TUNING FUZZY PD^{α} SLIDING-MODE CONTROLLER USING PSO ALGORITHM FOR TRAJECTORY TRACKING OF A CHAOTIC SYSTEM

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***Abstract**: This paper deals with the comparison between fuzzy sliding mode controller (FSMC) and novel approach of FSMC using a PD^{α} sliding surface, for a trajectory tracking of a chaotic system. In order to alleviate the chattering phenomenon due to the discontinuity in the signum function, a Takagi-Sugeno fuzzy logic controller is used, and to ensure optimal performance in the closed loop system, the PSO algorithm is also used.*
Finally the effectiveness of the proposed approach of $FPD^{\alpha}SMC$-based PSO algorithm is demonstrated by simulation results.

Key words: *SMC, FSMC, $FPD^{\alpha}SMC$, PSO, chaotic system.*

1. Introduction

Fractional-order calculus is an area of mathematics; it has 300 years of history that deals with derivatives and integrals from non-integer orders. In the last two decades, fractional calculus has been rediscovered by scientists and engineers. It has been applied in a many number of fields, namely in the area of control theory such as, A Fractional Order PID Tuning Algorithm for A Class of Fractional Order Plants [1], A novel fractional order fuzzy PID controller and its optimal time domain tuning based on integral performance indices [2], Observer Based Control of a Class of Nonlinear Fractional Order Systems using LMI [3], Optimized wave-absorbing control: Analytical and experimental results [4].

Besides, the SMC for example was largely proved its efficiency through the reported theoretical studies [5], [6].

The first step of SMC design is to select a sliding surface that models the desired closed-loop performance in state variable space. The second step is to design the equivalent and a hitting control law such as the system state trajectories forced toward the sliding surface and slides along it to the desired attitude. In the literature, several methods for selecting sliding surface have been reported. The approach in [7, 8, 9, 10] uses a proportional-derivative type sliding surface, where the order of derivation is an integer. In [11] a nonlinear sliding surface for the coupled tanks system is adopted and given best results. Due to the fact that the fractional order calculus plays an important role in various domains, a PD^{α} sliding surface is proposed in [12] [29], and a novel fractional integral terminal sliding mode concepts for the output tracking problem of relative-degree-one systems with uncertainty and disturbance is presented in [30]. Also authors in [31] have proposed a novel fractional-order integral type sliding surface for robust stabilization/ synchronization problem of a class of fractional-order chaotic systems in the presence of model uncertainties and external disturbances.

Motivated by the above discussion this paper designs a sliding surface based on the fractional order proportional-derivative (PD^{α}) [12], the best choice of the order of the sliding surface can results a small output response, improves settling time and stability of the system.

Then, to make the developed surface globally attractive and invariant, the control law is designed.

An advantage of these methods of control (SMC) is their robustness to parameter perturbations and bounded external disturbances. The robustness is attributed to the discontinuous term in the control input. However, this discontinuous term also causes an undesirable effect called chattering.

Sometimes this discontinuous control action can even cause the system performance to be unstable. To reach a better compromise between small chattering and good tracking precision, various compensation strategies have been proposed.

For example, integral sliding control [13, 14, 15], a fuzzy sliding mode control strategy [16]. Though introducing a fuzzy logic controller and taking off the *sign* function in the hitting control law of SMC may reduce the chatter amplitude.

The selection of suitable parameters of fuzzy PD^{α}

sliding mode controller (FPD$^{\alpha}$SMC) is a significant problem, that it can be solved either by manually changing the values or to use some optimization methods [36],[37], in this paper we are interested by the particle swarm optimization algorithm (PSO).
The rest of this article is organized as follows. Basic definitions of fractional calculus are described in Section 2. The Fuzzy PD$^{\alpha}$ sliding mode controller design, in Section 3. After that the PSO approach is described in Section 4. The optimization of FPD$^{\alpha}$SMC with PSO in Section 5. And finally the simulation results and conclusion are given in Sections 6 and 7, respectively.

