Speed Sensorless Control of PMSM Via Linear Kalman Filtering

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***Abstract* - This paper presents a novel PMSM sensorless control scheme based on a linear Kalman filter (LKF) observer. The LKF algorithm estimates the rotor flux components in the stator reference frame by measuring stator currents and voltages, and using a mathematical model based on rotor equations of the motor, and an observation model based on the imaginary power equation. In this way, the LKF algorithm is completely independent on stator resistance, and therefore rotor position estimates are not affected by even strong resistance variations. The proposed observer is linear but time-varying because the rotor speed is treated as a model parameter to be on-line updated. Estimation of rotor speed is performed in two steps: firstly a rough speed estimation is obtained by processing the estimates of rotor position; then a simple speed observer is used to improve the accuracy of speed estimates. The observer output, i.e. the final speed estimate, is fed back to both the speed control loop and LKF algorithm.**

***Index Terms* – Sensorless control, PMSM drives, Kalman filtering.**

I. INTRODUCTION

The field oriented control of a permanent magnet synchronous motor (PMSM) requires the knowledge of rotor position to perform an effective current (torque) control. Moreover, the speed control needs the rotor speed information. Therefore, high-performance PMSM drives are usually equipped with electromechanical sensors (i.e. resolver, absolute encoder) mounted on the rotor shaft.

Elimination of electromechanical sensors and cabling for the measurement of position and speed leads to great advantages in term of cost reduction, reliability, and noise immunity, because the sensor performance degrades due to vibration or humidity. Thus sensorless control strategies of PMSM have received much attention in industrial applications in the last years [1]. The main target of such methods is to extract the rotor position and angular speed information from the power leads of the machine, without using electromechanical transducers.

Most sensorless control schemes proposed in the literature for the estimation of rotor position and speed belongs to one of the following two categories:

- methods that depend on the fundamental excitation of the machine and speed-dependent phenomenon (back emf of motor phase),
- methods dependent on the magnetic saliency of the machine, due to saturation or geometric construction.

Sensorless control schemes belonging to the first category are generally not suitable for zero or very-low-speed operation [2-4], but efficient over a medium to high-speed range. This is due to the difficulty to extract position information from back emf having very small amplitude and frequency in presence of the system noise produced by the switching devices.

Accurate estimation of rotor position at low and zero speed has been successfully performed by injection and back-processing of high-frequency stator voltage or current components [5]. The technique gives good results in presence of marked magnetic anisotropy, and therefore it is less effective for surface-mounted PMSM, where the very low magnetic anisotropy needs highly accurate measurements and signal processing, and good noise-rejection characteristics.

The first-category sensorless control schemes can be further divided into three main classes. In the first one the motor equations are manipulated to express rotor speed and position as functions of terminal quantities (stator voltages and/or currents). This approach is very sensitive to motor parameter variations. The second class uses state observers to realise sensorless estimation schemes, but they generally do not guarantee the overall stability due to linearization of observer equations along the nominal state trajectory. In the third class the extended Kalman filter is used to estimate rotor position, speed, and model parameters [6-7]. The EKF-based approach is well suited to obtain high-accuracy estimates of state variables and model parameters in presence of strong measurement noise. Unfortunately, full-order EKFs are characterised by high computational efforts, while reduced-order ones have high sensitivity to parameter variations that cause a degradation of the estimation accuracy. Moreover, convergence and stability problems may arise when unsuitable initial conditions are selected.

In [9-10] the Authors proposed to estimate rotor speed and position of a surface-mounted PMSM by means of an EKF algorithm having a state-space model based on stator voltage equations. The approach is strongly dependent on electrical parameters like as stator resistance and PM flux linkage. Parameter uncertainties and variations cause errors in speed and position estimates, so that parameter online tuning becomes necessary. Moreover, a dedicated start-up procedure was proposed to avoid misconvergence of the estimated couple $\left(\hat{\omega}_r, \hat{\theta}_r\right)$ to the wrong couple $\left(-\omega_r, \theta_r + \pi\right)$ instead of the correct one $\left(\omega_r, \theta_r\right)$. This happens because the motor voltage equations formally admit two solutions, the correct one and the wrong one. At last, the EKF algorithm proposed in [9-10] is robust against mechanical parameter variations, since both the mathematical model and the observation equation of EKF do not depend on these parameters.

