PMSM Speed Controller based on Nonlinear Adaptive Backstepping

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Abstract—**In this paper, the speed control of a permanent magnet synchronous motor using an adaptive backstepping control design based on filed orientation is proposed. Firstly, the indirect field oriented control of permanent magnet synchronous motor (PMSM) is derived. Then, a novel adaptive backstepping control design technique is investigated to achieve a speed tracking objective under parameter uncertainties and disturbance of load torque. The effectiveness of the proposed control scheme is verified by numerical simulation. Simulation results clearly show that the proposed control scheme can track the speed reference signal generated by a reference model successfully under parameter uncertainties and load torque disturbance.**

Index Terms —**Permanent magnet synchronous motor, vector control, adaptive backstepping control, parameters uncertainties.**

I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are frequently used in industrial applications. Especially their compact design, high efficiency, high power/weight and torque/inertia ratios can be shown as the most important advantages of PMSMs. On the other hand, the high cost and their time-varying magnetic characteristics are the disadvantages of PMSMs [1–3]. The dynamic model of a PMSM is highly nonlinear because of the coupling between the motor speed and the electrical quantities, such as the d–q axis currents. The model parameters such as the stator resistance and the friction coefficient may also not be exactly known. Even worse, the load torque is always unknown. All these factors make controller design for a PMSM difficult when high speed and high precision are required in the real application.

The vector control technique (field-oriented control) is one of the most important closed loop techniques for AC machines in variable speed applications. Using this control technique, the torque and flux can be decoupled so each can be controlled separately. A PMSM under vector control has the dynamic performances capabilities of a separately excited dc machine while still retaining the advantages of ac over dc motors [1, 2, 4, 5]. However, the performance is sensitive to the variation of motor parameters, especially the rotor time-constant, which varies with the temperature and the saturation of the magnetizing inductance. Recently, much attention has been given to the possibility of identifying the changes in motor parameters of PMSM while the drive is in normal operation. This stimulated a significant research activity to develop PMSM vector control algorithms using nonlinear control theory in order to improve performances, achieving speed (or torque) and flux tracking, or to give a theoretical justification of the existing solutions [1, 2, 5].

Due to new developments in nonlinear control theory, several nonlinear control techniques have been introduced in the last two decades. One of the nonlinear control methods that have been applied to AC machines is the backstepping design [6, 7, 8]. Backstepping is a systematic and recursive design methodology for nonlinear feedback control. This approach is based upon a systematic procedure for the design of feedback control strategies suitable for the design of a large class of feedback linearisable nonlinear systems exhibiting constant uncertainty, and it guarantees global regulation and tracking for the class of nonlinear systems transformable into the parametric-strict feedback form. The backstepping design alleviates some limitations of other approaches [5, 6, 7, 8, 9, 10]. It offers a choice of design tools to accommodate uncertainties and nonlinearities and can avoid wasteful cancellations. The idea of backstepping design is to select recursively some appropriate functions of state variables as pseudo-control inputs for lower dimension subsystems of the overall system. Each backstepping stage results into a new pseudo-control design, expressed in terms of the pseudo-control designs from the preceding design stages. When the procedure terminates, a feedback design for the true control input results and achieves the original design objective by virtue of a Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage [5, 9, 10, 11, 12].

In this paper, an adaptive backstepping control design based on field orientation for PMSM speed control is proposed. The proposed controller is adopted to derive the control scheme, which is robust to the parameter uncertainties and load torque disturbance. The rest of this paper is organized as follows. Section 2 reviews the principle of the indirect field-oriented control (IFOC) of permanent magnet synchronous motor. Section 3 shows the development of the adaptive backstepping controller design for PMSM speed control. Section 5 gives some simulation results. Finally, some conclusions are drawn in section 6.

