BIFURCATION ANALYSIS OF BUCK-BOOST CONVERTER USING SYMBOLIC SEQUENCE METHOD AND COMPLEXITY MEASURES

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Abstract: *In this study, the complex behavior of buck-boost converter is investigated in detail using symbolic sequences and complexity measures. Input voltage is taken as bifurcation parameter for analysis. Primary and secondary symbolic sequences are built and utilized to categorize different types of smooth and non-smooth bifurcations. The concept of weight complexity and Lempel and Ziv (L-Z) complexity are implemented to quantify the bifurcation phenomenon of the system. The stability of the switching systems is known from the stability index based on weight and L-Z complexities. Applying block entropy measure, bifurcation types are identified and measured. The simulations and programming are performed using MATLAB software.*

Keywords: *Bifurcation, Block entropy, Border collision, Lempel–Ziv (L–Z) complexity, Switching block, Symbolic sequence.*

1. Introduction

Extensive researches on power electronics in renewable energy systems are emerging recently [1]. Besides, acquiring optimized output from power converters using various control schemes [2, 3] are the main concern of engineers today. Such switching powers are the rich source of exhibiting nonlinearity. For past few decades, nonlinear behavior such as bifurcations and chaos has been elaborated in those switching power converters [4-8]. It has become vital to analyze such unexpected behavior of switching

systems for engineers to design the stable system.

The abnormal dynamics in dc-dc converters are examined using many tools, including power spectrum [9], Lyapunov exponents [10] and fractal dimension. But they are too tough and time-consuming for the engineering applications. Initially, symbolic sequence is used to recognize border collision and bifurcation phenomena in power converters [11–13]. However, the previous literature involves waveform observation of the system to estimate the type of bifurcation. Most recently, detecting all types of bifurcation using duplicate symbolic sequence [14] has been reported. Moreover, a limited number of works has been performed using symbolic sequence method for bifurcation analysis. In this paper, a new thought of secondary symbolic block and the respective sequence is generated. And along with the primary symbolic sequence, all the bifurcation types are detected.

L-Z complexity [15, 16] and weight complexity measures based on the finite symbolic sequences has been utilized in this work, which concretely exposes the system behavior. As design engineers are more concerned about the system stability, the stability index based on weight and L-Z complexities is presented [14]. Also, various periodic motions and chaos are detected using block entropy.

This paper is organized as follows: section 2 deals with the concept of generating the primary and

secondary sequences. In Section 3, by utilizing some norms, the types of standard bifurcations and border collision bifurcations are detected. In section 4, bifurcation analysis of the system is explained based on the concept of weight and L-Z complexity. Section 5 explains the procedure for estimating the block entropy to examine the bifurcation and chaotic behavior of the system. To show the truthfulness of the concepts, in section 6, simulations results of system is illustrated. Finally, the conclusion of this study is described in section 7.

2. Primary and secondary symbolic sequences
2.1 Primary symbolic sequence
Primary switching block is generated by inspecting the operating modes in one switching period and the corresponding symbolic sequence is obtained by following primary switching block over infinite switching period. In dc–dc switching converters having two switching elements (switch and diode), the switching states that exist includes (on, off), (off, on) and (off, off). For discontinuous conduction mode as seen in Fig. 1(a), the sequence of switching states is (on, off) $\rightarrow$ (off, on) $\rightarrow$ (off, off) during one switching period, whereas the switching state (off, off) does not appear in continuous conduction mode as shown in Fig. 1(b).