In this paper the Authors propose to estimate rotor position and speed of a PMSM with surface-mounted magnets by using a new sensorless algorithm that is absolutely unaffected by stator resistance and PM flux linkage variations. The new algorithm is based on a Linear Kalman Filter (LKF) stochastic observer [8], and its robustness against electrical parameters is due to the choice of a state-space model of LKF based on the PM flux linkage equation in stator reference frame, and an observation model based on the imaginary electric power equation. The measured quantities are the stator currents and the reference voltage vector that controls the pulse-width modulator. Rotor position detection is performed by using the estimates of $\alpha-\beta$ components of PM flux linkage. Rotor speed is obtained firstly by processing position estimates, and then using a simple speed observer as a filter to reduce noise due to the derivative operation. Therefore misconvergence of the estimated couple $\left(\hat{\omega}_r, \hat{\theta}_r\right)$ to the wrong couple $\left(-\omega_r, \theta_r + \pi\right)$ is avoided, and a dedicated start-up procedure is no more necessary. Speed and position estimates track the actual values very fast at start-up, even when initial position estimate error is large. The speed estimate is fed back to both the speed control loop and LKF algorithm. In fact the LKF estimator is linear but time variant, since the rotor speed enters as a time-varying parameter in its mathematical model. The estimated-speed feedback in the LKF can cause divergence of position estimate and instability of the sensorless algorithm at low speed, less than 10% of rated one

II. Mathematical Model of PMSM

In the stationary reference frame $\alpha-\beta$ the stator equation of a surface-mounted PMSM is as follows:

$$\bar{v}_{s\alpha\beta} = \left(R_s + L_s p\right)\bar{i}_{s\alpha\beta} + p\bar{\lambda}_{r\alpha\beta} \tag{1}$$

where

$$p\bar{\lambda}_{r\alpha\beta} = j\omega_r \bar{\lambda}_{r\alpha\beta} = j\omega_r \lambda_f \, e^{j\theta_r} \tag{2}$$

$\bar{v}_{s\alpha\beta}$ is the stator voltage vector, $\bar{i}_{s\alpha\beta}$ is the stator current vector, R_s is the stator resistance, L_s is the stator inductance, ω_r is the electrical rotor speed, λ_f is the stator magnet flux linkage, and p is the derivative operator.

The expression of the imaginary electric power is also needed, because of its use in the observation model of LKF algorithm:

$$P_r = \frac{3}{2} Im\left\{\bar{v}_{s\alpha\beta}\, \bar{i}^{*}_{s\alpha\beta}\right\} = \frac{3}{2}\left(v_{s\beta}\, i_{s\alpha} - v_{s\alpha}\, i_{s\beta}\right) \tag{3}$$

From (1) and (3) it results:

$$\begin{aligned} Im\left\{\bar{v}_{s\alpha\beta}\, \bar{i}^{*}_{s\alpha\beta}\right\} &= Im\left\{R_s i_s^2 + L_s \bar{i}^{*}_{s\alpha\beta} p\bar{i}_{s\alpha\beta} + \bar{i}^{*}_{s\alpha\beta} p\bar{\lambda}_{r\alpha\beta}\right\} \\ &= L_s\left(i_{s\alpha} p i_{s\beta} - i_{s\beta} p i_{s\alpha}\right) + i_{s\alpha} p\lambda_{r\beta} - i_{s\beta} p\lambda_{r\alpha} \end{aligned} \tag{4}$$

In the synchronously rotating d-q reference frame the voltage equation (1) becomes:

$$\bar{v}_{sdq} = (R_s + L_s p)\bar{i}_{sdq} + j\omega_r L_s \bar{i}_{sdq} + j\omega_r \lambda_f \qquad (5)$$

To enhance d- and q-axis current control the coupling terms of (5) and back electromotive force (b.e.m.f.) have to be compensated through the feedforward injection of the voltage vector

$$\bar{v}_{sdq,comp} = j\omega_r L_s \bar{i}_{sdq} + j\omega_r \lambda_f \qquad (6)$$

The d- and q-axis current components are then indirectly controlled by the following voltage vector:

$$\bar{v}'_{sdq} = \bar{v}_{sdq} - \bar{v}_{sdq,comp} = (R_s + L_s p)\bar{i}_{sdq} \qquad (7)$$

The mathematical model of PMSM is completed by the mechanical equation:

$$\frac{J}{n_p} p\omega_r = T_e - T_L = \left(\frac{3}{2} n_p \lambda_f i_{sq}\right) - T_L \qquad (8)$$

where J is the motor and load inertia, n_p is the number of pole pairs, T_L is the load torque, and T_e is the electromagnetic torque.