II. MATHEMATICAL MODEL OF THE PMSM

The model of a typical surface-mounted PMSM can be described in the well known (d–q) frame through the Park transformation as follows: the stator d, q equations in the rotor frame are expressed as follows [1, 2, 5, 11, 12]:

$$u_d = R_s i_d + p\phi_d - \omega\phi_q$$
$$u_q = R_s i_q + p\phi_q + \omega\phi_d \qquad (1)$$

Where

$$\phi_d = L_d i_d \qquad (2)$$
$$\phi_q = L_q i_q + \phi_f \qquad (3)$$

ϕ_f is the magnet flux linkage,

Thus the dynamic model of a surface-mounted PMSM can be described as follows:

$$\frac{di_d}{dt} = -\frac{R_s}{L_d}i_d + \frac{P\omega L_q}{L_d}i_q + \frac{1}{L_d}u_d$$

$$\frac{di_q}{dt} = -\frac{R_s}{L_q}i_q + \frac{P\omega L_d}{L_q}i_d - \frac{P\phi_f}{L_q}\omega + \frac{1}{L_q}u_q \qquad (4)$$

$$\frac{d\omega}{dt} = \frac{3P\phi_f}{2J}i_q + \frac{3P}{2J}(L_d - L_q)i_d i_q - \frac{B}{J}\omega - \frac{T_L}{J}$$

Where i_d and i_q are the d–q axis currents, u_d and u_q are the d–q axis voltages, R_s is the stator resistance, L_d, L_d are d- and q- axes inductances, is the stator inductor, P is the pole pairs, J is the rotor moment of inertia, B is the viscous friction coefficient, T_L is the load torque, ω is the rotor angle speed in angle frequency, ϕ_f is the rotor magnetic flux linking the stator.

According to the model given in (4), it can be seen that the speed control can be achieved by controlling voltage u_q of q-axis. As long as the d-axis current i_d is maintained at zero, then the mathematical model of PMSM can be shown as

$$\frac{di_q}{dt} = -\frac{R_s}{L_q}i_q - \frac{P\phi_f}{L_q}\omega + \frac{1}{L_q}u_q$$

$$u_d = -P\omega L_q i_q \qquad (5)$$

$$T_{em} = \frac{3P\phi_f}{2J}i_q$$

III. ADAPTIVE BACKSTEPPING CONTROLLER DESIGN

The theory of backstepping design algorithm is a systematic approach to construct Lyapunov equation and unperturbed controller, so that the system is uniformly asymptotic stable at equilibrium point [5, 6, 11, 12]. In the design, constructing suitable function enables system design to achieve expected purpose. The objection to design this controller is to obtain the PMSM control voltages so as to achieve high-quality speed tracking performance. Based on Lyapunov stability principle and adaptive backstepping approach, with the corresponding Lyapunov function and virtual control function, we can get a satisfied controller. As there are various disturbed parameters in different combinations according to different situations, and then the adaptive update law can be designed correspondingly [11, 12].

In order to track the speed of PMSM, the design of the proposed controller involves the following steps.

Step 1: define the speed tracking error as

$$e_1 = \omega^* - \omega \qquad (6)$$

Where ω^* is the desired reference trajectory of the rotor speed. And the speed error dynamic is given by

$$\dot{e}_1 = \dot{\omega}^* - \dot{\omega}$$

$$\dot{e}_1 = \dot{\omega}^* - \left(\frac{3P\phi_f}{2J}i_q + \frac{3P}{2J}(L_d - L_q)i_d i_q - \frac{B}{J}\omega - \frac{T_L}{J}\right)$$

$$= \dot{\omega}^* - \left(\frac{3P\phi_f}{2J}i_q + \frac{3P}{2J}(L_d - L_q)i_d i_q + F\omega + \Gamma\right) \qquad (7)$$

Where, $F = -\frac{B}{J}$ and $\Gamma = -\frac{T_L}{J}$.

As the speed error needs to be reduced to zero, the d-q axes current component i_d and i_q are identified as the virtual control elements to stabilize the motor speed. To determine the stabilizing function, the following Lyapunov function is defined as

$$V = \frac{1}{2}e_1^2 \qquad (8)$$

We differentiate to get

$$\dot{V}_1 = e_1 \dot{e}_1$$

$$= e_1\left(\dot{\omega}^* - \left(\frac{3P\phi_f}{2J}i_q + \frac{3P}{2J}(L_d - L_q)i_d i_q + F\omega + \Gamma\right)\right)$$

$$= -k_1 e_1^2 + e_1 \cdot \left(k_1 e_1 + \dot{\omega}^* - \frac{3P\phi_f}{2J}i_q - F\omega - \Gamma\right)$$

$$- \frac{3P}{2J}(L_d - L_q)i_d i_q e_1 \qquad (9)$$

Where, k_1 is the closed-loop feedback constant. The speed control tracking is achieved if one defines the following stabilizing functions:

