

# Disturbance observer based iterative learning control for robot manipulators

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**Abstract:** In this paper, using a Lyapunov-like function, we derive a disturbance observer based iterative learning control scheme for the trajectory tracking problem of rigid robot manipulator. In this control scheme, the whole control law consists of two parts, the feedback control law, plus an iteratively updated term represents the estimated disturbance. The feedback control law using in this paper is a computed torque control, without compensating for the gravity forces. Using Lyapunov method, the asymptotic stability of the whole system is guaranteed, and the external disturbances with the gravity forces are compensated. Simulation results on the PUMA 560 robot manipulator, show the asymptotic convergence of tracking error, when the Coulomb and Viscous friction is considered as an external disturbance.

**Keywords:** Disturbance observer, iterative learning control, Lyapunov method, robot manipulator.

## 1. Introduction

During the last decade the class of rigid robot systems has been the subject of intensive research in the field of systems and control theory, particularly owing to the inherent nonlinear nature of rigid robots. For the same reason, these systems have widely been used to exemplify general concepts in nonlinear control theory. As a result of this excessive research activity a large variety of control methods for rigid robot systems have been proposed [1,2,3].

Since robot manipulators are generally used in repetitive tasks, one should take advantage of the fact that the reference trajectory is repeated over a given operation time. In this context, iterative learning control 'ILC' is an attractive technique when it comes to dealing with systems that execute the same task repeatedly over a finite time interval. The key feature of this technique is to use information from the previous operation in order to enable the controlled system to perform progressively better from operation to operation. This technique has been the center of interest of many researchers over the last two decades (see for instance [4,5,6,7,8]). On the other hand, another type of ILC algorithms has been developed using Lyapunov-like methods. In fact, in [9], Xu and Qu utilised a Lyapunov-based approach to illustrate how

an ILC can be combined with a variable structure controller to handle a broad class of non linear systems, in [10], Ham *et al.* utilised Lyapunov-based techniques to develop an ILC that is combined with a robust control design to achieve global uniformly ultimately bounded link position tracking for robot manipulators, the applicability of this design was extended to a broader class of nonlinear systems in [11]. Using Lyapunov-like function, Tayebi derived in [12] an adaptive ILC scheme for the trajectory tracking problem of rigid robot manipulators.

The control problem for a nonlinear system under disturbances has been developed and applied in engineering over two decades. Nakao *et al.* [13] proposed firstly the concept of disturbance observer 'DO' as compensating unknown disturbance. Furthermore, friction is a common phenomenon in mechanical systems. One of the most promising methods is observer-based control, where a variable structure DO has been proposed [14], and a nonlinear observer for a special kind of friction, i.e., Coulomb friction, has been proposed by Friedland and Park [15]. It has been further modified and implemented on robotic manipulators by Tafazoli *et al.* [16]. In [17] a DO based control approach for nonlinear systems under disturbances has been proposed, but only semiglobal stability condition of the composite controller-observer has been established.

To cancel out the disturbance, we present in this paper a disturbance observer based learning control. The proposed control scheme comprises a feedback control law, and an iteratively updated term represents the estimated disturbance. The asymptotic stability condition of the proposed controller is established, this result is based on Lyapunov theory. Simulation results on the PUMA 560 robot manipulator show the asymptotic convergence of tracking error, when the Coulomb and Viscous friction is considered as an external disturbance.

## 2. Dynamic equations for robot manipulators

We consider a robot manipulator that is composed of serially connected rigid links. The

motion of the manipulator with n-links is described by the following dynamic equation

$$\tau_k = M(q_k)\ddot{q}_k + C(q_k, \dot{q}_k)\dot{q}_k + G(q_k) + d(t) \quad (1)$$

where  $t$  denotes the time and the nonnegative integer  $k$  denotes the operation or iteration number. The signals  $q_k(t)$ ,  $\dot{q}_k(t)$ ,  $\ddot{q}_k(t) \in R^n$  denote the link position, velocity, and acceleration vectors, respectively.  $M(q_k(t)) \in R^{n \times n}$  represents the link inertia matrix,  $C(q_k(t), \dot{q}_k(t)) \in R^{n \times n}$  represents centripetal-Coriolis matrix,  $G(q_k(t)) \in R^{n \times 1}$  represents the gravity effects,  $\tau_k(t) \in R^{n \times 1}$  represents the torque input vector, and  $d(t) \in R^{n \times 1}$  is a disturbance torque or force vector. It should be noted that  $d(t)$  has different meanings in different observer applications. For example, it can be friction in friction compensation, reaction torque or force in force control, and unmodeled dynamics in independent joint control. In this paper, all of them are considered as disturbances, because, a general observer will be derived.