To realise speed sensorless control of PMSM drives rotor position and speed have to be estimated somehow. Errors in rotor position estimation cause wrong variable transformations and, consequently, the drive performance deteriorates.

III. LKF ALGORITHM FOR POSITION ESTIMATION

In the sensorless control scheme proposed in this paper the rotor position detection is realised by using a LKF observer, able to estimate the rotor flux components in the stationary reference frame $\alpha-\beta$. The discrete-time mathematical model of the stochastic observer is derived from equation (2). It results:

$$\begin{bmatrix}\lambda_{r\alpha}(n+1)\\ \lambda_{r\beta}(n+1)\end{bmatrix} = \begin{bmatrix}1 & -\omega_r^e(n)T_c\\ \omega_r^e(n)T_c & 1\end{bmatrix}\begin{bmatrix}\lambda_{r\alpha}(n)\\ \lambda_{r\beta}(n)\end{bmatrix} + \begin{bmatrix}w_\alpha(n+1)\\ w_\beta(n+1)\end{bmatrix} \qquad (9)$$

and in matrix form:

$$\boldsymbol{X}_{n+1} = \Phi_n \boldsymbol{X}_n + \boldsymbol{W}_{n+1} \qquad (10)$$

where T_c is the sampling time, and the terms w_α and w_β represent the model errors, assumed to be white and Gaussian uncorrelated noises, with zero mean values and covariance matrix $\boldsymbol{Q}$. The system noise takes into account system disturbances and model inaccuracies. It has to be noted that the rotor speed ω_r^e is a time-varying parameter to be on-line updated after it has been estimated somehow.

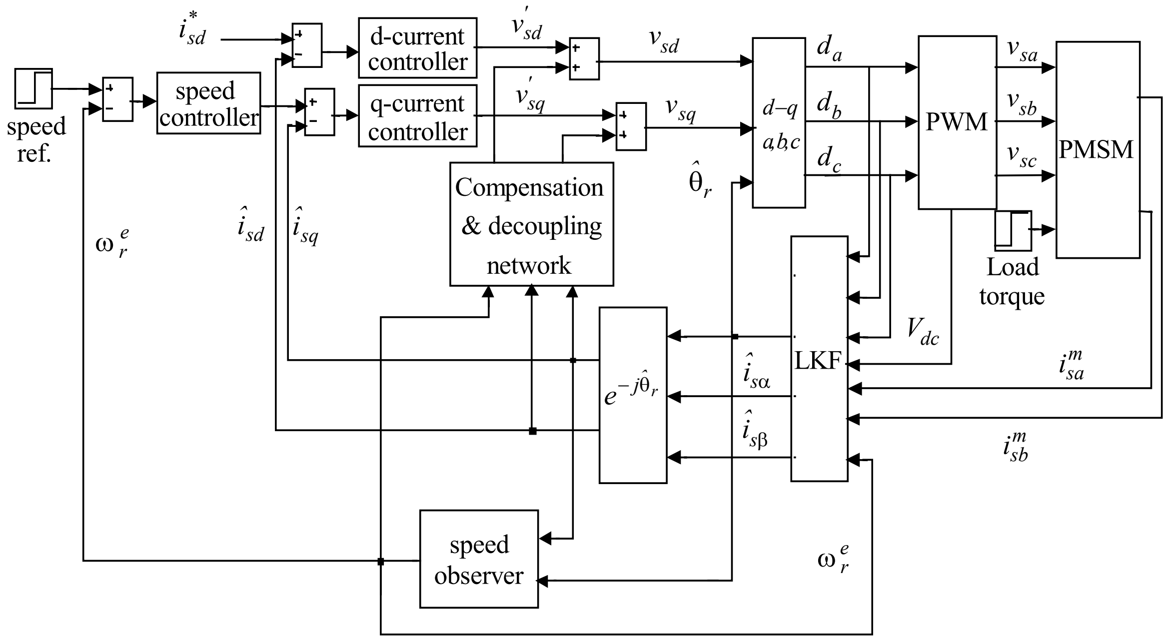


Fig. 1: Simulink block diagram of the PMSM sensorless drive.