$$i_q^* = \frac{2J}{3P\phi_f}(k_1 e_1 + \dot{\omega}^* - F\omega - \Gamma) \qquad (10)$$

$$i_d^* = 0 \qquad (11)$$

i_d^* and i_q^* are the references currents. Substituting (10) and (11) back into (9) yields

$$\dot{V}_1 = -k_1 e_1^2, \qquad (12)$$

Thus the virtual control is asymptotically stable. Since the parameters J, F and Γ are unknown we must use their estimate values ($\hat{J}$, $\hat{F}$ and $\hat{\Gamma}$) in (). Thus, let us define

$$\hat{i}_q^* = \frac{2\hat{J}}{3P\phi_f}(k_1 e_1 + \dot{\omega}^* - \hat{F}\omega - \hat{\Gamma}) \qquad (13)$$

Step 2: Now we go to next step and try to make the signals i_d^* and i_q^* behave as desired. So we define again error signals involving the desired variables

$$e_2 = \hat{i}_q^* - i_q$$
$$= \frac{2\hat{J}}{3P\phi_f}\left(k_1 e_1 + \dot{\omega}^* - \hat{F}\omega - \hat{\Gamma}\right) - i_q \tag{14}$$

$$e_3 = i_d^* - i_d$$
$$= -i_d \tag{15}$$

Then the error equation (7) can be expressed as

$$\dot{e}_1 = -k_1 e_1 + \frac{3P\phi_f}{2J}e_2 + \frac{3P}{3J}\left(L_d - L_q\right)e_3 i_q$$
$$-\frac{\tilde{J}}{J}\left(k_1 e_1 + \dot{\omega}^* - \hat{F}\omega - \hat{\Gamma}\right) + \tilde{F}\omega + \tilde{\Gamma} \tag{16}$$

Where $\tilde{J} = \hat{J} - J$, $\tilde{F} = \hat{F} - F$ and $\tilde{\Gamma} = \hat{\Gamma} - \Gamma$ are the parameter estimation errors. To stabilize the current components i_d and i_q, we define now the current error dynamics as

$$\dot{e}_2 = \dot{\hat{i}}_q^* - \dot{i}_q = \phi_1 + \phi_2\tilde{F} + \phi_3\tilde{\Gamma} - \phi_4\tilde{J} - \frac{1}{L_q}u_q \tag{17}$$

$$\dot{e}_3 = -\dot{i}_d = \frac{R_s}{L_d}i_d - \frac{P\omega L_q}{L_d}i_q - \frac{1}{L_d}u_d$$

$$\dot{e}_3 = -\phi_6 - \frac{1}{L_d}u_d \tag{18}$$

Where,

$$\phi_1 = \frac{2\hat{J}}{3P\phi_f}\left(k_1 e_1 + \dot{\omega}^* - \hat{F}\omega - \hat{\Gamma}\right) + \frac{\dot{\hat{J}}}{J}e_2 + \frac{2\hat{J}}{3P\phi_f}$$
$$\left(-k_1^2 e_1 + \ddot{\omega}^* + \frac{3Pk_1}{J}\left(L_d - L_q\right)e_3 i_q\right) - \frac{2\hat{J}}{3P\phi_f}\left(\dot{\hat{F}}\omega + \dot{\hat{\Gamma}}\right)$$
$$-\frac{2\hat{J}}{3P\phi_f}\hat{F}\left(\frac{3P}{2J}\left(\phi_f i_q + \left(L_d - L_q\right)i_d i_q\right) - F\omega - \Gamma\right) - \phi_5$$

$$\phi_2 = \frac{2\hat{J}}{3P\phi_f}k_1\omega$$

$$\phi_3 = \frac{2\hat{J}}{3P\phi_f}k_1$$

$$\phi_4 = -\frac{2\hat{J}}{3P\phi_f}\frac{k_1}{J}\left(k_1 e_1 + \dot{\omega}^* - \hat{F}\omega - \hat{\Gamma}\right)$$

$$\phi_5 = -\frac{R_s}{L_q}i_q + \frac{P\omega L_d}{L_q}i_d - \frac{P\phi_f}{L_q}\omega$$

$$\phi_6 = \frac{R_s}{L_d}i_d - \frac{P\omega L_q}{L_d}i_q$$

Now, since the actual control inputs u_d and u_q have appeared in the above equations, we can go to the final step where both control and parameter updating laws are determined.

