}\, i_{sd}) - \frac{f}{J} w_r - \frac{1}{J} T_L \\ 0 \\ w_r \end{bmatrix} \tag{2}$$

$$x_e = [i_{sd} \quad i_{sq} \quad \Psi_{rd} \quad \Psi_{rq} \quad \omega_r \quad T_L \quad \Theta_r]^T \tag{3}$$

And

$$B = \begin{bmatrix} \frac{1}{\sigma Ls} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sigma Ls} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T U = \begin{bmatrix} U_{ds} \\ U_{qs} \end{bmatrix} \tag{4}$$

$$\gamma = \left(\frac{1}{\sigma\tau_s} + \frac{1-\sigma}{\sigma\tau_r}\right), \quad k_s = \frac{M}{\sigma L_s L_r}, \quad \tau_r = \frac{L_r}{R_r},$$

$$\tau_s = \frac{L_s}{R_s}, \quad \sigma = 1 - \frac{M^2}{L_s L_r}$$

## 3. Input-output linearization control design based on a newly extended MVT observer

### 3.1. Feedback linearizing controller for IM

#### 3.1.1. The choice of outputs

The choice of outputs is according to the objectives of control. The rotor speed is chosen as the first output while the second output selected is the square of the rotor flux so as for tracking the purposed control trajectory.

$$y(x) = \begin{bmatrix} h_1(x_e) \\ h_2(x_e) \end{bmatrix} = \begin{bmatrix} w_r \\ (\Psi_{rd}^2 + \Psi_{rq}^2) \end{bmatrix} = \begin{bmatrix} w_r \\ \Psi_r^2 \end{bmatrix} \tag{5}$$

#### 3.1.2. Coordinates transformation

Defining the following transfer of coordinates that's given the next equations system:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} h_1(x_e) \\ \dot{h}_1(x_e) \\ h_2(x_e) \\ \dot{h}_2(x_e) \end{bmatrix} = \begin{bmatrix} \omega_r \\ \frac{n_p M}{J L_r}(\Psi_{rd}\, i_{sq} - \Psi_{rq}\, i_{sd}) - \frac{f}{J} w_r - \frac{1}{J} T_L \\ \Psi_r^2 \\ \frac{2}{\tau_r}\left(M(\Psi_{rd}\, i_{sq} - \Psi_{rq}\, i_{sd}) - (\Psi_{rd}^2 + \Psi_{rq}^2)\right) \end{bmatrix} \tag{6}$$

We can obtain the nonlinear state feedback control of IM as follow (for more details, see [2, 17])

$$\begin{bmatrix} U_{ds} \\ U_{qs} \end{bmatrix} = D(x_e)^{-1}\left[-A(x_e) + \begin{matrix} v_d \\ v_q \end{matrix}\right] \tag{7}$$

Where:

$$D(x_e) = \frac{1}{L_s - \frac{M}{L_r}} \begin{bmatrix} \frac{d\Psi_{rq}}{dt} & \frac{d\Psi_{rd}}{dt} \\ \frac{2M}{\tau_r}\frac{d\Psi_{rd}}{dt} & \frac{2M}{\tau_r}\frac{d\Psi_{rq}}{dt} \end{bmatrix} \quad (8)$$

And

$$A(x_e) = \begin{bmatrix} \left(\frac{4}{\tau_r^2} + \frac{2k_s}{\tau_r^2} M\right)(\Psi_{rd}^2 + \Psi_{rq}^2) - \left(\frac{6M}{\tau_r^2} + \frac{2\gamma M}{\tau_r}\right)(\Psi_{rd}^2\, i_{sd} - \Psi_{rq}\, i_{sq}) + \frac{2n_p\, w_r M}{\tau_r}(\Psi_{rd}^2\, i_{sq} - \Psi_{rq}\, i_{sd}) + \frac{2M^2}{\tau_r^2}(i_{sq} - i_{sd}) \\ \frac{n_p\, M}{JL_r}\left((\Psi_{rd}^2\, i_{sq} - \Psi_{rq}^2\, i_{sd}) + n_p\, w_r(\Psi_{rq}\, i_{sd} - \Psi_{rq}\, i_{sq}) + n_p\, k_s w_r(\Psi_{rd}^2 - \Psi_{rq}^2)\right) - \frac{f}{J^2}(T_L - f w_r) \end{bmatrix} \quad (9)$$

