

# ROBUST INDUCTION MOTOR CONTROL USING ADAPTIVE FUZZY SYNERGETIC CONTROL

L. MEDJBEUR M.N. HARMAS

QUERE Laboratory, Electrical Engineering Department, Ferhat Abbas University of Sétif  
lemya\_medjbeur@yahoo.fr, mharmas@yahoo.fr

S. BENAGGOUNE

LRE Laboratory, Electrical Engineering Department, Hadj Lakhdar University of Batna, Algeria  
s\_benaggoune@yahoo.fr

**Abstract:** A new control technique of an induction motor is undertaken through a robust approach tagged synergetic control. Like the sliding mode (SMC) approach the system state trajectories are forced to evolve on a designer chosen manifold according to performance specifications. But unlike SMC, synergetic control relies on a continuous control law thus preventing unwanted chattering to occur. Fuzzy sets are used to approximate unknown system functions and system stability conditions are derived.

**Key words:** indirect adaptive, fuzzy, synergetic control, induction motor, Lyapounov.

## I. INTRODUCTION

Sliding mode control has been extensively used in robust control approaches in many non linear applications ranging from inverted pendulum to power system stabilizers [1-5] and a large effort has been directed to address its main drawback: dangerous chattering ever present in SMC due to the discontinuous law component [2]. Many approaches have been proposed to reduce the latter but mostly at the expense of robustness performance [6-7]. Synergetic control like sliding mode is based on the basic idea that if we could force a system to a desired manifold with designer chosen dynamics using continuous control law, we should achieve similar performance as SMC without its main inconvenient: chattering phenomenon. To achieve this goal one has to choose a pertinent macro-variable first and then elaborate a manifold which enables the desired performance to be reached. Macro-variables can be a function of two or more system state variables [8]. Although similarities with sliding mode technique include system order reduction and decoupling, its chatter free operation makes it a sound and motivating approach easily implementable.

## II. SYNERGETIC CONTROL BASICS

Introduced in the last decades, synergetic control [9-10] is rapidly gaining acceptance not only by the robust control community but also by the industrial partners as well as illustrated by its implementation in power electronics [11-13] and its industrial application in battery charging [14].

We briefly introduce the basics of synergetic control synthesis for an n-order non linear dynamic system described by (1) :

$$\frac{dx(t)}{dt} = f(x,u,t) \quad (1)$$

where  $x$  represents the system state space vector and  $u$  its control. Although it could be easily extendable to multi-variable system, we will consider in this paper a single input single output case for simplicity. Control synthesis begins by a suitable choice of pertinent macro-variable function of two or more state variables given by (2):

$$\psi = \psi(x,t) \quad (2)$$

Where  $\psi$  and  $\psi(x,t)$  designate designer chosen macro-variable and a corresponding a state variables and time dependent function. Next a desirable manifold (3) is chosen on which the system will be forced to remain even in presence of unwanted disturbances or parameters fluctuations just as on a sliding mode surface.

$$\psi = 0 \quad (3)$$

A large choice is available to the designer in selecting the macro-variables features accordingly with the control objectives and practical physical constraints. The macro-variable, may be a simple linear combination, is forced to evolve accordingly to the designer imposed constraint of the general following equation:

$$T\dot{\psi} + \psi = 0, \quad T > 0 \quad (4)$$

Control parameter  $T$  dictates convergence rate towards the selected manifold given by (3). The appropriate control law is obtained using straightforward mathematical following steps:

$$\frac{d\psi(x,t)}{dt} = \frac{d\psi(x,t)}{dx} \cdot \frac{dx}{dt} \quad (5)$$

Using (1) and (2) in (4) leads to (6):

$$T \frac{d\psi(x,t)}{dx} f(x,u,t) + \psi(x,t) = 0 \quad (6)$$

Resolving (6) for  $u$  gives the control law as:

$$u = g(x, \psi(x,t), T, t) \quad (7)$$

As can be seen, control law  $u$  depends not only on system variables but on parameter  $T$  and macro-variable  $\psi$  as well giving the designer latitude to choose controller features acting upon the full non linearized system model.

An appropriate designer choice of the macro-variables and judicious manifolds lead to closed-loop system global stability and invariance to parameter fluctuation [15-16] for when the system reaches the pre-specified manifold it remains on it.