Step 3: To design the control and parameter updating laws, we extend the Lyapunov function in (8) to include the state variables e_2, e_3 and the parameter estimation errors $\tilde{J}$, $\tilde{F}$ and $\tilde{\Gamma}$ as

$$V_2 = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + \frac{1}{\gamma_1}\tilde{J}^2 + \frac{1}{\gamma_2}\tilde{\Gamma}^2 + \frac{1}{\gamma_3}\tilde{F}^2\right) \tag{19}$$

Where γ_1, γ_2 and γ_3 are positive design constants of adaptive gains.

Now we differentiate the Lyapunov function V_2 in (19) and substitute all error dynamics equations to get

$$\dot{V}_2 = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \frac{1}{\gamma_1}\tilde{J}\dot{\hat{J}} + \frac{1}{\gamma_2}\tilde{\Gamma}\dot{\hat{\Gamma}} + \frac{1}{\gamma_3}\tilde{F}\dot{\hat{F}}$$

$$= e_1\left(-k_1 e_1 + \frac{3P\phi_f}{2J}e_2 + \frac{3P}{3J}\left(L_d - L_q\right)e_3 i_q\right.$$
$$\left.-\frac{\tilde{J}}{J}\left(k_1 e_1 + \dot{\omega}^* - \hat{F}\omega - \hat{\Gamma}\right) + \tilde{F}\omega + \tilde{\Gamma}\right) + e_2\left(\phi_1\right.$$
$$\left.+\phi_2\tilde{F} + \phi_3\tilde{\Gamma} - \phi_4\tilde{J} - \frac{1}{L_q}u_q\right) + e_3\left(-\phi_6 - \frac{1}{L_d}u_d\right)$$
$$+\frac{1}{\gamma_1}\tilde{J}\dot{\hat{J}} + \frac{1}{\gamma_2}\tilde{\Gamma}\dot{\hat{\Gamma}} + \frac{1}{\gamma_3}\tilde{F}\dot{\hat{F}} \tag{20}$$

If the following d-q axes control voltage are selected

$$u_q = R_s i_q + P\omega L_d i_q + P\phi_f\omega + L_q\left[k_2 e_2 + \frac{2\hat{J}}{3P\phi_f}\right.$$
$$\left(k_1 e_1 + \hat{F}\omega + \hat{\Gamma} + \dot{\omega}^*\right) + \frac{2\hat{J}}{3P\phi_f}\left(\dot{\hat{F}}\omega + \dot{\hat{\Gamma}} + \ddot{\omega}^* + k_1\dot{\omega}^*\right)$$
$$+\left(\hat{F} - k_1\right)\left(1 + \frac{\left(L_d - L_q\right)i_d}{\phi_f}\right)i_q$$
$$\left.-\frac{2\hat{J}}{3P\phi_f}\left(\hat{F} - k_1\right)\left(\hat{F}\omega + \Gamma\right)\right] \tag{21}$$

$$u_d = R_s i_d - P\omega L_q i_q - \frac{3P}{2J}\left(L_d - L_q\right)L_d i_q e_1 - k_3 L_d e_3 \tag{22}$$

Equation (21) results in the following expression:

$$\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + \frac{3P\phi_f}{2J}e_1 e_2 + \tilde{J}\left[-k_1 e_1^2\right.$$
$$+\hat{F}e_1\omega + \hat{\Gamma}e_1 + \ddot{\omega}^* - \left(\hat{F} - k_1\right)\left(1 + \frac{\left(L_d - L_q\right)}{\phi_f}\right)i_q e_2$$
$$\left.+\frac{1}{\gamma_1}\dot{\tilde{J}}\right] + \tilde{\Gamma}\left[e_1 - \frac{2\hat{J}}{3P\phi_f}\left(\hat{F} - k_1\right)e_2 + \frac{1}{\gamma_2}\dot{\tilde{\Gamma}}\right]$$
$$+\tilde{F}\left[e_1\omega - \frac{2\hat{J}}{3P\phi_f}\left(\hat{F} - k_1\right)\omega e_2 + \frac{1}{\gamma_3}\dot{\tilde{F}}\right] \tag{23}$$