Primary switching block is defined by representing the three switching states in one switching cycle using binary digits b_1, b_2, and b_3. Let $b_i = 0$ or 1 denotes whether the corresponding switching state exists or not during one switching cycle. Primary switching block is given as follows:
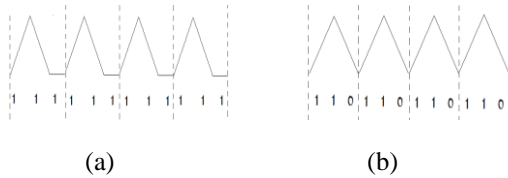

(a)   (b)
Fig. 1. Primary symbolic sequence for
(a) Discontinuous mode (b) Continuous mode

$$(b_1 b_2 b_3)_2 = (O)_{10} \qquad (1)$$

Where, O is a decimal value in the range of (0-7). The string of successive primary switching blocks over infinite switching periods, form primary symbolic sequence:

$$O = (O_0 O_1 O_2 \dots O_n \dots) \qquad (2)$$

2.2 Secondary symbolic sequence
The primary symbolic sequence reveals only the occurrence of border collision bifurcation. In particular, since the primary symbolic sequence only provides partial information about the dynamics of the system. To further identify the different types of standard and border collision bifurcation, the periodicity of waveforms is required. Hence, the concept of the secondary symbolic block and the respective sequence is established.

The inductor current i_L is sampled at integral multiples of switching period and the maximum inductor current is considered as a border current I_b. With reference to I_b as a threshold value, the secondary switching block is obtained. The secondary switching block at nth switching instant is defined as

$$l_n = \begin{cases} 1, & i_L < I_b \\ 0, & i_L \geq I_b \end{cases} \qquad (3)$$

And the corresponding secondary symbolic sequence established over infinite switching period is given by,

$$L = (l_0 l_1 l_2 \dots l_n \dots) \qquad (4)$$

The periodicity of the generated sequence is then determined for bifurcation analysis.

3. Bifurcation analysis based on symbolic sequences
With reference to literature [13], a dc-dc switching converter is defined by the iterative map $X_{n+1} = f(X_n, a)$, where a is a parameter, and X_n state variable sampled at integer multiples of switching time. When the converter experiences standard bifurcation as the parameter a is varied, the form of f remains unaltered before and after the bifurcation. Whereas, when the converter exhibits border collision bifurcation, the form of f alters as a is varied [13]. In the symbolic sequence analysis, as the primary symbolic sequence is generated based on the form of f, the onset of border collision bifurcation is easily detected by inspecting the primary symbolic sequence.

Further to distinguish some standard bifurcation types such as period doubling bifurcation and border collision bifurcation from period-n to period-n, period-m or chaos, the secondary symbolic sequence is generated, and it reveals the periodicity of the

waveform. Moreover, the order in which the sequences are analyzed to classify the bifurcation types is $O \rightarrow L$.

Suppose $a_1 < a_c < a_2$, where a_c signifies the critical parameter value. P_{oi} and P_{Li} denotes the periodicity of the primary and secondary symbolic sequences respectively for $a = a_i$ with $i = 1$ or 2. Then, by applying the norms as tabulated in Table I, all kinds of bifurcation are determined [14].

Table I Detecting Bifurcation types based on Symbolic Sequences

Standard Bifurcation	Period Doubling	$O_1=O_2$ $P_{O1}=P_{O2}$	$L_1 \neq L_2$ $2P_{L1}=P_{L2}$
	Saddle Node	$O_1=O_2$ $P_{O1}=P_{O2}$	$L_1 \neq L_2$ $P_{L1} >> P_{L2}$
	Hopf	$O_1=O_2$ $P_{O1}=P_{O2}$	$L_1 \neq L_2$ $P_{L1} << P_{L2}$
Border collision Bifurcation	Period-n to Period-n	$O_1 \neq O_2$ $P_{O1} \neq P_{O2}$	$L_1=L_2$ $P_{L1}=P_{L2}$
	Period-n to Period-m	$O_1 \neq O_2$ $P_{O1} \neq P_{O2}$	$L_1 \neq L_2$ $P_{L1} \neq P_{L2}$
	Period-n to chaos	$O_1 \neq O_2$ $P_{O1} << P_{O2}$	$L_1 \neq L_2$ $P_{L1} << P_{L2}$

4. Bifurcation analysis based on complexity measures

The concept of L–Z complexity and weight complexity is explored to study the nonlinear behavior of dc–dc switching converters. As the primary switching block reveals the information of complexity, the weight complexity is utilized to analyze primary symbolic sequence. Moreover, the secondary switching block is defined from the current level and represented as symbols; hence L-Z complexity is used to inspect secondary symbolic sequence.