In the sequel,  $q_d(t)$ ,  $\dot{q}_d(t)$ ,  $\ddot{q}_d(t) \in R^n$  denote the desired link position, velocity, and acceleration vectors, respectively. The norm of a vector  $X$  is defined as

$$\|X\| = \sqrt{X^T X} \quad (2)$$

and the norm of a matrix  $A$  as

$$\|A\| = \sqrt{\lambda_{\max}(A^T A)} \quad (3)$$

with  $\lambda_{\max}(\cdot)$  denotes the maximum eigenvalue of  $A$ .

The dynamic equation of (1) has the following properties [18,19,20] that will be used in the controller development and analysis.

**P1:** The inertia matrix  $M(q_k(t))$  is symmetric, positive definite and bounded as

$$0_n < \beta_1 I \leq \|M(q_k)\| \leq \beta_2 I \quad (4)$$

where  $q_k \in R^n$ , and  $\beta_2 > \beta_1 > 0$ .

**P2:**  $M(q(t))$  is globally Lipschitz continuous in their arguments as follows

$$\|M(q_{k+1}) - M(q_k)\| \leq l_m \|q_{k+1} - q_k\| \quad (5)$$

where  $l_m$  a positive constant.

**P3:**  $G(q_k)$  is globally Lipschitz continuous in their arguments and bounded as

$$\|G(q_{k+1}) - G(q_k)\| \leq g_m \|q_{k+1} - q_k\| \quad (6)$$

$$\|G(q_k)\| \leq l_g \quad (7)$$

where  $g_m$  and  $l_g$  denote known positives bounding constants.

In this paper, the following lemma is used

*Lemma*[19] The inertia matrix  $M(q_k)$  has the following property

$$\|M^{-1}(q_{k+1}) - M^{-1}(q_k)\| \leq l_m \beta_1^{-2} \|q_{k+1} - q_k\|. \quad (8)$$

*Proof*

$$M^{-1}(q_{k+1}) - M^{-1}(q_k) = -M^{-1}(q_{k+1})(M(q_{k+1}) - M(q_k))M^{-1}(q_k). \quad (9)$$

From properties 1 and 2, we ave

$$\|M^{-1}(q_{k+1}) - M^{-1}(q_k)\| \leq l_m \beta_1^{-2} \|q_{k+1} - q_k\|. \quad (10)$$

The following assumptions are imposed.

**A1:** The disturbance  $d(t)$  is repetitive and bounded as follows

$$\|d(t)\| \leq l_d, \forall t \in [0, T]. \quad (11)$$

**A2:** The reference trajectory and its first and second time-derivative are bounded, and the resetting condition is satisfied, i.e.

$$\dot{q}_d(0) - \dot{q}_k(0) = q_d(0) - q_k(0) = 0, \forall k \in N \quad (12)$$

Our objective is to design a control law  $\tau(t)$  guaranteeing the convergence of  $q_k(t)$  and  $\dot{q}_k(t)$  to the desired trajectory  $q_d(t)$  and desired velocity  $\dot{q}_d(t)$ , respectively, for all  $\forall t \in [0, T]$  when  $k$  tends to infinity.

In the following, we will introduce a tracking control algorithm which is referred to as the disturbance observer based learning control, and the asymptotic stability is guaranteed.

### 3. Disturbance observer based iterative learning control

We propose the following control

$$\tau(t) = M(q)[\ddot{q}_d(t) + K_v \dot{e} + K_p e] + C(q, \dot{q})\dot{q} + \hat{d}(t) \quad (13)$$

where  $e(t) = q_d(t) - q(t)$  is the tracking error vector,  $\dot{e}(t) = \dot{q}_d(t) - \dot{q}(t)$  is the velocity error vector,  $K_v = k_v I_{n \times n}$ ,  $K_p = k_p I_{n \times n}$ ,  $k_p = \sigma k_v$  with  $k_p$ ,  $k_v$  and  $\sigma$  are positive scalar constants, and  $I_{n \times n} \in R^{n \times n}$  is an identity matrix.

For symmetric matrix  $H \in R^{n \times n}$ ,  $\lambda_{\max}(H)$  and  $\lambda_{\min}(H)$  are maximum and minimum eigenvalues of  $H$  respectively.