From (24), the following update laws can be derived as:

$$\dot{\hat{J}} = \gamma_1\left[k_1 e_1^2 - \hat{F}e_1\omega - \hat{\Gamma}e_1 - \ddot{\omega}^*\right.$$
$$\left.+\left(\hat{F} - k_1\right)\left(1 + \frac{\left(L_d - L_q\right)}{\phi_f}\right)i_q e_2\right] \tag{24}$$

$$\dot{\hat{\Gamma}} = \gamma_2\left[-e_1 + \frac{2\hat{J}}{3P\phi_f}\left(\hat{F} - k_1\right)e_2\right] \tag{25}$$

$$\dot{\hat{F}} = \gamma_3\left[e_1\omega - \frac{2\hat{J}}{3P\phi_f}\left(\hat{F} - k_1\right)\omega e_2\right] \tag{26}$$

Therefore, one can obtain the following expression for the derivative of the Lyapunov function (24) as

$$\dot{V}_2 = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + \frac{3P\phi_f}{2J} e_1 e_2 \leq 0 \qquad (27)$$

For sufficiently large k_1 and k_2, then proved that equation (28) guarantees asymptotic stability in the complete system.

IV. SIMULATION RESULTS

To prove the rightness and effectiveness of the proposed control scheme, digital simulations have been performed using Matlab/Simulink software. Numerous simulations were performed, and sample results are shown here.

The machine parameters are given in table 1. The parameter used in simulation are chosen as $k_1 = 200$, $k_2 = 300$, $k_3 = 300$, $\gamma_1 = 0.001$, $\gamma_2 = 0.3$ and $\gamma_3 = 1030$.

The drive system is started at a constant load 0f 1.2N.m with speed reference set at 150 rad/s. a step change in load torque from 1.2Nm to 3.6Nm was applied at 1.5s. The actual speed converges with reference speed in very short time with a negligible overshoot and no steady state error. The adaptive backstepping controller rejects the load disturbance rapidly and converges back to the reference speed. Figure 2 shows the corresponding d-q axes motor current along with the command q-axis current. The q-axis stator current swiftly reaches to its new value corresponding to the load torque applied. This shows the capability of the proposed controller in terms of disturbance rejection. Figure 3 shows the stator current i_a which increases when the load disturbance is applied. A comparison between the proposed controller (adaptive backstepping) and the conventional backstepping (with no adaptive states and parameters) is shown in Fig. 4. In Figs. 4, it can be observed that the speed response of the adaptive backstepping controller present better tracking characteristics. Figure 6 and 7 shows a zoomed response of the speed under inertia moment variation for the proposed controller and the classical one. As shown in fig. 6 and 7, the actual speeds converge to the references. It is evident from these figures that the proposed controller can handle the parameter variation (inertia motor) without any deviation in sped. No noticeable variation in speed is present; therefore the controller is insensitive to parameter variation. Overall, the speed tracking performance of the complete drive system is found robust and is more robust than the conventional backstepping controller.

Table 1

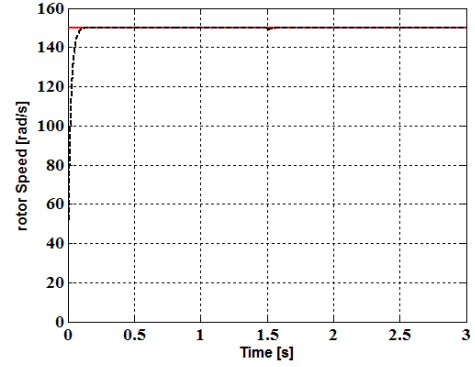| Motor rated power	3-phase, $1hp$
Rated voltage	208V
Rated current	3A
Rated frequency	50Hz
Pole pair number P	2
d-axis inductance, L_d	42.44 mH
q-axis inductance, L_q	79.57 mH
Stator resistance, R_s	1.9Ω
Motor inertia, J	0.003 kgm^2
Friction coefficient, B	0.001$Nm/rad/s$
Magnetic flux constant, ϕ_f	0.311 Wb

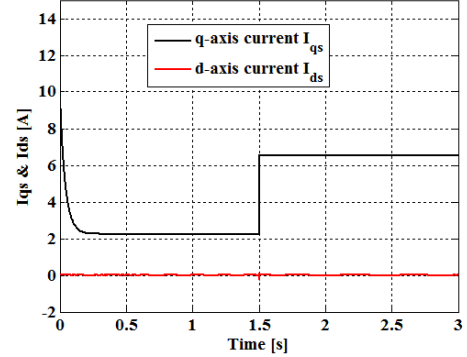Fig. 1: simulated response of the rotor speed for the proposed controller.