4. 1 Lempel–Ziv complexity

Lempel and Ziv complexity measure C for a given finite secondary symbolic sequence $L = l_0 l_1 l_2 \dots l_{n-1} l_n$ is obtained by following the procedure given below [15, 16].

1) Let R and S denoted two subsequences of L, and RS signifies the concatenation R and S, while sequence $RS\pi$ is produced by excluding the last character from RS sequence (π denotes the operation of deleting the last character in the sequence). Let $V(RS\pi)$ denote the vocabulary of all different subsequences of $RS\pi$.

2) Let $R = l_0 l_1 l_2 \dots l_{r-1} l_r$ and $S = l_{r+1}$, in general, then $RS\pi = l_0 l_1 l_2 \dots l_r$; if S is a part of $V(RS\pi)$, then S is a subsequence of $RS\pi$, not a new sequence.

3) Renew S to be $l_{r+1} l_{r+2}$, and check whether S belongs to $V(RS\pi)$ or not.

4) Repeat the previous steps until S does not belong to $V(RS\pi)$. Now, $S = l_{r+1} l_{r+2} l_{r+3} \dots l_{r+i}$ is not a subsequence of $V(RS\pi) = l_0 l_1 l_2 \dots l_r$, so the separator • is included to separate the sequence as $RS = l_0 l_1 l_2 \dots l_r \bullet l_{r+1} l_{r+2} l_{r+3} \dots l_{r+i}$ and C is incremented by one.

5) Then, R is renewed to be $R = l_0 l_1 l_2 \dots l_{r+i}$ and $S = l_{r+i+1}$.

6) The aforementioned procedure is repeated till S is the last character. The number of different subsequences in L is the measure of complexity C.

4.2 Weight complexity

The weight complexity C_d is obtained by finding the sum of the maximal primary switching blocks of each subsequence in the new primary symbolic sequence [14]. The concept of weight complexity is enlightened with primary symbolic sequence $O = (676767676 \cdots) = (67)\infty$ as an example.

1) Initially, $O = (676767676 \cdots)$ is transformed as $O = (76767676 \cdots)$.

2) Next, take $R = O_1 = 7$, $S = O_2 = 6$, $RS = 76$ and $RS\pi = 7$, so S is not the subsequence of RSπ, and then, introduce the separator • on RS sequence, so $RS = 7 \bullet 6 \bullet$.

3) After that, let $R = O_1 O_2 = 76$, $S = O_3 = 7$, $RS = 767$ and $RS\pi = 76$, so S is the subsequence of $RS\pi$ and $RS = 7 \bullet 6 \bullet 7$.

4) Then, let $R = O_1 O_2 = 76$, $S = O_3 O_4 = 76$, $RS = 7676$ and $RS\pi = 767$, so S is the subsequence of $RS\pi$ and $RS = 7 \bullet 6 \bullet 76$.

5) Steps 2 and 3 are repeated, and then, finally get $O = 7 \bullet 6 \bullet 76767676 \cdots$.

6) So, this primary symbolic sequence has three subsequences. The maximal blocks in the three subsequences are 7, 6 and 7, respectively. Thus, the weight complexity of the primary symbolic sequence is $C_d = 7 + 6 + 7 = 20$.

4.3 Conditions based on complexity measures to differentiate bifurcation types

With the aid of complexity measures C and C_d, the bifurcation types are categorized with the following conditions [14].