$\hat{d}(t)$  represents the estimated disturbance, it is given by the learning law as follows

$$\hat{d}_{k+1}(t) = M_{k+1} \sum_{j=1}^k \eta z_j(t) \quad (14)$$

where  $z_k(t) = \dot{e}_k(t) + \sigma e_k(t)$ ,  $\eta = \mu K_v$ .

Then the following theorem can be proved.

### Theorem

Given the robot dynamics (1) with the tracking controller (13), where  $\hat{d}(t)$  is given by the iterative learning law (14), and let assumptions (A1-A2) be satisfied.

If

1.  $\sigma^2 \geq 2\psi$ ,
2.  $\lambda_{\min}(K_v - \sigma I_n) \geq \frac{1}{2}\lambda_{\max}(\eta)$ .

Then

$$\lim_{k \rightarrow \infty} e_k(t) = \lim_{k \rightarrow \infty} \dot{e}_k(t) = 0 \quad \forall t \in [0, T], \quad (15)$$

$$\lim_{k \rightarrow \infty} \hat{d}_k(t) = d(t) + G(t) \quad \forall t \in [0, T]. \quad (16)$$

Where  $\psi = \beta_1^{-1} g_m + \beta_1^{-2} l_m(l_d + l_g)$ , and  $\eta = \mu K_v$ .

### Proof

From (14), we have at  $k^{\text{th}}$  iteration

$$\hat{d}_k(t) = M_k \sum_{j=1}^{k-1} \eta z_j(t) \quad (16)$$

hence

$$\sum_{j=1}^{k-1} \eta z_j(t) = M_k^{-1} \hat{d}_k(t) \quad (17)$$

where  $M_k$  is nonsingular matrix.

From (14) and (17), we can write at  $(k+1)^{\text{th}}$  iteration

$$\hat{d}_{k+1} = M_{k+1} M_k^{-1} \hat{d}_k + M_{k+1} \eta z_k. \quad (18)$$

Substituting (13) to (1), we obtain at  $k^{\text{th}}$  iteration

$$\ddot{e}_k + K_v \dot{e}_k + K_p e_k = M_k^{-1} [d(t) + G_k(q) - \hat{d}_k(t)]. \quad (19)$$

From (18) and (19), we obtain at  $(k+1)^{\text{th}}$  iteration

$$\ddot{e}_{k+1} + K_v \dot{e}_{k+1} + K_p e_{k+1} = M_{k+1}^{-1} [d(t) + G_{k+1}(q)] - M_{k+1}^{-1} \hat{d}_k(t) - \eta z_k(t). \quad (20)$$

Subtracting (19) from (20), we have

$$\begin{aligned} \ddot{\tilde{z}}_k + (K_v - \sigma I) \dot{\tilde{z}}_k + \sigma^2 \tilde{e}_k &= (M_{k+1}^{-1} - M_k^{-1}) d \\ &+ (M_{k+1}^{-1} - M_k^{-1}) G_k \\ &+ M_{k+1}^{-1} (G_{k+1} - G_k) - M_{k+1}^{-1} \eta z_k \end{aligned} \quad (21)$$

where  $\tilde{e}_k = e_{k+1} - e_k$  and  $\tilde{z}_k = z_{k+1} - z_k$ .

Consider the Liapunov function candidate

$$V_k(t) = \int_0^t (z_k(\tau)^T \eta z_k(\tau)) d\tau \quad \text{for } t \in [0, T] \quad (22)$$

hence

$$V_{k+1} = V_k + \int_0^t [\tilde{z}_k^T \eta \tilde{z}_k + 2 \tilde{z}_k^T \eta z_k] d\tau. \quad (23)$$

Let's define

$$\Delta V_k = V_{k+1} - V_k. \quad (24)$$

From (21), (22), (23) and (24), we can write

$$\Delta V_k = \int_0^t [\tilde{z}_k^T \eta \tilde{z}_k - 2 \tilde{z}_k^T \tilde{e}_k - 2 \tilde{z}_k^T (K_v - \sigma I_n) \tilde{z}_k - 2 \tilde{z}_k^T \sigma^2 \tilde{e}_k$$

$$\begin{aligned} &+ 2 \tilde{z}_k^T M_{k+1}^{-1} (G_{k+1} - G_k) + 2 \tilde{z}_k^T (M_{k+1}^{-1} - M_k^{-1}) d \\ &+ 2 \tilde{z}_k^T (M_{k+1}^{-1} - M_k^{-1}) G_k] d\tau. \end{aligned} \quad (25)$$