2. Basic definitions of fractional calculus

The fractional differ-integral operators denoted by ${}_aD_t^{\alpha}$ (Fractional calculus) are a generalization of integration and differentiation of the operators of a non integer order. In the literature we find different definitions of fractional differ-integral, but the commonly used are:
The Riemann-Liouville (RL) definition:

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \qquad (1)$$

The Caputo's definition:

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_a^t \frac{f^m(\tau)}{(t-\tau)^{1-(m-\alpha)}} d\tau \qquad (2)$$

Where $m-1<\alpha<m$ and $\Gamma(\cdot)$ is the well-known Euler's gamma function, and its definition is:

$$\Gamma(x) = \int_0^{\infty} e^{-t} t^{(x-1)} dt, \; x>0 \qquad (3)$$

Where, x is the order of the integration.
On the other hand, Grunwald-Letnikov (GL) reformulated the definition of the fractional order derivative as follows:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h \to 0} \frac{1}{h^{\alpha}} \sum_{k=0}^{(t-a)/h} (-1)^k \binom{\alpha}{k} f(t-kh) \qquad (4)$$

Because the numerical simulation of a fractional differential equation is not simple as that of an ordinary differential equation, so the Laplace transform method is often used as being a tool for the resolution of the problems arising in engineering [17, 18].
In the following section, we give the Laplace transforms of the fractional order derivative given previously.

The Laplace transform of RL definition is as follows [17],[32]:

$$\int_0^{\infty} e_0^{st} D_t^{\alpha} f(t) dt = s^{\alpha} F(s) - \sum_{k=0}^{m-1} {}_0D_t^{\alpha-k-1} f(t)_{t=0} \qquad (5)$$

Where $s=jw$ denotes the Laplace operator. For zero initial conditions, the Laplace transform of fractional derivative of Riemann-Liouville, caputo and Grunwald-Letnikov reduce to (6) [32],[33].

$$L\left({}_0D_t^{\alpha} f(t) \right) = s^{\alpha} F(s) \qquad (6)$$

In this paper the fractional order element s^{α} is approximated with Oustaloup's filter [19]. The Oustaloup's filter is based on the approximation of a function of the form:

$$G(s) = s^{\alpha}, \quad \alpha \in R^{+} \qquad (7)$$

By a rational function:

$$\hat{G}(s) = s^{\alpha} = K \prod_{k=-N}^{N} \frac{s+w_k'}{s+w_k} \qquad (8)$$

Where the parameters of this function (zeros, poles, and gain) can be determined by the following formulas:

$$w_k' = w_b \cdot \left({}^{w_h}\!/\!{}_{w_b} \right)^{(k+N+0.5(1-\alpha))/(2N+1)}$$
$$w_k = w_b \cdot \left({}^{w_h}\!/\!{}_{w_b} \right)^{(k+N+0.5(1+\alpha))/(2N+1)}, K = w_h^{\alpha} \qquad (9)$$

$(2N+1)$ is the order of the filter, w_b and w_h are respectively the low and high transient-frequencies. In this paper we consider the 5th order Oustaloup's rational approximation for the FO element within the frequency range $w \in \{10^{-2}, 10^{2}\}$.

3. Fuzzy PD$^{\alpha}$ sliding mode controller design

We consider the following stat-space representation of the second-order nonlinear system:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + b(x).u \end{aligned} \qquad (10)$$

Where $x = [x_1 \; x_2]^T$ is the state vector, $f(x)$ and $b(x)$ are nonlinear functions, u is the control input designed to track a command x_{1d}(t) closely. Without losing generality, assume $b(x) > 0$ for all x.

For this kind of system, we find several control methods, such as, fuzzy control, PID control, sliding mode control,…etc.

A. PD^{α} sliding mod controller (PD^{α}SMC)

For the system presented in Eq. (10), firstly we use the following PD^{α} sliding surface using Caputo's definition as:

$$S = D_t^{\alpha}(e) + k_p.e \tag{11}$$

Remark: it is clear that selecting $\alpha = 1$, a classical PD sliding surface can be recovered.