### III. PROBLEM STATEMENT

Considering the following  $n$ -order non linear SISO system:

$$\begin{aligned} x^{(n)} &= f(x) + g(x)u \\ y &= x \end{aligned} \quad (8)$$

$f, g$  Represent system unknown continuous functions  $u \in \mathfrak{R}$  and  $y \in \mathfrak{R}$  are input and output system respectively. The system state vector is given as:

$$x = (x_1 \ x_2 \ \dots \ x_n)^T = (x \ \dot{x} \ \dots \ x^{(n-1)}) \in \mathfrak{R}^n$$

The error vector  $e$  is defined as (9):

$$\bar{e} = y_m - x = [e \ \dot{e} \ \dots \ e^{(n-1)}]^T \in \mathfrak{R}^n \quad (9)$$

In the error state – space, the macro-variable is defined as:

$$\psi(e) = c_1 e + c_2 \dot{e} + \dots + c_{n-1} e^{(n-2)} + e^{(n-1)} = \bar{c}^T \bar{e} \quad (10)$$

Where  $\bar{c} = [c_1 \ c_2 \ \dots \ c_{n-1}]^T$  is chosen such that  $h(\lambda)$  is Hurwitz:

$$h(\lambda) = \lambda^{n-1} + c_{n-1} \lambda^{n-2} + \dots + c_1$$

In the trivial case where in (8)  $f$  and  $g$  are known, the control law (11) is easily obtained:

$$u = \frac{1}{g(x)} \left[ -f(x) + y_m^{(n)} - e^{(n)} \right] \quad (11)$$

Using the synergetic approach, equation (4) can be expressed as:

$$\begin{aligned} \begin{bmatrix} \dot{e} \\ \vdots \\ e^{(n-1)} \\ e^{(n)} \end{bmatrix} &= -\frac{1}{T} \psi(e) \\ \Rightarrow e^{(n)} &= -\sum_{i=1}^{n-1} c_i e^{(i)} - \frac{1}{T} \psi(e) \end{aligned} \quad (12)$$

Making use of (12) in equation (11) lead to the synergetic control signal:

$$\tilde{u} = \frac{1}{g(x)} \left[ -f(x) + y_m^{(n)} + \sum_{i=1}^{n-1} c_i e^{(i)} + \frac{1}{T} \psi(e) \right] \quad (13)$$

Asymptotic stability is obtained using the Lyapounov function candidate:  $V = \frac{1}{2} \psi(e)^2$  which leads, after differentiation, to:

$$\begin{aligned} \dot{V} &= \psi(e) \dot{\psi}(e) \\ \dot{V} &= -\frac{1}{T} \psi^2(e) \leq 0 \end{aligned} \quad (14)$$

### IV. FUZZY SYSTEMS SYNTHESIS

Fuzzy systems with singleton fuzzification, center average defuzzifier and inference product are functions  $f$  such that:  $f: U \subset \mathfrak{R}^n \rightarrow \mathfrak{R}$  and can be expressed in the following form [18-20]:

$$y(x) = \frac{\sum_{l=1}^M \bar{y}^l \left( \prod_{i=1}^n \mu_{F_i^l}(x_i) \right)}{\sum_{l=1}^M \left( \prod_{i=1}^n \mu_{F_i^l}(x_i) \right)} \quad (15)$$

With  $x = (x_1 \dots x_n)^T \in U$  is the input vector,  $\bar{y}$  represent membership function centers,  $\mu_{F_i^l}(x_i)$  corresponds to the membership function of input  $x_i$  for the fuzzy rule  $l$ , in which ‘AND’ is realized by inference product. Fuzzy system (15) can be expressed as:

$$y(x) = \theta^T \xi(x) \quad (16)$$

In which  $\theta = (\bar{y}^1 \dots \bar{y}^M)^T$  is a parameter vector and  $\xi(x) = (\xi^1(x) \dots \xi^M(x))^T$  is a regressive vector where the regressor  $\xi^l(x)$  represent the fuzzy basis function defined by [19]:

$$\xi^l(x) = \frac{\prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^M \left( \prod_{i=1}^n \mu_{F_i^l}(x_i) \right)} \quad (17)$$