1) If C changes and C_d does not change, the system undergoes standard bifurcations. To further identify the different types of standard bifurcation, the variation in C is analyzed. When the growth of C is less, the system will experience flip (period doubling) bifurcation. However, when C falls abruptly, the system will exhibit saddle-node bifurcation. Then, if C increases and the increment of C is more, the systems will exhibit slow frequency oscillation via Hopf bifurcation.
2) If C does not change but C_d changes, the system exhibit border collision bifurcation and jumps from a periodic motion to another periodic motion with a same periodic number of the former.
3) If both C and C_d changes, the converter is said to experience border collision bifurcation and jumps from a one periodic orbit to another periodic orbit with a different periodic number of the former or chaotic orbit.

4.4 Stability index of DC–DC switching converters based on complexity measures

The primary switching block shows the converter switching states in one switching period, which also infers the degree of the system stability. That is, weight complexity C_d reflects the stability of the system, and it is inversely proportional to the stability degree. When C_d is low, the system's stability is high and vice-versa. The L–Z complexity derived from secondary switching sequence also has a similar relation to the stability degree. With the variation of bifurcation parameter beyond the critical value, C is high, which further imply that the system losses its stability i.e., stability degree is low. The stability index is given as follow [14]:

$$SI = \frac{C_O\,C_{do}}{C\,C_d} \qquad (5)$$

Where, C_0 and C_{d0} are the values of C and C_d, when the system functions in stable period-1operation. The period-1 orbit has the highest stability $SI = 1$.

5. Analysis of chaotic behavior using block entropy

Block entropy scheme is implemented in the secondary symbolic sequence to analyze the dynamics of the system. The algorithm for computing block entropy is illustrated in Fig. 2. Initially, the symbolic sequence is grouped to form a subsequence of a suitable length L, and then, such a combination of symbolic codes of the same length is moved as shown in the Fig. 2. Next, each subsequence is converted into decimals, and the probabilities of various decimal

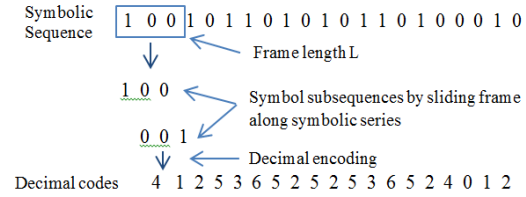codes representing subsequences in series are found [17].

Fig. 2. Block entropy algorithm

Let P_i denotes the probability of a decimal code i in a series

$$P_i = \frac{number\ of\ deciaml\ code\ i}{Total\ number\ of\ decimal\ codes} \qquad (6)$$

According to Shannon's measure, the uncertainty of the system operation is quantified by the block entropy H_L which is given as follows [17]:

$$H_L = -\sum_{i=1}^{N} P_i\, log_2 P_i \qquad (7)$$

Where, $N = 2^L$. Furthermore, the block entropy provides the quantitative descriptions of a sequence.

6. Applied example

The circuit diagram of the current mode controlled buck-boost converter is shown in Fig. 3. To analyze the chaotic behavior of the system, the parameters chosen is tabulated in Table II. The investigation of bifurcation phenomena is carried out by keeping input voltage (E) as the bifurcation parameter. Varying E in the range of (10-50) V, a bifurcation diagram is plotted and the onset of border collision bifurcation is also depicted in the Fig. 4. From this diagram, it is observed that the

Table II
Specification of current controlled buck-boost converter

Parameters	Values
Supply Voltage	50V
Switching Frequency	20 kHz
Load Resistance	20
Output Voltage	35V
Duty Ratio(Buck mode)	0.4
Load current	(2-4)A
Inductance	0.5mH
Capacitance	4µF

converter exhibits period doubling cascade as the value of E is varied from 45V to 24.4V. Further, the converter bifurcates to chaos if $E < 24.4$V. When E is in the range of 12.2V to 11.8V, a small periodic window is examined between the chaotic regions,
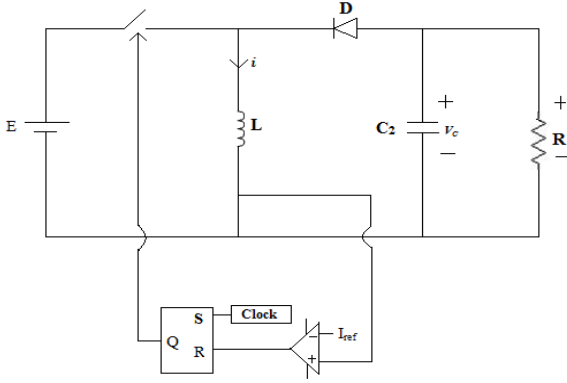
Fig. 3. Circuit diagram of a current controlled buck-boost