The fractional derivatives Caputo right hand definition (RHD) [34] of function $f(t)$ gives, $D_t^{\alpha} f(t) = D_t^{(\alpha-m)} \frac{d^m}{dt^m} f(t)$, where m is an integer greater than α. From this we can write the sliding surface S as follows:

$$S = D_t^{\alpha-1}(\dot{e}) + k_p.(e) \tag{12}$$

Where $e = x_1 - x_{1d}$, and k_p is a positive constant.

It is obvious from (11) that keeping system states on the sliding surface $S(x,t)$, $\forall\ t > 0$ will guarantee that the tracking error vector asymptotically approach to zero. The corresponding sliding condition is:

$$\frac{1}{2}\frac{d}{dt}S^2 = S.\dot{S} \le 0 \tag{13}$$

The general control structure that satisfies the stability condition of the sliding motion can be written as:

$$\begin{aligned} u &= u_{eq} + \frac{1}{b(x)} D_t^{(1-\alpha)}(u_h) \\ &= u_{eq} + \frac{1}{b(x)} D_t^{(1-\alpha)}(-K_s.\mathrm{sgn}(S)) \end{aligned} \tag{14}$$

Where u_{eq} is called the equivalent control law that is derived by setting $\dot{S} = 0$ and K_s is a positive constant. We refer to [32] for more details. Differentiating both sides of Eq (12) to the order unity yields the equality in (15)

$$\dot{S} = D_t^{\alpha-1}\ddot{e} + k_p.(\dot{e}) = D_t^{\alpha-1}(\ddot{x}_1 - \ddot{x}_{1d}) + k_p(\dot{e}) \tag{15}$$

From Eq(15) one can conclude that:

$$D_t^{(1-\alpha)}(\dot{S}) = (\ddot{x}_1 - \ddot{x}_{1d}) + k_p D_t^{(1-\alpha)}(\dot{e}) \tag{16}$$

By setting $\dot{S} = 0$, and substituting $\ddot{x}_1 = \dot{x}_2$, the equivalent control is obtained, and it has the flowing formula:

$$u_{eq} = \frac{-1}{b(x)}\left[f(x) - \ddot{x}_{1d} + k_p.D_t^{(1-\alpha)}(\dot{e}) \right] \tag{17}$$

To verify the stability analysis, substituting Eq(14) into Eq(10) yields:

$$\dot{x}_2 = \ddot{x}_1 = \ddot{x}_{1d} - k_p.D_t^{(1-\alpha)}(\dot{e}) - K_s.D_t^{(1-\alpha)}(\mathrm{sgn}(S)) \tag{18}$$

Eq(18) becomes

$$\ddot{x}_1 - \ddot{x}_{1d} + k_p.D_t^{(1-\alpha)}(\dot{e}) = -K_s.D_t^{(1-\alpha)}(\mathrm{sgn}(S)) \tag{19}$$

By using Eq (16), the Eq (19) can be rewritten as follows:

$$D_t^{(1-\alpha)}(\dot{S}) = -K_s.D_t^{(1-\alpha)}(\mathrm{sgn}(S)) \tag{20}$$

Differentiate (20) to the order $(\alpha - 1)$. Since $(\alpha - 1) \prec 0$ this indeed corresponds to fractional order integration, corresponding to negative valued α in Caputo's definition in (2), and taking into account the property of Caputo's derivative ${}_aD_t^{-\alpha}({}_aD_t^{\alpha} f(t)) = f(t) - \sum_{k=0}^{m-1} \frac{f^k(a)}{k!}(t-a)^k$; with $\dot{S}(0) = 0$; $\frac{d}{dt}(\mathrm{sgn}(s(0))) = 0$ for $m = 1$. This lets us have:

$$\dot{S} = -K_s.(\mathrm{sgn}(S)) + K_s.(\mathrm{sgn}(S(0))) \tag{21}$$

For $t = 0$, we have $\dot{S} = 0$ and for $t \succ 0$ we have $\dot{S} = -K_s.(\mathrm{sgn}(S))$