The LKF algorithm also needs an observation model to relate the observed quantities to the state variables. In our case, the observed variable is obtained from the imaginary electric power, taking into account the stator voltage equations. We have the following:

$$\begin{aligned} Z^m(n+1) &= \left[v_{s\beta}^m(n+1)\,i_{s\alpha}^m(n+1) - v_{s\alpha}^m(n+1)\,i_{s\beta}^m(n+1)\right]T_c \\ &= \left[-i_{s\beta}^m(n+1)\lambda_{r\alpha}(n+1) + i_{s\alpha}^m(n+1)\lambda_{r\beta}(n+1)\right] \\ &\quad -\left[-i_{s\beta}^m(n+1)\lambda_{r\alpha}(n) + i_{s\alpha}^m(n+1)\lambda_{r\beta}(n)\right] \\ &\quad + L_s\left[i_{s\alpha}^m(n)\,i_{s\beta}^m(n+1) - i_{s\beta}^m(n)\,i_{s\alpha}^m(n+1)\right] \end{aligned} \tag{11}$$

where the apex (m) denotes measured quantities. The observation model is therefore:

$$\begin{aligned} Z^m(n+1) &= \begin{bmatrix} -i_{s\beta}^m(n+1) \\ i_{s\alpha}^m(n+1) \end{bmatrix}^T \left\{ \begin{bmatrix} \lambda_{r\alpha}(n+1) \\ \lambda_{r\beta}(n+1) \end{bmatrix} - \begin{bmatrix} \lambda_{r\alpha}(n) \\ \lambda_{r\beta}(n) \end{bmatrix} \right\} \\ &\quad + L_s\left[i_{s\alpha}^m(n)\,i_{s\beta}^m(n+1) - i_{s\beta}^m(n)\,i_{s\alpha}^m(n+1)\right] + \nu(n+1) \end{aligned} \tag{12}$$

The measurement error ν is assumed to be white and Gaussian noise, uncorrelated with the model one, with zero mean value and variance R. If we put:

$$y^m(n+1) = Z^m(n+1) - L_s\left[i_{s\alpha}^m(n)\,i_{s\beta}^m(n+1) - i_{s\beta}^m(n)\,i_{s\alpha}^m(n+1)\right] \tag{13}$$

the observation model becomes:

$$y^m(n+1) = \begin{bmatrix} -i_{s\beta}^m(n+1) \\ i_{s\alpha}^m(n+1) \end{bmatrix}^T \left\{ \begin{bmatrix} \lambda_{r\alpha}(n+1) \\ \lambda_{r\beta}(n+1) \end{bmatrix} - \begin{bmatrix} \lambda_{r\alpha}(n) \\ \lambda_{r\beta}(n) \end{bmatrix} \right\} + \nu(n+1) \tag{14}$$

and assumes the structure of a delayed-state Kalman filter, since the observed variable y^m at time n+1 depends on both the state at sampling instants n and n+1. Equation (15) can be written in matrix form as follows:

$$\boldsymbol{Y}_{n+1} = \boldsymbol{H}_{n+1}\boldsymbol{X}_{n+1} + \boldsymbol{J}_{n+1}\boldsymbol{X}_n + \boldsymbol{v}_{n+1} \tag{15}$$

where $\boldsymbol{H}_{n+1} = -\boldsymbol{J}_{n+1} = \begin{bmatrix} -i_{s\beta}^m(n+1) \\ i_{s\alpha}^m(n+1) \end{bmatrix}^T$.