To ensure a faultless tracking of the rotor speed and the square of the rotor flux trajectories, respectively. The variables $v_d$ and $v_q$ are calculated through [17]:

$$\begin{aligned} v_d &= -k_{11}(w_r - w_{r\,ref}) - k_{12}\left(\frac{dw_r}{dt} - \frac{dw_{r\,ref}}{dt}\right) \\ v_q &= -k_{21}(\Psi_r^2 - \Psi_{r\,ref}^2) - k_{22}\left(\frac{d\Psi_r^2}{dt} - \frac{d\Psi_{r\,ref}^2}{dt}\right) \end{aligned} \quad (10)$$

Where $k_{11}$, $k_{12}$, $k_{21}$, and $k_{22}$ could be determined so as to make assured the closed loop system stable and to have a fast response in trajectory tracking.

## 3.2. New extended MVT observer design

### 3.2.1. problem declaration

In this section, we present an effectual method for designing nonlinear systems' observers. Considering the following nonlinear system.

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u \\ y = Cx(t) \end{cases} \quad (11)$$

We can write (11) in the Lipchitzien form as (where this passage is well illustrated in [1, 18, 19] ):

$$\begin{cases} \dot{x}(t) = A_0 x(t) + B_0 u(t) + \sum_{i=1}^{r} \mu_i(x(t))\left(\bar{A}_i x(t) + \bar{B}_i u(t)\right) \\ y = Cx(t) \end{cases} \quad (12)$$

We can also present the state equation of the observer as follow:

$$\dot{\hat{x}}(t) = A_0\hat{x}(t) + B_0 u(t) + L_0(y(t) - \hat{y}(t)) + \sum_{i=1}^{r} \mu_i(\hat{x}(t))\left(\bar{A}_i\hat{x}(t) + \bar{B}_i u(t)\right) \quad (13)$$

The dynamic of the state error ($e(t) = x(t) - \hat{x}(t)$) can be written as:

$$\dot{e}(t) = (A_0 - L_0 C)e(t) + (\emptyset(x,u) - \emptyset(\hat{x},u)) \quad (14)$$

Where:

$$\emptyset(x,u) = \sum_{i=1}^{r} \mu_i(x(t))\left(\bar{A}_i x(t) + \bar{B}_i u(t)\right) \quad (15)$$

The stability analysis of (14) can't directly be affected. The aim is to find the gain $L_0$ of the observer (13) that stabilizes the dynamic of the state estimation error equation (14).

### 3.2.2. Mean value theorem

In this part, the MVT approach is presented so as to develop the extended observer gain $L_0$. The MVT approach has been well illustrated in[1, 18]. Appending the MVT approach to (14) the dynamics of the observer errors can be exhibited as follows:

$$\dot{e}(t) = \left((A_0 - L_0 C) + \sum_{i=1}^{n}\sum_{j=1}^{n} e_n(i)\, e_n^T(j)\frac{\partial \emptyset_i}{\partial x_j}(\xi^i)\right)e(t) \quad (16)$$

Combining the sector nonlinearity with the MVT approach, the dynamics of the observer errors (16) becomes as (for more details, see references [1, 18] ):

$$\dot{e}(t) = \sum_{r=1}^{r} \mu_i\big(\xi(t)\big)(A_0 - L_0C + A_i^*)e(t) \qquad (17)$$

Where: $r \leq 2^{n^2}$ and $\xi(t) \in [x, \hat{x}]$ .