Thus by using (13) we can obtain:

$$\begin{aligned} S.\dot{S} &= S.(-K_s.(\mathrm{sgn}(S))) \\ &= -K_s.|S| \le 0 \end{aligned} \tag{22}$$

Lemma 1. [28] Consider the following autonomous linear fractional-order system:

$${}_0D_t^{\alpha} x(t) = A.x(t),\ x(0) = x_0 \tag{23}$$

Where $x \in R^n$, $A = (a_{ij}) \in R^{n\times n}, 0 \prec \alpha \prec 1$, is asymptotically stable if and only if (see figure 1):

$$|\arg(eig(A))| > \alpha.\frac{\pi}{2} \tag{24}$$

In this case, the components of the state decay towards 0 like $t^{-\alpha}$.

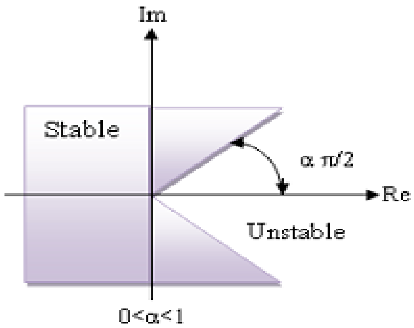


Fig. 1. Stable domain of fractional order system in s^{α} plane

Proof. When the sliding mode occurs, system (11) can be represented as follow:

$$D_t^{\alpha}(e) + k_p.e = 0 \qquad (25)$$

Therefore, the sliding mode dynamics is obtained by the following equation:

$$D_t^{\alpha}(e) = -k_p.e \qquad (26)$$

It can be seen that $|\arg(-k_p)| = \pi$, so the sliding surface parameter k_p is selected to be positive to satisfy the stability condition of Lemma 1.

In summary the used PD^{α} sliding surface can guarantee the stability in the sense of the Lyapunov theorem.

However, a large control gain *Ks* often causes the chattering effect. In order to tackle this problem, a continuous fuzzy logic control term Δu is used to approximate u_h.

B. Fuzzy PD$^{\alpha}$ Sliding Mode Controller (FPD$^{\alpha}$SMC)

The FPD$^{\alpha}$SMC is a hybrid controller; it can be regarded as a fuzzy regulator that controls the variable *S* approach to zero.

The structure of a fuzzy controller design consists of: 1) the definition of input-output fuzzy variables; 2) decision-making related to fuzzy control rules; 3) fuzzy inference logic; and 4) defuzzification.

For the proposed FPD$^{\alpha}$SMC we used the sliding surface *S* as the input at the fuzzy controller, and Δu is the fuzzy controller output. The structure is shown in figure 2:

Where:

$$\begin{aligned} u &= u_{eq} + u_f \\ &= u_{eq} + \frac{1}{b(x)} D_t^{(1-\alpha)}(\Delta u) \end{aligned} \qquad (27)$$

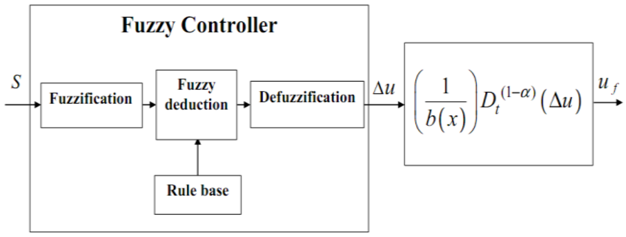


Fig. 2. Structure of the Fuzzy controller

Assuming that the input and output of the fuzzy controller has five level language variables, its membership function is shown in figure. 3.

Where ϕ and K are used to expand or shrink the divisions of the membership functions along the universes of discourse, *r* is a coefficient to be used to adjust the width of the input membership function of the linguistic variable Zero [24].