Table III
Primary and Secondary Symbolic Sequences

Input voltage E (V)	Primary Sequence	P_o	Secondary Sequence	P_L
(50-42.5)	$(6)\infty$	1	$(0)\infty$	1
(42.4-28.5)	$(6)\infty$	1	$(01)\infty$	2
(28.4-25.3)	$(46)\infty$	2	$(01)\infty$	2
(25.2-24.4)	$(6664)\infty$	4	$(1011)\infty$	4
(24.3-23.5)	$(6664)\infty$	4	$(10111111)\infty$	8
(23.4-12.3)	∞	$+\infty$	∞	$+\infty$
12.2	$(466)\infty$	3	$(101)\infty$	3
12	$(466)\infty$	3	$(101111)\infty$	6
11.8	$(446466)\infty$	6	$(101111)\infty$	6
$E<11.8$	∞	$+\infty$	∞	$+\infty$

where the converter undergoes period-3 to period-6 operation.

The simulated result showing the generation of primary and secondary symbolic sequences for different values of the input voltage is illustrated in the Fig. 5 and Fig. 6, respectively.

6.1 Analysis of bifurcation behavior based on primary and secondary symbolic sequences

The primary and secondary symbolic sequences and their periodicity for different ranges of input voltages are identified and tabulated in Table III. It can be inferred from the table that for the range of (50-42.5) V, the converter exhibits period-1 operation. As the input voltage (E) is decreased from 42.5V to 42.4V, there is no change in the periodic number of primary sequence (P_o) i.e., $P_{O1} = P_{O2}$ but periodicity of secondary sequence (P_L) is found to be doubled i.e., $2P_{L1} = P_{L2}$. This implies that period doubling, a type of standard bifurcation has occurred. When E is reduced from 28.5V to 28.4V, $P_{O1} \neq P_{O2}, P_{L1} = P_{L2}$. This shows the occurrence of period-2 to period-2 border collision bifurcation. It is depicted in Fig. 6(c), and also confirms from the bifurcation diagram shown in Fig. 4(a). When E is changed from 25.3V to 25.2V, $P_{O1} \neq P_{O2}$ and $P_{L1} \neq P_{L2}$, it communicates that border collision bifurcation has occurred, and jumps from period-2 orbit to period-4 orbit. Moreover, when E is further reduced from 24.4V to 24.3V, $P_{O1} = P_{O2}$ and $2P_{L1} = P_{L2}$, exemplifies that converter exhibits period doubling bifurcation, and forks from period-4 orbit to period-8 orbit.

The converter enters chaotic region when $E < 23.5V$. After the intermittent chaos, a small periodic window is observed, which also exhibits period

doubling cascade, is embedded in the chaotic region. In the periodic window, the system shows period-3 operation at input voltage $E = 12.2V$. As E is decreased to 12V, $P_{O1} = P_{O2}$ and $2P_{L1}=P_{L2}$ implying period doubling bifurcation have occurred as it jumps from period-3 orbit to period-6 orbit. When $E = 11.8V$, $P_{O1} \neq P_{O2}$ and $P_{L1} = P_{L2}$, indicates that system exhibits border collision bifurcation and forks from period-6 orbit to another period-6 orbit. For further decrement of E below 11.8V, both the sequences become more random. This shows that the system enters into the chaotic region again.
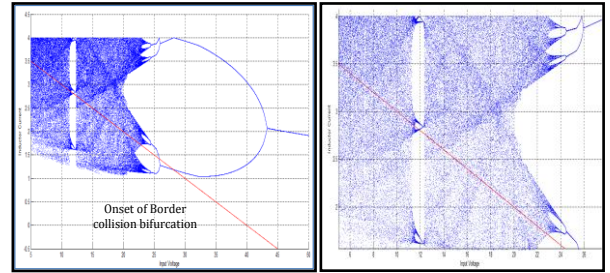