Beginning from an initial state estimate $\hat{\boldsymbol{X}}_0(+)$ and covariance matrix of estimation errors $\boldsymbol{P}_0(+)$, the recursive LKF algorithm is composed by the following steps:

1) state prediction:

$$\hat{\boldsymbol{X}}_n(-) = \hat{\Phi}_{n-1}\,\hat{\boldsymbol{X}}_{n-1}(+) \tag{16}$$

where $\hat{\Phi}_{n-1}$ is the transition matrix at time (n-1)T_c computed using the speed $\omega_r^e(n-1)$ estimated by the observer described in the next section;

2) innovation:

$$\boldsymbol{In}_n = \boldsymbol{Y}_n^m - \boldsymbol{H}_n\,\hat{\boldsymbol{X}}_n(-) - \boldsymbol{J}_n\,\hat{\boldsymbol{X}}_{n-1}(+) \tag{17}$$

3) covariance matrix of prediction errors:

$$\boldsymbol{P}_n(-) = \hat{\Phi}_n\,\boldsymbol{P}_{n-1}(+)\hat{\Phi}_n^T + \boldsymbol{Q}_n \tag{18}$$

4) Kalman gain matrix:

$$\boldsymbol{G}_n = \left[\boldsymbol{P}_n(-)\boldsymbol{H}_n^{\mathrm{T}} + \hat{\Phi}_{n-1}\,\boldsymbol{P}_{n-1}(+)\boldsymbol{J}_n^{\mathrm{T}}\right]\boldsymbol{L}_n^{-1} \tag{19}$$

where

$$\begin{aligned} \boldsymbol{L}_n &= \boldsymbol{H}_n\,\boldsymbol{P}_n(-)\boldsymbol{H}_n^{\mathrm{T}} + \boldsymbol{R}_n + \left[\boldsymbol{J}_n + \boldsymbol{H}_n\,\hat{\Phi}_{n-1}\right]\boldsymbol{P}_{n-1}(+)\boldsymbol{J}_n^{\mathrm{T}} \\ &\quad + \boldsymbol{J}_n\,\boldsymbol{P}_{n-1}(+)\hat{\Phi}_{n-1}^{\mathrm{T}}\boldsymbol{H}_n^{\mathrm{T}} \end{aligned}$$

5) state estimation:

$$\hat{\boldsymbol{X}}_n(+) = \hat{\boldsymbol{X}}_n(-) + \boldsymbol{G}_n\,\boldsymbol{In}_n \tag{20}$$

6) covariance matrix of estimation errors:

$$\boldsymbol{P}_n(+) = \boldsymbol{P}_n(-) - \boldsymbol{G}_n\,\boldsymbol{L}_n\,\boldsymbol{G}_n^T \tag{21}$$

7) update of the transition matrix $\hat{\Phi}(n)$ by means of rotor speed estimate $\omega_r^e(n)$.

Once the α–β rotor flux components are estimated, rotor position can be computed as follows:

$$\hat{\theta}_r(n) = arctan\left(\frac{\hat{\lambda}_{r\beta}(n)}{\hat{\lambda}_{r\alpha}(n)}\right) \tag{22}$$

IV. Rotor Speed Estimation

Operation of LKF observer needs the rotor speed to be estimated, because it is treated as a time-varying parameter in the LKF model. Speed estimation is performed by using the functions $\sin(\hat{\theta}_r)$ and $\cos(\hat{\theta}_r)$ at times (n) and (n+1) as follows:

$$\hat{\omega}_r(n+1) = cos(\hat{\theta}_r(n+1))\frac{sin(\hat{\theta}_r(n+1)) - sin(\hat{\theta}_r(n))}{T_c}$$
$$- sin(\hat{\theta}_r(n+1))\frac{cos(\hat{\theta}_r(n+1)) - cos(\hat{\theta}_r(n))}{T_c} \quad (23)$$

However, this speed estimate is highly corrupted by noise due to the derivative operations, and therefore has to be suitably filtered using the speed observer shown in fig. 2. The electromagnetic torque, fed in the speed observer, is estimated by using its expression in d-q reference frame $\hat{T}_e = (3/2)n_p \lambda_f \hat{i}_{sq}$.

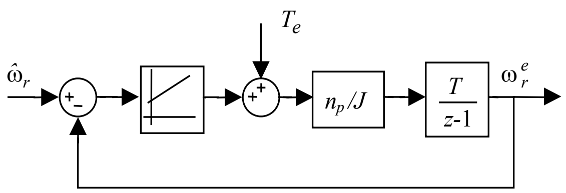


Fig. 2: Block scheme of the speed observer.