Using assumption (A2), we have

$$\begin{aligned} \Delta V_k &= -\tilde{z}_k^T \tilde{z}_k - \tilde{e}_k^T \sigma^2 \tilde{e}_k \\ &+ \int_0^t [\tilde{z}_k^T \eta \tilde{z}_k - 2 \tilde{z}_k^T (K_v - \sigma I_n) \tilde{z}_k - 2 \tilde{e}_k^T \sigma^3 \tilde{e}_k \\ &+ 2 \tilde{z}_k^T M_{k+1}^{-1} (G_{k+1} - G_k) + 2 \tilde{z}_k^T (M_{k+1}^{-1} - M_k^{-1}) d \\ &+ 2 \tilde{z}_k^T (M_{k+1}^{-1} - M_k^{-1}) G_k] d\tau. \end{aligned} \quad (26)$$

According to properties (1, 3), using assumptions (A1-A2), and lemma, we obtain

$$\begin{aligned} \Delta V_k &\leq -\tilde{z}_k^T \tilde{z}_k - \tilde{e}_k^T \sigma^2 \tilde{e}_k + \int_0^t \left[ -\left( 2\lambda_{\min}(K_v - \sigma I_n) - \lambda_{\max}(\eta) \right) \|\tilde{z}\|^2 \right. \\ &+ 2 \left( \beta_1^{-1} g_m \|\tilde{e}_k\| + \beta_1^{-2} l_m(l_d + l_g) \|\tilde{e}_k\| \right) \dot{\tilde{e}}_k \\ &\left. + 2\sigma \left( \beta_1^{-1} g_m \|\tilde{e}_k\| + \beta_1^{-2} l_m(l_d + l_g) \|\tilde{e}_k\| \right) \tilde{e}_k - 2\sigma^3 \|\tilde{e}_k\|^2 \right] d\tau \end{aligned} \quad (27)$$

Hence

$$\begin{aligned} \Delta V_k &\leq -\|\tilde{z}_k\|^2 - (\sigma^2 - 2\psi) \|\tilde{e}_k\|^2 \\ &- \int_0^t \left[ \left( 2\lambda_{\min}(K_v - \sigma I_n) - \lambda_{\max}(\eta) \right) \|\tilde{z}_k\|^2 + (\sigma^2 - \psi) \|\tilde{e}_k\|^2 \right] d\tau \end{aligned} \quad (28)$$

where  $\psi = \beta_1^{-1} g_m + \beta_1^{-2} l_m(l_d + l_g)$ .

If

1.  $\sigma^2 \geq 2\psi$ ,
2.  $\lambda_{\min}(K_v - \sigma I_n) \geq \frac{1}{2}\lambda_{\max}(\eta)$ .

Then

$$\Delta V_k \leq 0 \quad t \in [0, T]. \quad (29)$$

Therefore  $V_k(t)$  is bounded  $\forall k \in N$  and  $t \in [0, T]$ .

Consequently, one can conclude that

$\lim_{k \rightarrow \infty} e_k(t) = \lim_{k \rightarrow \infty} \dot{e}_k(t) = 0$ ,  $t \in [0, T]$ , and from (19), (16) is satisfied. Which completes the proof.

## 4. Numerical simulation results

Consider the first three joints (waist  $q_1$ , shoulder  $q_2$ , elbow  $q_3$ ) of the PUMA 560 arm. The dynamic model for PUMA 560 can be written as (1).

The elements of  $M(q)$ ,  $C(q, \dot{q})$  and  $G(q)$  are given in appendix.

The Coulomb and Viscous friction is considered as an external disturbance.

### A. Friction simulation

The external disturbance considered is Coulomb and Viscous friction, given by

$$d(\dot{q})=c_1 \text{sign}(\dot{q})+c_2 \dot{q} \quad (30)$$

The parameters for first, second and third links in the simulation are given by

$$c_1=[100 \ 100 \ 100]^T N.m \quad (31)$$

$$c_2=[30 \ 30 \ 30]^T N.m/rad/s. \quad (32)$$

There are some problems in using the friction model (30) in simulation directly. One is due to the discontinuity of the friction characteristics at zero velocity, a very small step size is required for testing zero velocity. The other is that when the velocity is zero, or the system is stationary, the friction is indefinite and depends on the controlled torque. In the simulation, to improve the numerical efficiency, a revised friction model, which is modified from [21], is adopted. The revised friction model can be described by

$$d_r=d+(T_r-d)e^{-(\dot{q}/l)^2} \quad (33)$$

where  $d$  is given by (30),  $d_r$  is the revised friction,  $l$  is a small positive scalar, and  $T_r$  is given by

$$T_r(t)=\begin{cases} K & t>K \\ t & -K\leq t\leq K \\ -K & t<-K \end{cases} \quad (34)$$

where  $K$  is a positive scalar.