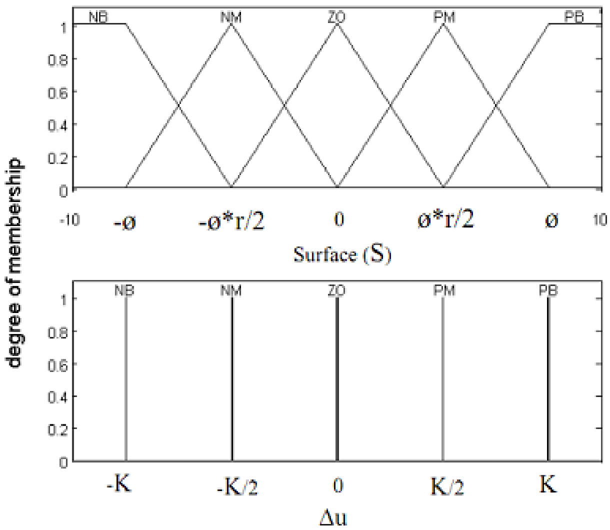
Such linguistic expressions can be used to form the fuzzy control rules as below:

Rule 1: IF S is **NB**, THEN Δu is **PB**.
Rule 2: IF S is **NM**, THEN Δu is **PM**.
Rule 3: IF S is **ZO**, THEN Δu is **ZO**.
Rule 4: IF S is **PM**, THEN Δu is **NM**.
Rule 5: IF S is **PB**, THEN Δu is **NB**.

Where **NB** denotes "Negative Big", **NM** denotes "Negative Mid", **ZO** denotes "Zero", **PB** denotes "Positive Big", and **PM** denotes "Positive Mid".

The FLC output (Δu) is determined using the weighted average method [11].

Fig. 3. Membership functions of input variable (S) and



FLC output (Δu) for the FPD$^{\alpha}$SMC

4. Particle swarm optimization (PSO)

Particle swarm optimization (PSO) is an evolutionary computation technique developed by Kennedy and Eberhart in 1995 [20]. The inspiration underlying the development of this algorithm was the social behaviour of animals, such as the flocking of birds and the schooling of fish, and the swarm theory. It has been proven to be efficient in solving optimization problem especially for nonlinearity and non differentiability, multiple optimum, and high dimensionality [21, 22].

In PSO, the velocity of each particle is modified iteratively by its individual best position (*pbest*), and the global best position (*gbest*) found by particles in its neighborhood. As a result, each particle searches around a region defined by its individual best position (*pbest*) and the global best position (*gbest*) from its neighborhood. Henceforth we use *Vi* to denote the velocity of the i^{th} particle in the swarm, *pi* denote its position. At each step (or iteration) *n*, by using the individual best position, (*pbest*), and global best position, (*gbest*), the velocity and position of each particle are updated by the following tow equations:

$$V_i(n) = W.[V_i(n-1) + c_1 r_1 (pbest_i - p_i(n-1)) + c_2 r_2 (gbest - p_i(n-1))] \quad (28)$$

$$p_i(n) = p_i(n-1) + V_i(n) \quad (29)$$

Where r_1 and r_2 are random numbers between 0 and 1; c_1 and c_2 are positive constant learning rates; *W* is called the constriction factor [23] and is defined by (30):

$$W = \frac{2}{\left|2 - c - \sqrt{c^2 - 4.c}\right|}, c = c_1 + c_2, c > 4 \quad (30)$$

In each step *n* the position is confined within the range of [p_{min}; p_{max}]. If the position violates these limits, it is forced to its proper values [21].

$$p_i = \begin{cases} p_{\min} & if \quad p_i < p_{\min} \\ p_i & if \quad p_{\min} < p_i < p_{\max} \\ p_{\max} & if \quad p_i > p_{\max} \end{cases} \quad (31)$$

Changing position by this way enables the i^{th} particle to search around its individual best position *pbest*, and global best position, *gbest*.

The following shows the design step for implementing the PSO algorithm [21].

Step 1. Initialize particles with random position and velocity on dimension in the problem space.

Step 2. If a prescribed number of iterations (generations) is achieved, and then stop the algorithm.