The PI controller parameters of the speed observer are tuned according to the absolute value optimum criterion, setting the time constant of the observer equal to 4 ms.

V. SIMULATION TESTS

Simulation tests have been performed both at high and low speed by using SIMULINK program of MATLAB. The parameters of the PMSM used in the tests are as follows: stator resistance R_S = 5.2 Ω, stator inductance L_S = 4.35 mH, magnet flux linkage λ_f = 0.1 Wb, rated torque T_n = 0.6 Nm, system inertia $J = 0.86*10^{-4}$ kg m^2, viscous friction coefficient $B = 2.94*10^{-5}$ Ns/rad, rated current I_n = 1 A, DC link voltage V_{DC} = 300 V, rated electrical speed ω_n = 1675 rad/s, pole pairs n_p = 4. The phase voltages are reconstructed from DC-bus voltage and duty cycle, and motor currents are filtered by a first-order low pass filter with cut-off frequency of 5 kHz, and then sampled at 10 kHz. In the LKF mathematical model the rated motor parameters are used.

The LKF has been started assigning the following initial state-estimate vector and initial estimate-error covariance matrix:

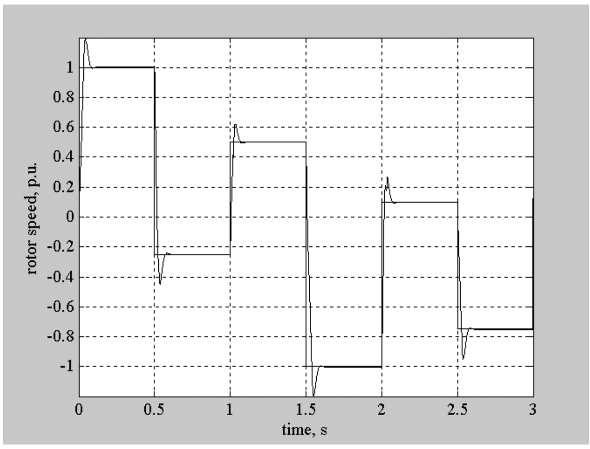
$\hat{\boldsymbol{X}}_0(+) = [\lambda_f \quad 0]$, $\boldsymbol{P}_0(+) = 10^{-2}\boldsymbol{I}$.

Taking into account the uncertainties on the observed speed, the covariance matrix of the model errors has been chosen as $\boldsymbol{Q} = 10^{-2}\ \boldsymbol{I}$. White noise sequences with normal distribution have also been added to the voltage and current inputs to simulate measurement errors. The noise variance of the observed variable y^m is $R = 10^{-6}$ [Wb^{2}A^2].

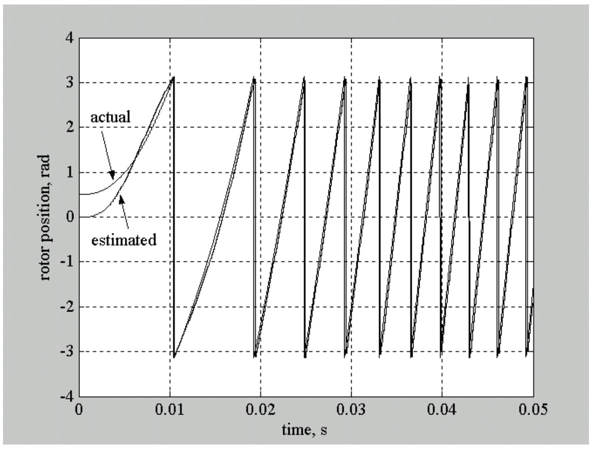
Figures from 3(a) to 3(f) show the test results obtained at different speed-reference step signals (both high and low speed), when the stator resistance and PM flux linkage are perfectly known and equal to their rated values, and when the initial position estimate-error is 30 electrical degrees. It is evident that the high convergence rate and accuracy of the position estimates, at both motor start-up and speed inversion, guarantee a correct field orientation and good speed estimation in the range from 10% of rated speed to full speed. To demonstrate the insensitivity of the proposed sensorless algorithm against stator resistance and PM flux linkage variations, other tests have been performed with the following parameter detuning in the motor model: $R_s = 1.3\, R_{sn}$, $\lambda_f = 0.8\, \lambda_{fn}$, where the index n denotes nominal values. These test results are shown in figures from 4(a) to 4(c). It has to be pointed out that the steady-state q-axis current component at full load is no more equal to 1 p.u. but, as expected, to 1.25 p.u. because of the PM flux linkage reduction.