Fig. 4. (a) Bifurcation diagram (b) Zoomed form of (a), showing a periodic window

6.2 Analysis of bifurcation behavior based on complexity measures

The weight complexity measure C_d, L-Z complexity measure C, and stability Index (SI) for various range of
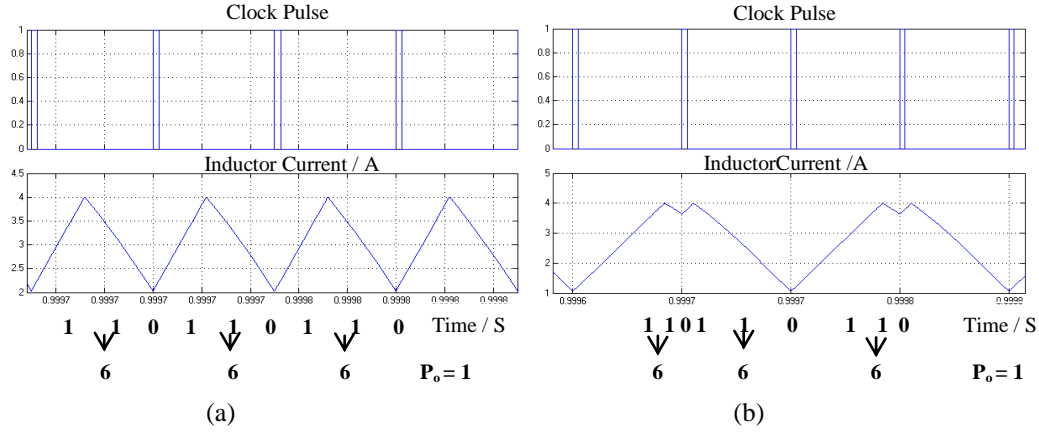
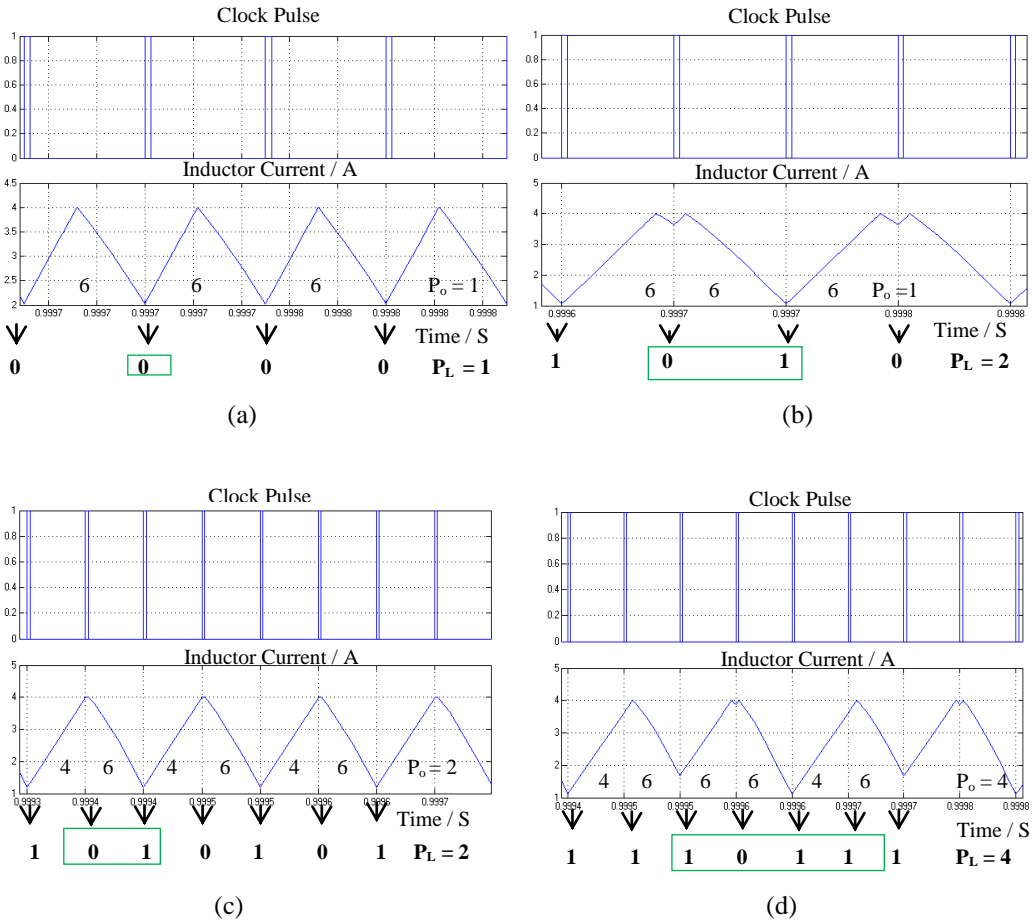Fig. 5. Simulated results showing primary symbolic sequence (a) Period-1 operation (b) Period-2 operation.