Step 3. For each particle, evaluate the desired optimization fitness function, and record each particle's best previous position (*pbest),* and global best position (*gbest*).

Step 4. Change the velocity and position according to equations (28) and (29) respectively, for each particle

Step 5. Check each particle's position using (31).

Step 6. Go back to Step 2.

5. Optimization of FPD$^{\alpha}$SMC with PSO

The design problem is defined as finding the optimum values of the fuzzy PD$^{\alpha}$ sliding mode controller parameters in the closed-loop system. The Parameters vector composed by the positions of the membership functions (when the conclusions are fixed), the gain k_p, and the fractional order α.

Let p_i = [ϕ, r, K, k_p, α] the vector of selective parameters of FPD$^{\alpha}$SMC, the regions of the decision variables are mentioned as follows.

$$0.1 < \phi < 10, \quad 0.1 < r < 1, \quad 0.1 < K < 20, \quad 0.01 < k_p < 20, \quad 0.1 < \alpha < 0.98$$

To converge toward the optimal solution, the PSO algorithm must be guided by the cost function. Hence, it should be properly defined before the PSO algorithm is executed.

In the present study, the used cost function (F_1) is defined by the following formula:

$$F_1 = \sum_{i=1}^{N} (\gamma_1 . |e(i)| + \gamma_2 . |u(i)|) \quad (32)$$

In order to compare the performances of the different controllers, we define the flowing cost functions:

$$F_2 = \sum_{i=1}^{N} (e^2(i)) \quad (33)$$

$$F_3 = \sum_{i=1}^{N} (u^2(i)) \quad (34)$$

Where *e(i)* is the trajectory error of i^{th} sample, *u(i)* is the control signal of i^{th} sample and *N* is the number of samples. The weighting factors γ_1 and γ_2 are used to give more flexibility to the designer depending on the nature of application and relative importance of low error and control signal.

6. Simulation results

In this section, we shall demonstrate that the FPD$^{\alpha}$SMC tuned with PSO is applicable to the problem of trajectory tracking control of a non-linear chaotic system. Tuning process with PSO is also applied to the Fuzzy Sliding Mode Controller (FSMC) using *PD* sliding surface.

For the robustness evaluation of the different controllers tuned by PSO algorithm, an external disturbances (d=0.2∗ $sin(t)$) is added to the system, and the parameters variation of the system is carried out, where the controllers' parameters tuned by PSO are kept unchanged.

The simulation is carried out using the"Matlab/Simulink" tools within 0.01 sample time. The population size of PSO algorithm is set to 20 particles. The parameters c_1; c_2 and W are set to 2.05, 2.05 and 0.7298 respectively, and the maximum number of iteration n is set to 40 iterations. The weighting factors γ_1 and γ_2 are set to 3 and 0.1 respectively. The cost function (32) is minimized for each of the FSMC and the FPD$^\alpha$SMC controllers with the corresponding controller parameters reported in Table 1.

Table 1
Optimal parameters for FSMC and FPD$^\alpha$SMC

		FSMC	**FPD$^\alpha$SMC**
Performance index	min(F_1)	113.9903	112.4680
	min(F_2)	0.5793	0.3064
	min(F_3)	2.6400×10^3	3.9092×10^3
Controller parameters	ϕ	0.1881	0.1000
	r	0.1000	0.3513
	K	12.5932	4.1112
	k_p	4.7645	10.9677
	α	-	0.9800

A. Example (Chaotic system)

In recent years, chaotic systems have attracted considerable interest and have been extensively investigated. An interesting subject in chaos theory is to eliminate the chaotic behavior by means of control systems [25], [26], [35], [38]. Consider a second-order chaotic system such as [27].