VI. CONCLUSIONS

The paper has presented a sensorless technique for PMSM drives consisting on rotor position estimation from a linear Kalman filter-based algorithm. The proposed observer is linear but time-varying because the rotor speed is treated as a model parameter to be on-line updated. Processing the estimates of rotor position and using a simple speed observer one can perform accurate speed estimation. The proposed technique guarantees good performance of the drive over a wide speed range, from 10% of rated speed to full speed, and also an effective maximum torque/current control of the drive because the field orientation is achieved. The algorithm needs low computational efforts and, consequently, is suitable for implementation on low cost μp.



(a)



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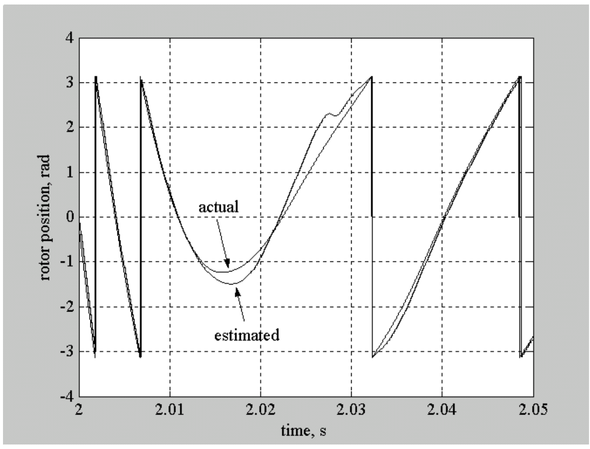

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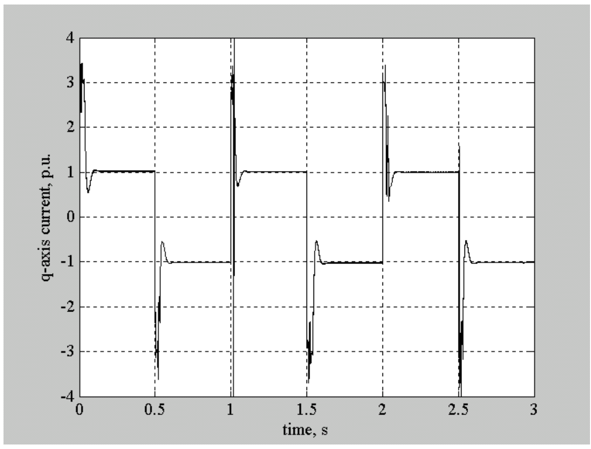

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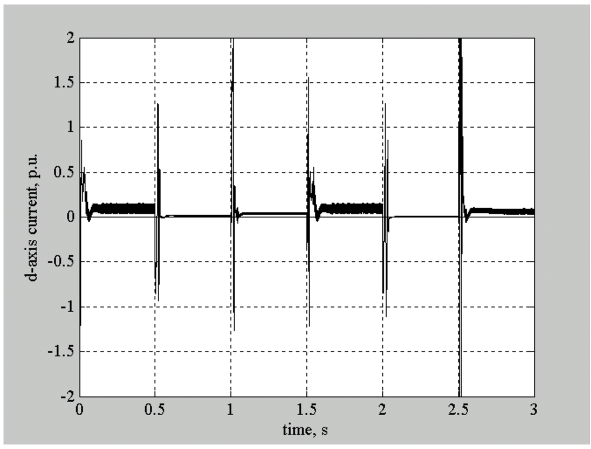