Fig. 6. Simulated results showing secondary symbolic sequence (a) Period-1operation (b) Period-2 operation (c) Period-2 to period-2 border collision (d) Period-4 operation
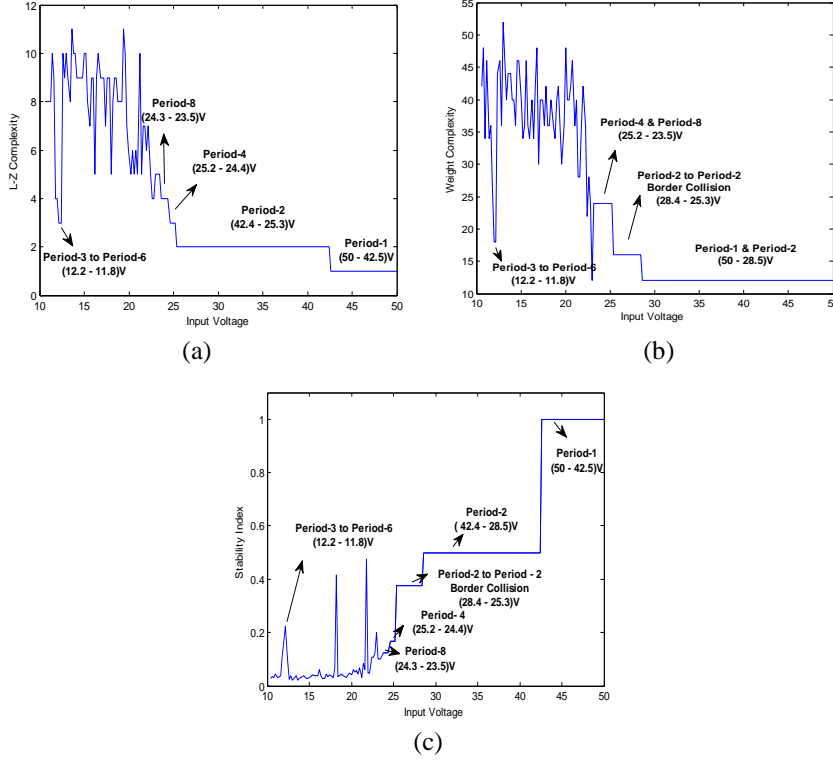
Fig. 7 (a) L-Z complexity plot (b) Weight complexity plot (c) Stability index plot