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.4x_2 + 1.1x_1 - x_1^3 + q.\cos(w_c.t) + u + d \qquad (35)$$

Where u the control signal and d is an unknown external disturbance assumed to be bounded as follows:

$$|d| \le D \qquad (36)$$

Simulation result of tracking control of the state $x_1(t)$ with a desired reference wave is shown in figure. 4 where q and w_c are set to 2.1 and 1.8 respectively. Figure. 5 shows the simulation result with parameters variation and adding external disturbance. The expression of the desired reference (x_{1d}) is given as follows:

$$x_{1d} = 0.12(\cos(\pi t) + \sin(\frac{2\pi t}{3})) \qquad (37)$$

Where the initial conditions ($x_1(0)$; $x_2(0)$) are set to (0.3, 0).

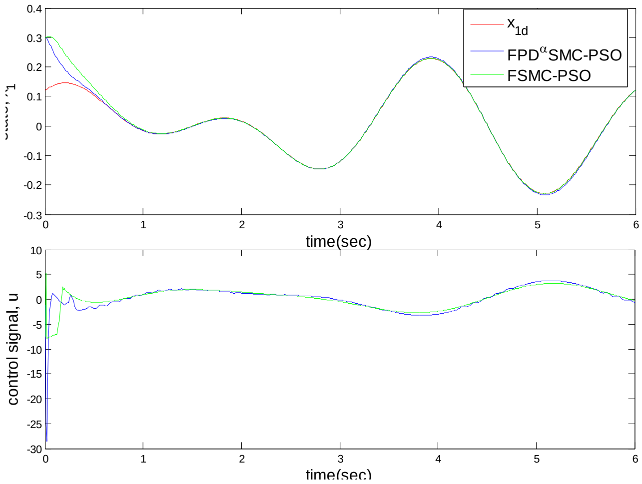


Fig. 4. Simulation result of the chaotic system.

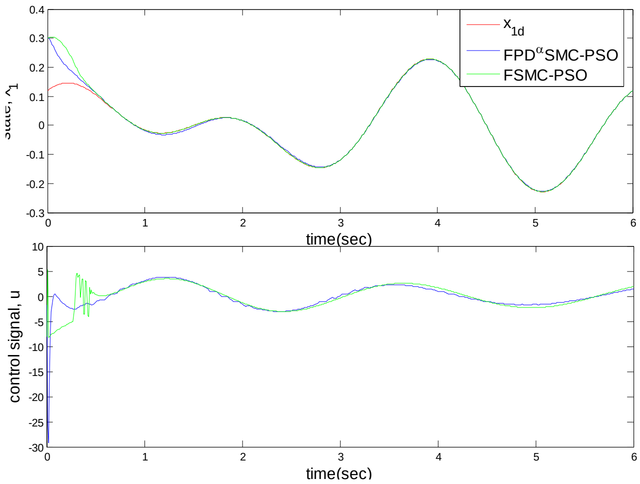


Fig. 5. Simulation result of the chaotic system, with parameters variation (q=3.2; w_c = 2.5) and adding external disturbance d.

From the simulation results, the FPD$^\alpha$SMC performs better control specification such as fast response compared with FSMC, also it is evident that the FPD$^\alpha$SMC outperforms the FSMC for trajectory tracking task.

However when compared with respect to small magnitude of control signal, the FSMC gives better results.

After adding external disturbance and changing in parameters of the system we could obviously find that the disturbance rejection ability of different controllers tuned with PSO.

7. Conclusion

In this paper a Fuzzy PD$^\alpha$ Sliding Mode Controller that combines the advantages in term of robustness of the fractional calculus, fuzzy logic for its ability to express the amount of ambiguity in human reasoning and sliding mod controller in term of robustness to parameters variation and external disturbances, is investigated for a chaotic system.

Firstly, PD$^{\alpha}$ surface sliding mode controller is used. The design yields an equivalent control term with an addition of fuzzy logic control to approximate the discontinuous control term and to alleviate the chattering phenomenon.

Then the application of the PSO method can perform an efficient search for the optimal parameters of both FSMC and FPD$^{\alpha}$SMC, and achieve good accuracy.

Finally, the simulation results show the effectiveness of the proposed controller algorithm for nonlinear chaotic systems.

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