When the velocity is within a very small area near zero, defined by  $l$ , the friction  $d_r$  is equal to the applied torque  $T$ . When the velocity is greater than this, the second term in the above expression vanishes and the friction  $d_r$  given by this revised model is equal to the friction given by (30). In the simulation,  $l$  is chosen as 0.1.

### B. Simulation results

Simulation parameters:  $\eta=\text{diag}\{4,4,4\}$ ,  $\sigma=25$ ,  $K_v=\text{diag}\{27,27,27\}$ ,  $K_p=\text{diag}\{675,675,675\}$ .

$g_m$  is given by [22] :

$$g_m \geq n \left( \sup_{i,j,q} \left| \frac{\partial g_i(q)}{\partial q_j} \right| \right).$$

Therefore, we find that  $g_m=89.5[kg.m^2/sec^2]$ ,

$l_g=45.6[m/sec^2]$ ,  $\beta_1=1.5[kgm^2]$ ,  $l_d=400[N.m]$ .

The desired trajectories are

$$q_{d1}(t)=\frac{\pi}{2} \sin(2\pi t)+\pi \text{ rad} ; \quad 0 \leq t \leq 3.$$

$$q_{d2}(t)=0.2\sin(2\pi t)+1 \text{ rad} ; \quad 0 \leq t \leq 3.$$

$$q_{d3}(t)=\frac{\pi}{2} \sin(2\pi t)+\pi \text{ rad} ; \quad 0 \leq t \leq 3.$$

Figure (1-6) show the simulation results of real and desired position trajectories for the 1<sup>st</sup> and 40<sup>th</sup> iteration of each joint. We can see that the real position follows the desired position, through learning iterations. In figure (7-9), it is shown that the position error of the 40<sup>th</sup> operation is reduced much in contrast to the first operation for each joint.

Figure (10-15) show the convergence of the real velocity to desired velocity, through iteration number. Therefore, the control algorithm works well.

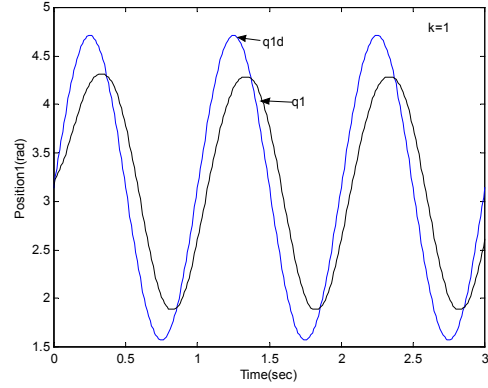


Fig.1 Real and desired position for first joint (k=1)

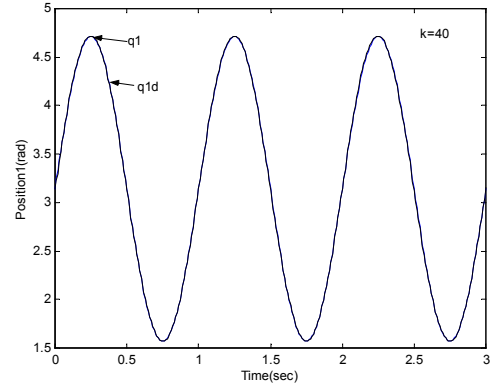


Fig.2 Real and desired position for first joint (k=40)

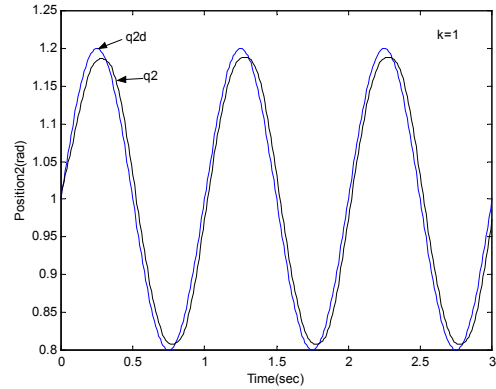


Fig.3 Real and desired position for second joint (k=1)