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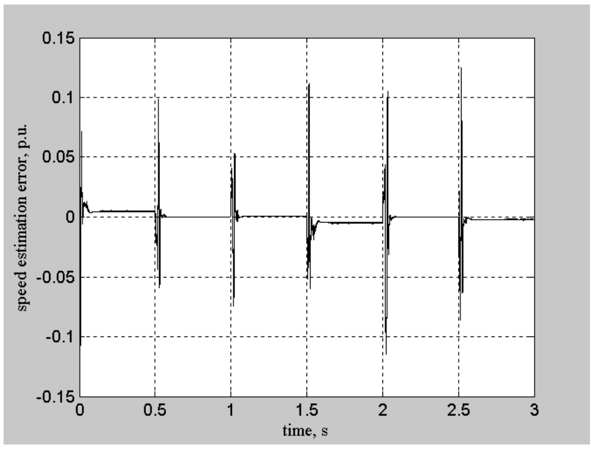
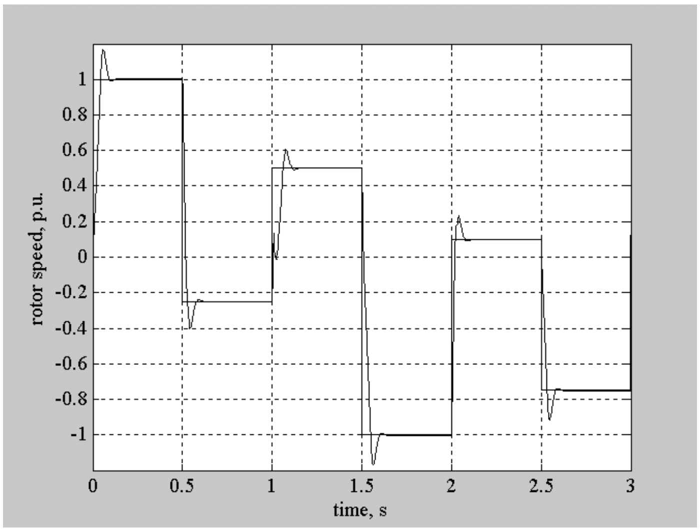


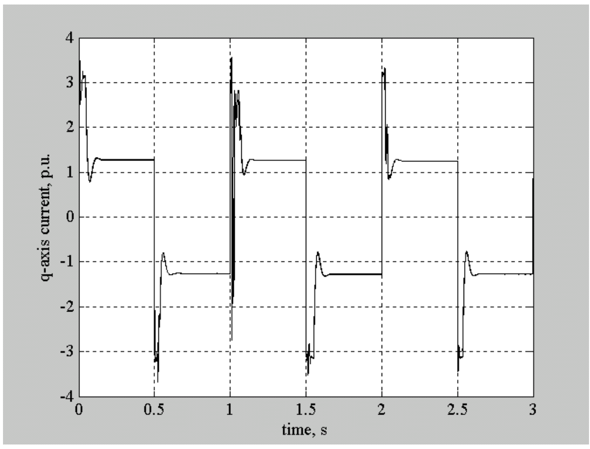
(f)

Figs. 3: Test results obtained with no detuning of stator resistance and magnet flux linkage:

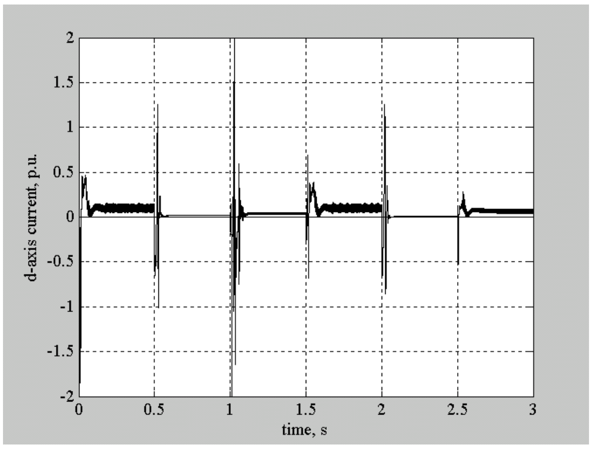
a) rotor speed response;
b) actual and estimated rotor position at start-up;
c) actual and estimated rotor position at speed inversion;
d) actual and estimated q-axis current responses;
e) actual and estimated d-axis current responses;
f) speed estimation errors.



(a)



(b)



(c)

Figs. 4: Test results obtained with detuning of stator resistance and magnet flux linkage:
a) rotor speed response;
b) actual and estimated q-axis current responses;
c) actual and estimated d-axis current responses.

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