The stability of the state estimation error (17) is studied so as to find the observer gain $L_0$ by applying the quadratic Lyapunov function that is given as follows:

$$V\big(e(t)\big) = e^T(t)Pe(t) \qquad (18)$$

The state estimation error asymptotically converges to zero if there exists a matrix $P = P^T > 0$ such as the following LMI be verified:

$$A_0^T P + PA_0 + A_i^{*T} P + PA_i - MC - M^T C^T + \alpha P < 0 \qquad (19)$$

For $\left(i = 1, \cdots, 2^{n^2}\right)$

Knowing that the extended observer gain is calculated as:

$$L_0 = P^{-1}M \qquad (20)$$

### 3.2.3. Extended observer design for IM

Firstly, the system is transformed in canonical form, then an observer is suggested to estimate the unknown extended states of the IM via the observer equation (13).

Secondly, it is assumed that the first, the second and the fifth components of the state vector are measured, those conduct to the output:

$$y(t) = Cx_e(t) \text{ such that } C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \qquad (21)$$

Where:

$$x_e = [i_{sd} \quad i_{sq} \quad \Psi_{rd} \quad \Psi_{rq} \quad \omega_r \quad T_L \quad \Theta_r]^T$$

The dynamics state estimation error is given as (17) where $A_i^*$ are obtained from:

$$\frac{\partial f_e}{\partial x}(\xi) = \sum_{i=1}^{5}\sum_{j=1}^{5} e_n^T(i)e_n(j)\frac{\partial f_{e_i}}{\partial x_j}(\xi^i) \qquad (22)$$

$$\frac{\partial f_e}{\partial x}(\xi) = \begin{bmatrix} -\gamma & \frac{\partial f_{e_1}}{\partial x_2}(\xi) & \frac{k_s}{\tau_r} & \frac{\partial f_{e_1}}{\partial x_4}(\xi) & \frac{\partial f_{e_1}}{\partial x_5}(\xi) & 0 & 0 \\ \frac{\partial f_{e_2}}{\partial x_1}(\xi) & \frac{\partial f_{e_2}}{\partial x_2}(\xi) & \frac{\partial f_{e_2}}{\partial x_3}(\xi) & \frac{k_s}{\tau_r} & \frac{\partial f_{e_2}}{\partial x_5}(\xi) & 0 & 0 \\ \frac{M}{\tau_r} & \frac{\partial f_{e_3}}{\partial x_2}(\xi) & -\frac{1}{\tau_r} & \frac{\partial f_{e_3}}{\partial x_4}(\xi) & 0 & 0 & 0 \\ 0 & \frac{\partial f_{e_4}}{\partial x_2}(\xi) & \frac{\partial f_{e_4}}{\partial x_3}(\xi) & -\frac{1}{\tau_r} & 0 & 0 & 0 \\ \frac{\partial f_{e_5}}{\partial x_1}(\xi) & \frac{\partial f_{e_5}}{\partial x_2}(\xi) & \frac{\partial f_{e_5}}{\partial x_3}(\xi) & \frac{\partial f_{e_5}}{\partial x_4}(\xi) & -\frac{f}{J} & -\frac{1}{J} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

With:

$$\frac{\partial f_{e_1}}{\partial x_2}(\xi) = n_p.\xi_5 + 2.k_1.\xi_2 \qquad \frac{\partial f_{e_1}}{\partial x_4}(\xi) = \frac{\partial f_{e_2}}{\partial x_3}(\xi) = k_s n_p.\xi_5$$

$$\frac{\partial f_{e_1}}{\partial x_5}(\xi) = k_s.n_p.\xi_4 + n_p.\xi_2 \qquad \frac{\partial f_{e_2}}{\partial x_1}(\xi) = -n_p.\xi_5 - k_1.\xi_2$$

$$\frac{\partial f_{e_2}}{\partial x_2}(\xi) = -\gamma + k_2\frac{\partial f_{e_5}}{\partial x_4}(\xi) = -\gamma - k_1.\xi_1 \qquad \frac{\partial f_{e_2}}{\partial x_5}(\xi) = -k_s.n_p.\xi_3 - n_p.\xi_1$$