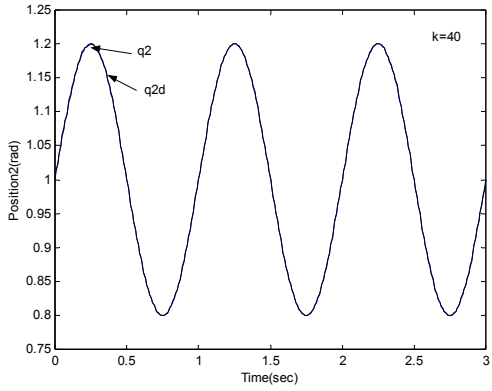


Fig.4 Real and desired position for second joint (k=40)

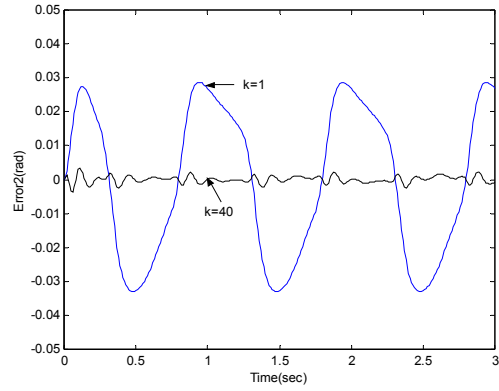


Fig.8 Position error for second joint (k=1 and k=40)

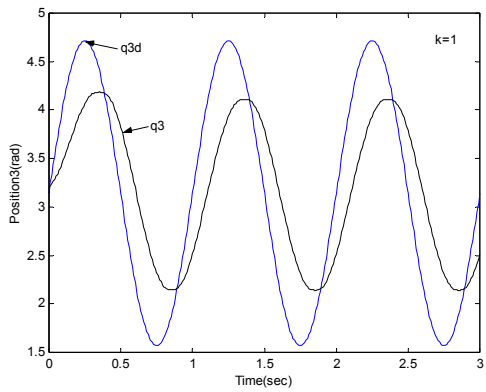


Fig.5 Real and desired position for third joint (k=1)

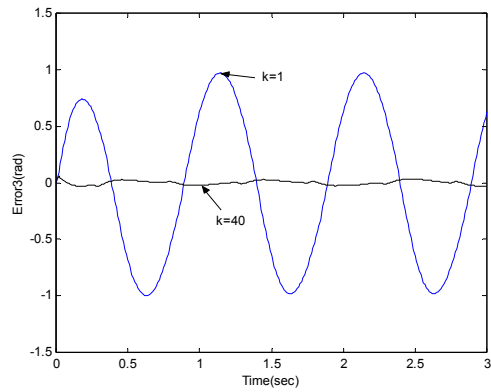


Fig.9 Position error for third joint (k=1 and k=40)

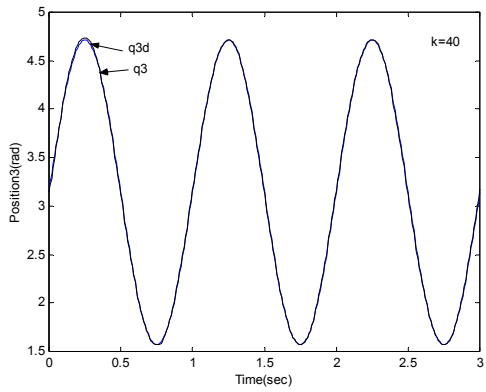


Fig.6 Real and desired position for third joint (k=40)

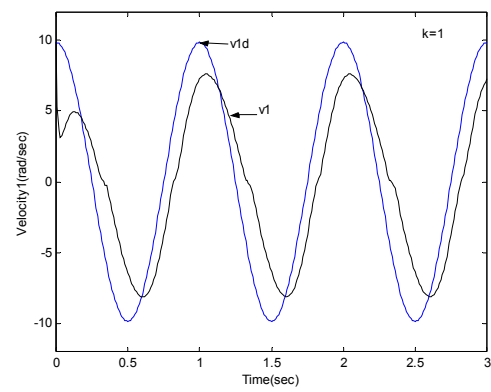


Fig.10 Real and desired velocity for first joint (k=1)