Table IV Complexity measures and *SI* for various values of *E*

E (V)	C	C_d	SI
(50-42.5)	1	12	1
(42.4-28.5)	2	12	0.5
(28.4-25.3)	2	16	0.375
(25.2-24.4)	3	24	0.1667
(24.3-23.5)	4	24	0.125
(23.4-12.3)	$+\infty$	$+\infty$	Decreases
12.2	3	18	0.2222
12	4	18	0.1667
11.8	4	26	0.1153
E < 11.8	$+\infty$	$+\infty$	Decreases

input voltage are presented in Table IV. When the input voltage is in the range of (50-42.5) V, the system is found to functions in the period-1 regime, and the corresponding values of C and C_d are taken as

C_o and C_{do}, respectively. With E decreased from 42.5V to 42.4V, it is observed that there is no change in C_d, but C increases. As the increment of C is less, it is understood that the system bifurcates to period-2 operation. When E is reduced from 28.5V to 28.4V, C_d changes, but C does not change. This reveals the occurrence of border collision (period-2 to period-2) at $E = 28.4$V. Both the measures C_d and C changes, when E is varied from 25.3V to 25.2V, showing the existence of period-2 to period-4 border collision bifurcation. Moreover, when E is further reduced from 24.4V to 24.3V, C_d remains the same but C increases by only one, it implies that system exhibits period doubling bifurcation, and branches from period-4 orbit to period-8 orbit. For $E < 23.5$V, the value of complexity measures increases abruptly, and this shows that the system enters the chaotic region. At $E = 12.2$ V, it is noticed that C_d decreases to 3, and C to 18 implying that period-3 operation has occurred after intermittent chaos. When $E = 12$V, C increases only by one, but C_d remains the same revealing that the

converter undergoes period doubling bifurcation as it jumps from period-3 orbit to period-6 orbit. For further decrement of E to 11.8V, C does not change and C_d changes. This infers that the system exhibits border collision bifurcation, and it jumps from period-6 orbit to another period-6 orbit. Below 11.8V, both the measures, again increases gruffly implying that the system enters into the chaotic region. The plots of complexity measures for variation in input voltage are shown in Fig. 7 (a, b).

Using equation (5), stability Index is calculated and plotted against bifurcation parameter E as depicted in Fig. 7(c). From this plot, it is clear that stability of the system is high (i.e., $SI = 1$) for stable operation and decreases gradually as the system bifurcates via period doubling cascade. During chaotic region SI is least. Study of stability of the system over the wide range of bifurcation parameter is beneficial for the design engineers.

6.3 Analysis of bifurcation behavior based on block entropy measure

With the help of block, entropy scheme applied on secondary symbolic sequence, the scrutiny of dynamics of the converter is carried out, and it highly correlate with results obtained based on L-Z and weight complexities. Using Shannon's measure (7), the block entropy algorithm is implemented and the entropy values are plotted against the bifurcation parameter as shown in Fig. 8. When the entropy value remains constant, the system is periodic. The entropy value rises with the evolution of chaotic behavior.
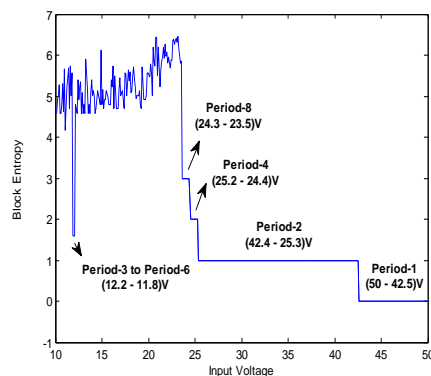

Fig. 8. Block entropy plot

7. Conclusion

In this work, the chaotic and bifurcation behavior of a current mode controlled buck-boost converter in continuous conduction mode is investigated using symbolic sequence method and complexity measures. It is shown that as the input voltage is varied, the system enters chaotic regime via period doubling route. With the help of primary and secondary sequences, the events of bifurcation phenomena are identified. The anticipated bifurcations scenario of the system is also confirmed using complexity measures. System stability is known from the stability index, which is inevitable for design engineers.

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