$$\frac{\partial f_{e_3}}{\partial x_2}(\xi) = -k_2\frac{\partial f_{e_5}}{\partial x_1}(\xi) = k_1.\xi_4 \qquad \frac{\partial f_{e_3}}{\partial x_4}(\xi) = -\frac{\partial f_{e_4}}{\partial x_3}(\xi) = k_2\frac{\partial f_{e_5}}{\partial x_3}(\xi) = -k_1.\xi_2$$

$$\frac{\partial f_{e_4}}{\partial x_2}(\xi) = \frac{M}{\tau_r} - k_2\frac{\partial f_{e_5}}{\partial x_2}(\xi) = -\frac{M(\Psi_{rd}+1)}{\tau_r\,\Psi_{rd}}.\xi_3$$

The MVT approach gives the following matrix gain $L_0$ guaranteeing the exponential convergence of the suggested extended observer. The observer gain $L_0$ is obtained from (20) by solving the LMI problem (19):

$$L_0 = \begin{pmatrix} 1.7888\times10^{7} & -3.1442\times10^{4} & 4.9969\times10^{3} \\ -2.5728\times10^{4} & 1.6267\times10^{7} & -2.9907\times10^{6} \\ 2.3671\times10^{6} & -7.1504\times10^{3} & 1.0246\times10^{3} \\ -721.9369 & 2.1529\times10^{6} & -3.9564\times10^{4} \\ -2.6528\times10^{3} & 3.575\times10^{5} & 9.7711\times10^{6} \\ 6.04\times10^{4} & -1.136\times10^{8} & -3.9638\times10^{6} \\ -2.7277\times10^{-5} & 9.419\times10^{-4} & 0.9997 \end{pmatrix}$$

## 4. Simulation results

The proposed control technique based on the MVT extended observer (global scheme is shown in Fig. 1) are implemented and the simulation results are obtained under Matlab/Simulink environment. This technique is applied to the IM which has the followings parameters[13]:

| | | |
|---|---|---|
| Pole Pair | $n_p$ | *2* |
| Rotor Inductance | $L_r$ | *0.4718 H* |
| Stator Inductance | $L_s$ | *0.4718 H* |
| Rotor Resistance | $R_r$ | *4.30 Ω* |
| Stator Resistance | $R_s$ | *9.65 Ω* |
| Mutual Inductance | **M** | *0.4475 H* |
| Moment of Inertia | **J** | *0.0293 $Kg.m^{-2}$* |

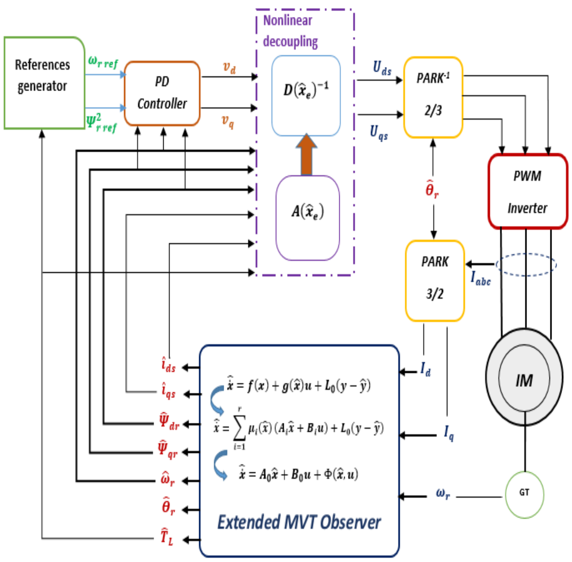


Fig. 1. Global diagram of the proposed controller based extended MVT observer applied to IM

The input-output linearizing controller based on the rotor flux( $\Psi_r$) which has a tracking state of 0.854 wb and the rotor speed ($\omega_r$) that has the desired trajectory begin by 50 rad/s and has increased to 100 rad/s at t=0.5s. The desired states are shown in blue in Fig. 2, 3. Initially, the motor is unloaded, after that, a load torque of 3Nm is applied to the IM at t = 0.3s that is offered in Fig. 7.