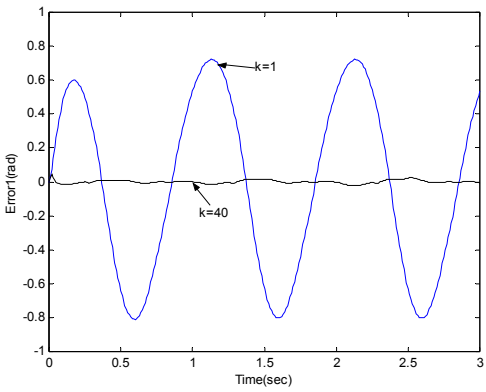


Fig.7 Position error for first joint (k=1 and k=40)

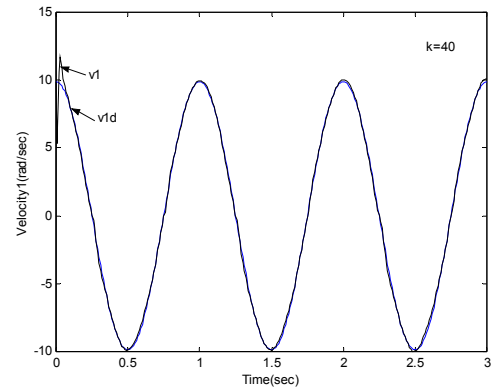


Fig.11 Real and desired velocity for first joint (k=40)

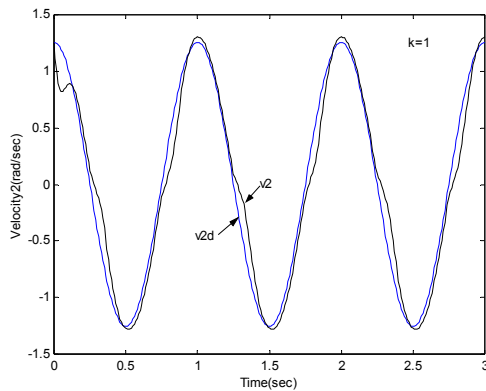


Fig.12 Real and desired velocity for second joint (k=1)

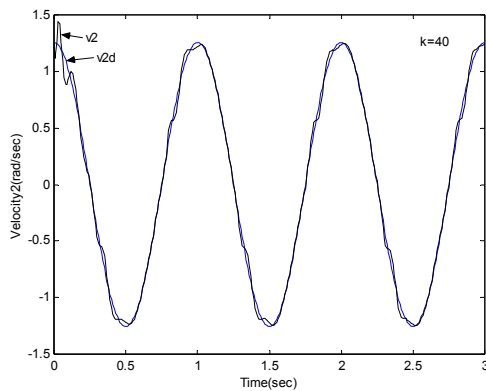


Fig.13 Real and desired velocity for second joint (k=40)

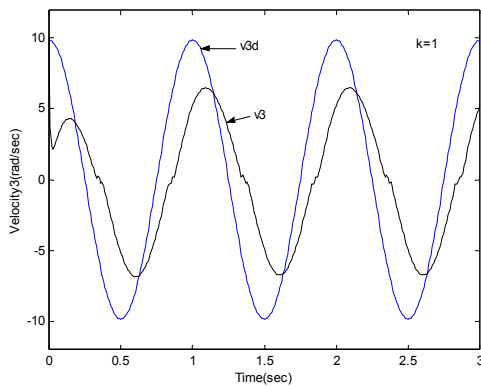


Fig.14 Real and desired velocity for third joint (k=1)

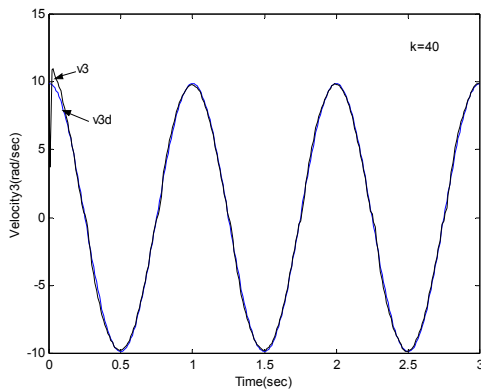


Fig.15 Real and desired velocity for third joint (k=40)

## 5. Conclusion

This paper has presented a disturbance observer based iterative learning control scheme for the position tracking problem of rigid robot manipulators with subject to external disturbances. The proposed controller is based upon a feedback controller, which is given by a computed torque control without compensating for the gravity terms, plus an iteratively term represents the disturbance estimated. Therefore, the external disturbances with the gravity forces are compensated, and the asymptotic stability is guaranteed. The proof of convergence is based upon the use of a Lyapunov-like positive definite sequence, which is shown to be monotonically decreasing under the proposed control scheme. Simulation results on the PUMA 560 robot manipulator show the asymptotic convergence of tracking error, when the Coulomb and Viscous friction is considered as an external disturbance.