## V. ADAPTIVE FUZZY SYNERGETIC CONTROL

The result in (13) is realizable only while  $f(x)$  and  $g(x)$  are well known. However,  $f(x)$  and  $g(x)$  are generally unknown and the ideal controller (13) cannot be implemented. We replace  $f(x)$  and  $g(x)$  by the fuzzy logic system (16). Hence, the resulting control law is as follows:

$$u = \frac{1}{\hat{g}(x/\theta_g)} [-\hat{f}(x/\theta_f) + y_m^{(n)} + \sum_{i=1}^{n-1} c_i e^{(i)} + \frac{1}{T} \psi(e)] \quad (18)$$

$$\hat{f}(x|\theta_f) = \theta_f^T \xi(x) \quad (19)$$

$$\hat{g}(x|\theta_g) = \theta_g^T \xi(x) \quad (20)$$

### A. Parameters Adaptation

First  $\hat{f}$  et  $\hat{g}$  are replaced by their corresponding fuzzy system estimate as in (16) and adaptation laws developed from classical Lyapounov synthesis procedure to ensure closed loop stability as well as rapid parameter convergence.

Defining:

$$\theta_f^* = \arg \min_{\theta_f \in \Omega_f} \left[ \sup_{x \in R^n} |\hat{f}(x/\theta_f) - f(x)| \right] \quad (21)$$

$$\theta_g^* = \arg \min_{\theta_g \in \Omega_g} \left[ \sup_{x \in R^n} |\hat{g}(x/\theta_g) - g(x)| \right] \quad (22)$$

$\Omega_f$  and  $\Omega_g$  Represent constraint sets for  $\theta_f$  and  $\theta_g$  respectively, based on expert information, defined by:

$$\Omega_f = \left\{ \theta : |\theta_f| \leq M_f \right\} \quad (23)$$

$$\Omega_g = \left\{ \theta_g : |\theta_g| \leq M_g \right\} \quad (24)$$

In which  $M_f$  and  $M_g$  are positive constants.

Let us introduce the minimum approximation error as:

$$w = (\hat{f}(x/\theta_f^*) - f(x)) + (\hat{g}(x/\theta_g^*) - g(x))u_c \quad (25)$$

Using (24) macro-variable (10) can be re-written as:

$$\dot{\psi}(e) = \sum_{i=1}^{n-1} c_i e^{(i)} - f(x,t) - g(x,t)u + y_m^{(n)} \quad (26)$$

$$\begin{aligned} \dot{\psi}(e) &= \sum_{i=1}^{n-1} c_i e^{(i)} + [\hat{f}(x, \theta_f) - f(x,t)] \\ &\quad + [\hat{g}(x, \theta_g) - g(x,t)]u - \sum_{i=1}^{n-1} c_i e^{(i)} - \frac{1}{T} \psi(e) \end{aligned}$$

$$\begin{aligned} \dot{\psi}(e) &= \left[ \hat{f}(x, \theta_f) - \hat{f}(x, \theta_f^*) \right] + \\ &\quad \left[ \hat{g}(x, \theta_g) - \hat{g}(x, \theta_g^*) \right] u - \frac{1}{T} \psi(e) + w \\ \dot{\psi}(e) &= \left( \theta_f^T - \theta_f^{*T} \right) \xi(x) + \left( \theta_g^T - \theta_g^{*T} \right) \xi(x) u - \frac{1}{T} \psi(e) + w \end{aligned} \quad (27)$$

where  $\phi_f = \theta_f - \theta_f^*$ ,  $\phi_g = \theta_g - \theta_g^*$

$$\dot{\psi}(e) = \phi_f^T \xi(x) + \phi_g^T \xi(x) u - \frac{1}{T} \psi(e) + w \quad (28)$$

Let's consider the Lyapounov function candidate  $V$ :

$$V = \frac{1}{2} \psi(e)^2 + \frac{1}{2\gamma_1} \phi_f^T \phi_f + \frac{1}{2\gamma_2} \phi_g^T \phi_g \quad (29)$$

Where  $\gamma_1$  and  $\gamma_2$  are positive constants.

Differentiating  $V$  with respect to time gives:

$$\begin{aligned} \dot{V} &= \psi(e) \phi_f^T \xi(x) + \frac{1}{\delta_1} \phi_f^T \dot{\phi}_f + \psi(e) \phi_g^T \xi(x) u \\ &\quad + \frac{1}{\delta_2} \phi_g^T \dot{\phi}_g - \frac{1}{T} \psi(e)^2 + \psi(e) w \end{aligned}$$

in which we make use of :  $\dot{\phi}_f = \dot{\theta}_f$  and  $\dot{\phi}_g = \dot{\theta}_g$  , to obtain:

$$\begin{aligned} \dot{V} = & \frac{1}{\delta_1} \phi_f^T (\delta_1 \psi(e) \xi(x) + \dot{\theta}_f) - \frac{1}{T} \psi(e)^2 \\ & + \frac{1}{\delta_2} \phi_g^T (\delta_2 \psi(e) \xi(x) + \dot{\theta}_g) + \psi(e)w \end{aligned} \quad (30)$$