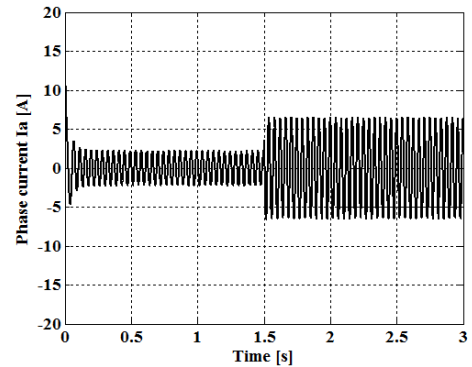Fig. 2: simulated response of the stator current i_d, i_q for the proposed controller.

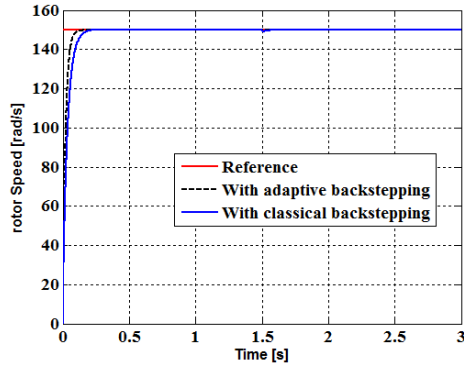Fig. 3: simulated response of the stator current i_a for the proposed controller.

Fig. 4: Simulated results of the comparison between the classical and the adaptive backstepping design for PMSM speed
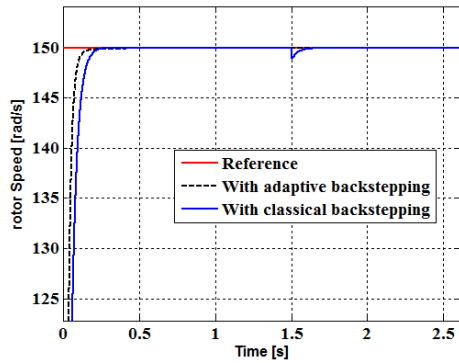

Fig. 5: Zoomed responses of speed control obtained by classical and adaptive backstepping control for PMSM
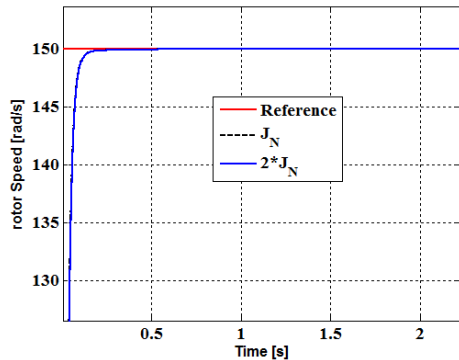

Fig. 6: Simulated results of the proposed adaptive backstepping controller for PMSM speed tracking with inertia motor variation
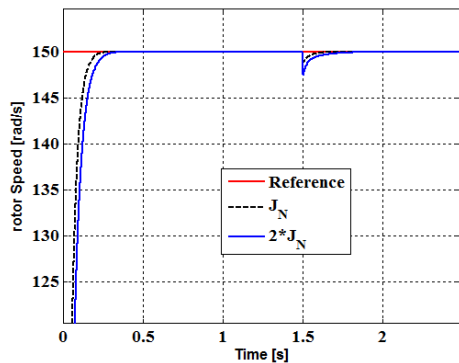

Fig. 7: Simulated results of the classical backstepping controller for PMSM speed tracking with inertia motor variation

V. CONCLUSION

In this work, we have presented a nonlinear controller based on adaptive backstepping technique in order to offer a choice of design tools to accommodate uncertainties and nonlinearities. This study has successfully demonstrated the design of the adaptive backstepping control for the speed control of a permanent magnet synchronous motor and the nonlinear field orientation control design. The control laws were derived based on the motor model incorporating the parameters uncertainties and the external disturbances. By recursive manner, virtual control states of the PMSM drive have been identified and stabilizing laws are developed subsequently using Lyapunov stability theory. The performance of the proposed adaptive controller has been investigated in simulation using Matlab/Simulink software. The simulation results show its effectiveness and robustness at tracking a reference speed under parameters uncertainties especially inertia motor variation.

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