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## Appendix

The dynamic model for PUMA 560 can be written as (1). We Consider the first three joints.

The elements of  $M(q)$  are given by

$$m_{11} \approx 2.57 + 1.38c_2^2 + 0.30s_2s_3 + 7.44 \times 10^{-1}c_2s_{23}$$

$$m_{12} = m_{21} \approx 6.90 \times 10^{-1}s_2 - 1.34 \times 10^{-1}c_{23} + 2.38 \times 10^{-2}c_2$$

$$m_{13} = m_{31} \approx -1.34 \times 10^{-1}c_{23} - 3.97 \times 10^{-3}s_{23}$$

$$m_{22} \approx 6.79 + 7.44 \times 10^{-1}s_3$$

$$m_{23} = m_{32} \approx 0.333 + 3.72 \times 10^{-1}s_3 - 1.10 \times 10^{-2}c_3$$

$$m_{33} \approx 1.16.$$

The elements of  $C(q, \dot{q})$

$$c_{11} \approx (-2.76s_2c_2 + 7.44 \times 10^{-1}c_{23} + 0.60s_2c_3$$

$$- 2.13 \times 10^{-2}(1 - 2s_2s_3))\dot{q}_2$$

$$c_{12} \approx (6.90 \times 10^{-1}c_2 + 1.34 \times 10^{-1}s_{23} - 2.38 \times 10^{-2}s_2)\dot{q}_2$$

$$+ (2.67 \times 10^{-1}s_{23} - 7.58 \times 10^{-3}c_{23})\dot{q}_3$$

$$c_{13} \approx (7.44 \times 10^{-1}c_2c_{23} + 0.60s_2c_3 + 2.20 \times 10^{-2}c_2s_{23}$$

$$- 2.13 \times 10^{-2}(1 - 2s_2s_3))\dot{q}_1$$

$$c_{21} \approx -0.5(-2.67s_2c_2 + 7.44 \times 10^{-1}c_{23} + 0.60s_2c_3$$

$$- 2.13 \times 10^{-2}(1 - 2s_2s_3))\dot{q}_1$$

$$c_{22} \approx (2.20 \times 10^{-2}s_3 + 7.44 \times 10^{-1}c_3)\dot{q}_3$$

$$c_{23} \approx 0.5(2.20 \times 10^{-2}s_3 + 7.44 \times 10^{-1}c_3)\dot{q}_3$$

$$c_{31} \approx -0.5(7.44 \times 10^{-1}c_2c_{23} + 0.60s_2c_3 + 2.20 \times 10^{-2}c_2s_{23}$$

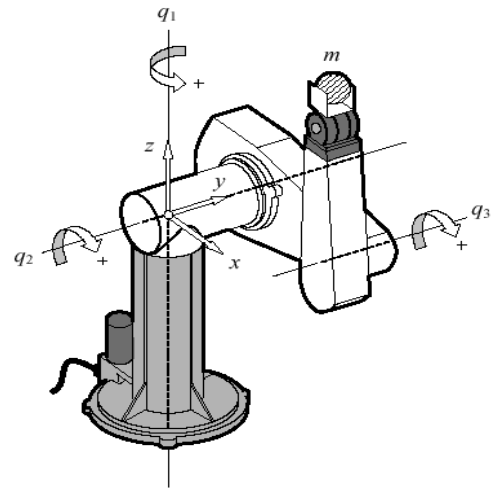
$$- 2.13 \times 10^{-2}(1 - 2s_2s_3))\dot{q}_1$$

$$c_{32} \approx -0.5(2.20 \times 10^{-2}s_3 + 7.44 \times 10^{-1}c_3)\dot{q}_3$$

$$c_{33} \approx 0$$

The elements of  $G(q)$ :  $g_1 \approx 0$

$$g_2 \approx -37.2c_2 - 8.4s_{23} + 1.02s_2, \quad g_3 \approx -8.4s_{23} - 0.25c_{23}.$$



PUMA 560 robot.