Choosing the following adaptation laws:

$$\begin{aligned} \dot{\theta}_f &= -\delta_1 \psi(e) \xi(x) \\ \dot{\theta}_g &= -\delta_2 \psi(e) \xi(x)u \end{aligned} \quad (31)$$

Equations (30) and (31) bring about the following result:

$$\dot{V} \leq -\frac{1}{T} \psi(e)^2 + \psi(e)w \quad (32)$$

The term  $\psi(e)w$  is very small due to the minimum in the approximation error introduced in (24). Fuzzy systems are known as universal approximators and therefore they can approximate  $f$  and  $g$  by their estimates  $\hat{f}$  and  $\hat{g}$  to any arbitrary accuracy [19]. Hence an adequate number of fuzzy rules in the estimation of  $\hat{f}$  and  $\hat{g}$  permit a very small value, leading to (33)

$$\dot{V} \leq -\frac{1}{T} \psi(e)^2 \leq 0 \quad (33)$$

since  $w$  is the minimum approximation error (33) is the best result that we can obtain. Therefore, all signals in the system are bounded. Obviously, if  $e(0)$  is bounded, then  $e(t)$  is also bounded for all  $t$ . Since the reference signal  $y_m$  is bounded, then the system states  $x$  are bounded as well. To complete the proof and establish asymptotic convergence of the tracking error, we need proving that:

$\psi \rightarrow 0$  as  $t \rightarrow \infty$ . Assume that  $|\psi| \leq \alpha$ , then (33) can be rewritten as:

$$\dot{V} \leq -\frac{1}{T} \alpha |\psi| + \alpha |w| \quad (34)$$

Integrating both sides of (34), we have

$$\int_0^t |\psi| d\tau \leq \frac{T}{\alpha} (|V(0)| + |V(t)|) + T \int_0^t |w| d\tau \quad (35)$$

Then we have  $\psi \in L_1$ . Form (35), we know that  $w$  is bounded and every term in (27) is bounded, hence,

$\psi, \dot{\psi} \in L_\infty$ , by use of Barbalat's lemma [20], we have:

$\psi(t) \rightarrow 0$  as  $t \rightarrow \infty$ , the system is therefore stable and the error will asymptotically converge to zero.

## VI. SIMULATION RESULTS

In this section, we introduce a brief description of the induction motor used in simulation followed by presentation of results principally a good tracking and a continuous control law therefore easy to implement.

In order to assess our approach we used a three phase star-connected four-pole 600W, 60Hz, induction servomotor drive described by (15) [17]:

$$J\ddot{\theta} + B\dot{\theta} + T_L = T_E \quad (15)$$

Where  $J$  is the moment of inertia,  $B$  is the damping coefficient,  $T_E$  represents the electric torque and  $T_L$  denotes the external load disturbance. The electric torque can be written as [17]:

$$T_E = K_T i_{qs}^* \quad (16)$$

$$K_T = \frac{3N_p}{2} \cdot \frac{L_m^2}{L_r} \cdot i_{ds}^* \quad (17)$$

Where  $K_T$  is the electric torque constant,  $i_{qs}^*$  and  $i_{ds}^*$  are respectively the torque current, and the flux current control,  $N_p$  is the number of pole pairs,  $L_m$  is the magnetizing inductance per phase and  $L_r$  is the rotor inductance per phase. Then the description of the dynamic structure of the control induction motor can be represented in the following form:

$$\ddot{\theta} = \frac{1}{J} \left[ -B\dot{\theta} + K_T i_{qs}^* \right] \quad (18)$$

Define  $x_1 = \theta$  to be the rotor angle of the induction motor and  $x_2 = \dot{\theta}$  the motor angular velocity. The dynamic equation of system (15) can be written as (19):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix} u + \begin{bmatrix} 0 \\ c \end{bmatrix} d \quad (19)$$

Where

$a = -B/J$ ,  $b = K_T/J$ ,  $c = -1/J$ ,  $d = T_L$  and  $u = i_{qs}^*$  is

the control signal. The control objective is to design a control law so that the rotor position tracks the desired trajectory. Assume that the parameters of the induction motor system are unknown.