The suggested extended observer design is applied to the IM machine so as to estimate all ordinary IM states ($i_{ds}$, $i_{qs}$, $\Psi_{rd}$, $\Psi_{rq}$ and $w_r$) and moreover the load torque ( $T_L$) and the rotor position ( $\theta_r$). In Figs. 2 to 8, the real states have the red line, while the dashed green line indicates the estimated states.

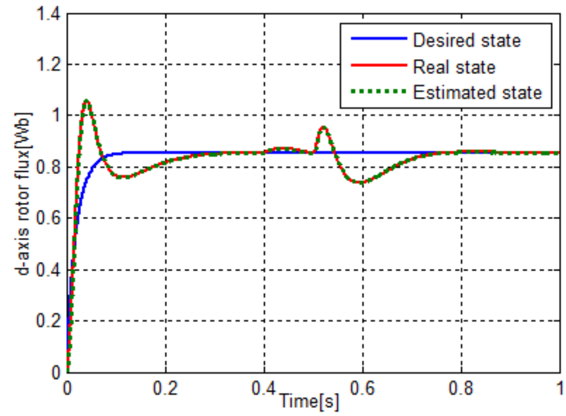


Fig. 2. The desired rotor flux (blue), d-axis rotor flux and its estimation

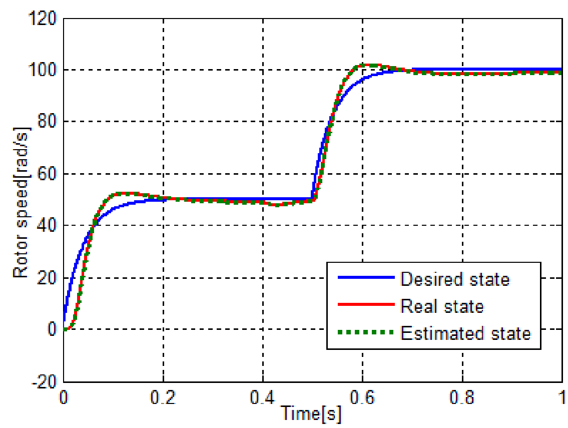


Fig. 3. Rotor speed, its desired and its estimation

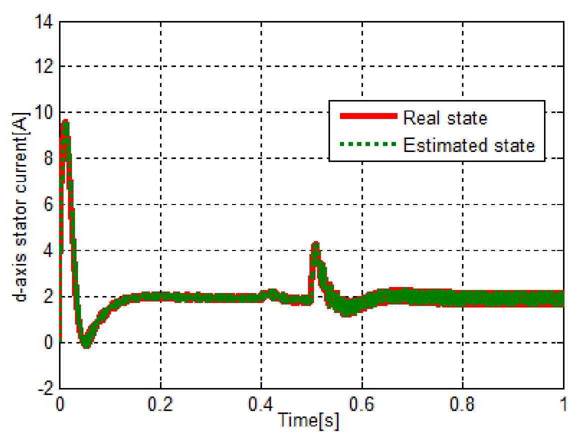


Fig. 4. d-axis stator current and its estimation

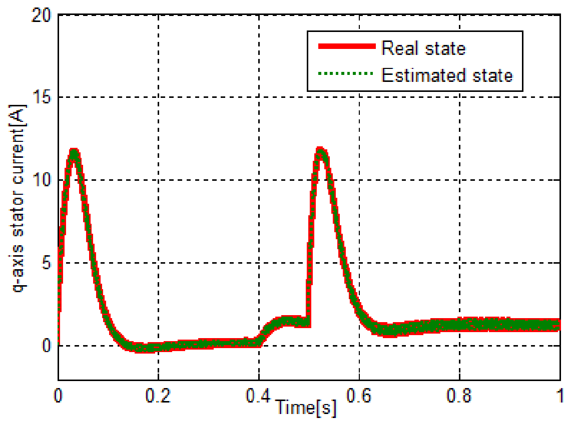

Fig. 5. q-axis stator current and its estimation

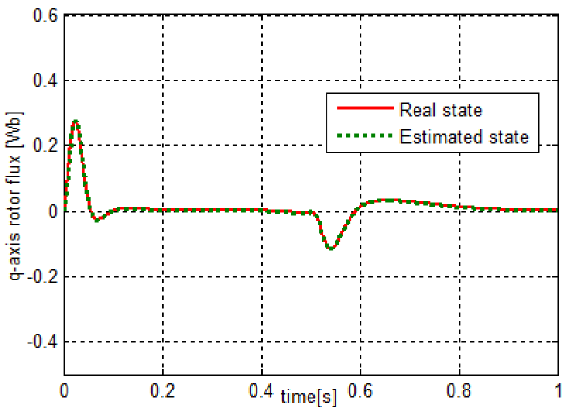

Fig. 6. q-axis rotor flux and its estimation

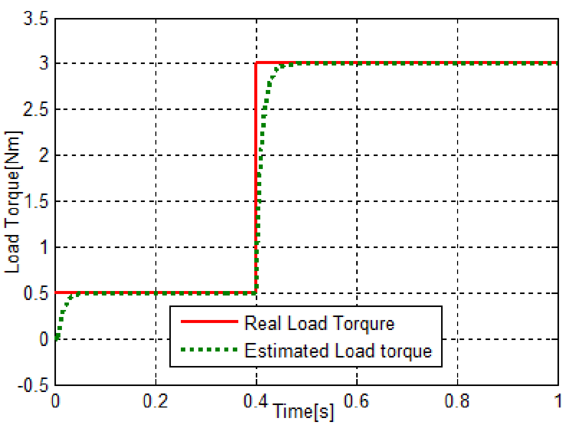

Fig. 7. Load torque and its estimation

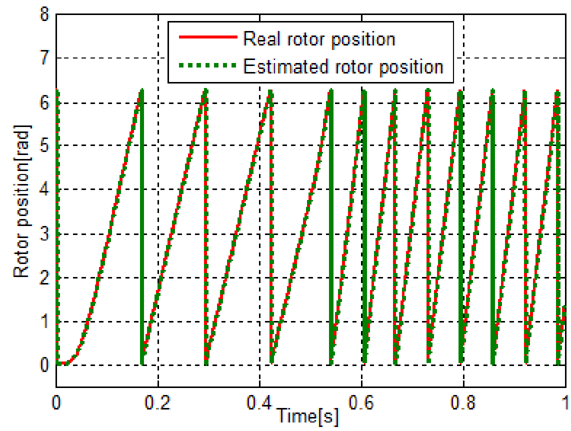

Fig. 8. Rotor position and its estimation.

By analyzing the simulation results, the obtained performance of the rotor speed and the rotor flux tracking are very appropriate (Fig2 and 3). The Fig.2 to 8 present that the estimation errors approximately converge to zero with a fast response time. The results described above prove the effectiveness of the MVT extended observer for a very complex nonlinear system that is the induction motor. However, low cost and fast digital signal processors making an easy implementation comparatively with the other approaches (sliding mode, MRAS, etc.).

## 5. Conclusion

The concept of input-output linearization and decoupling control is applied to the induction motor drive across a PWM inverter is distinctly presented. The new nonlinear extended observer based on MVT combined with the sector nonlinearity which estimates the extended IM machine states (the classical IM machine states, the load torque, and the rotor position) has been exhibited. The concept of the controller and the newest extended MVT observer have been implemented to the IM (powered by a PWM inverter) under the Matlab/Simulink environment. The numerical simulations those have been presented reveal the effectiveness of the suggested nonlinear control and the newly extended MVT observer. In future work, the testing of the suggested algorithm in real experimental trials will be taken into consideration.

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