For simulation purpose we use the following induction motor parameters [17]:

$$J = 4.78 \times 10^{-3} \text{ Nm} / \text{s}^2 ; B = 5.34 \times 10^{-3} \text{ Nms} / \text{rad}$$

$$K_T = 0.4851 \text{ Nm} / \text{A} ; T_L = 0.5 \text{ Nm}$$

It is desired for the rotor angle to track a sine-wave trajectory  $x_d = \theta_d = \frac{\pi}{30} \sin(t)$  and to assess robustness we apply an external load disturbance  $T_L$  at  $t=7\text{sec}$ .

We start by choosing the synergetic macro-variable

$$\psi(e) = c_1 e + \dot{e}$$

The membership functions for system states

$x_i$ ,  $i=1,2$  are selected as follows:

$$\begin{aligned} \mu_{F_i^1}(x_i) &= \exp \left[ - \left( \frac{x_i + \pi/6}{\pi/24} \right)^2 \right], \\ \mu_{F_i^2}(x_i) &= \exp \left[ - \left( \frac{x_i + \pi/12}{\pi/24} \right)^2 \right], \\ \mu_{F_i^3}(x_i) &= \exp \left[ - \left( \frac{x_i}{\pi/24} \right)^2 \right], \\ \mu_{F_i^4}(x_i) &= \exp \left[ - \left( \frac{x_i - \pi/12}{\pi/24} \right)^2 \right], \\ \mu_{F_i^5}(x_i) &= \exp \left[ - \left( \frac{x_i - \pi/6}{\pi/24} \right)^2 \right]. \end{aligned}$$

25 rules are used to approximate the system unknown functions. The initial consequent parameters of fuzzy rules are chosen randomly in the interval  $[0.5, 2]$ .

With learning rate  $\delta_1 = 8$  and  $\delta_2 = 1$  and  $x_0 = [0, 0]^T$

We observe in figure 1 that we have good tracking despite an external disturbance applied at  $t=7\text{secs}$  for it is perfectly handled by the continuous law shown in figure 2 which shows rapid suppression of the ef-

fect of the disturbance without showing any undesirable chattering.

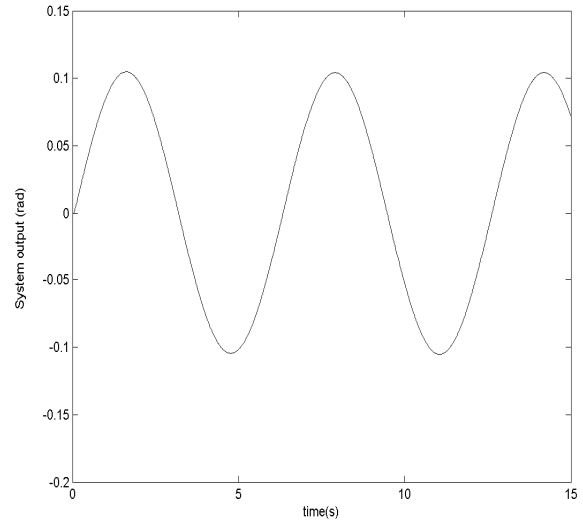


Figure 1 System output

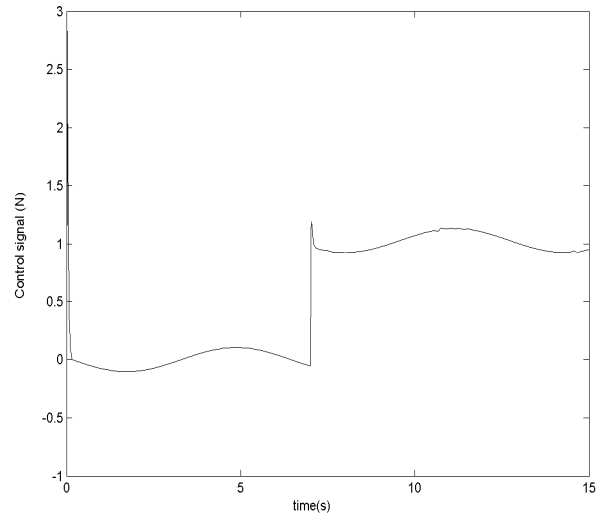


Figure 2 Control signal

## VII. CONCLUSION

A new robust indirect adaptive fuzzy synergetic controller has been presented with the development of a continuous control law easy to implement. Stability study and design details were given and a simple induction motor tracking a sine wave reference was used in simulation proving the soundness of the proposed approach.

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