STUDY AND ANALYSIS OF ADAPTIVE AIR SUSPENSION SYSTEM

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Abstract: *The increased concern on ride comfort and handling characteristics of an automobile have led to extensive research on automobile suspension systems. The authors of this article propose an adaptive air suspension system with LQR control strategy. The LQR controller is tuned by a combination of PSO and manual tuning. A dynamic model of the air suspension system used in passenger vehicles is designed and simulated for both passive and adaptive systems using MATLAB/Simulink. Air suspension is a non-linear system and thus the authors have derived a stiffness equation for the same with minimal assumptions. A comparative analysis is performed with the widely used PID controller to compare the efficiency of the proposed controller. The results are obtained for bumps, pot holes and ISO standard random road conditions. The settling time, peak displacement and tuning strategies are compared and analyzed. The results show that the adaptive system can achieve better vibration isolation compared to passive system. On comparing the analysis parameters, it is seen that the LQR controller has better potential to improve ride comfort by reducing the maximum displacement amplitude of the vehicle by 89.88% and provide better handling characteristics by reducing the settling time of the system by 85%.*

Keywords: *air suspension, PID, LQR, PSO, adaptive suspension, ride comfort.*

1. INTRODUCTION

The increase in demand for ride comfort calls for the use of active suspension systems in automobile. A good suspension system is expected to provide low suspension deflection transmissibility for handling and low vibration transmissibility for better ride comfort. The conflict between these two always exists in a passive suspension system which demands the use of an adaptive air suspension system whose stiffness can be manipulated with the air pressure inside the bellow based on load experienced.

Ride comfort is directly related to the acceleration sensed by passengers when travelling on a rough road. Suspension travel refers to the relative displacement between the sprung and the unsprung masses [1]. A suspension system is basically a nonlinear system. For the ease of analysis, an equivalent linear system should be considered [2]. An Active suspension system can provide better ride comfort and handling compared to its passive counterpart. For evaluating their performances, metrics like ride comfort, ease of handling, suspension deflection, actuator saturation and controller constrained information should be considered [3].

In passenger vehicles, especially while travelling on Indian roads, low frequency vibrations are experienced by the passengers. This is the same in case of farm vehicles too. Prolonged exposure of human body to these low frequency vibrations may have harmful and dangerous effects on human health. Thus, to attenuate prolonged low frequency vibrations, the authors propose an adaptive suspension system using an effective control strategy to provide a dynamic stiffness.

A leaf spring or a coil spring can provide necessary suspension and ensure road – vehicle contact which is the primary function. But, while travelling over an irregular road, they cannot adjust their stiffness in order to provide necessary ride comfort to the passenger. An airspring is non-linear by nature. Various factors such as change in pressure, volume and height of the bellow influence the stiffness directly apart from the load experienced from the road and passengers. The change in volume, variation in air mass and effective area can have an impact on the stability of the system. Increase in fixed volume and area, and increase in air mass flow rate of air spring, has positive effects on its stability [4]. The authors in this article have derived an equation for the air spring stiffness considering these vital parameters thereby ensuring the accuracy of measurement. The same is given as an input in Matlab/Simulink simulation for accurate results.

A PID controller is readily available in market whereas LQR control is a strategy and the controller can be designed and developed as per user specifications integrating the LQR algorithm. The LQR controller works on satisfying the Riccati equation with effective weighting functions. In this work, the LQR controller is tuned using a combination of Particle Swarm Optimization and manual tuning to obtain efficient weighting functions.

2. MATHEMATICAL MODELING

The full vehicle is a nonlinear system by design. It is a complicated and complex process to evaluate the complete vehicle. In the past, the vehicle suspension system was simplified as a nonlinear system with multiple frequency excitations [5]. For studying the chaotic motions in the vehicle, a quarter car model was considered. On application of LQ regulators in 2D models with preview control, ride and handling could be improved [6]. A 2DOF quarter car is modelled as shown in Fig. (1a), as a sprung mass mounted on a spring and damper (suspension system), carried by an unsprung mass mounted on a tire which ensures road-vehicle contact.

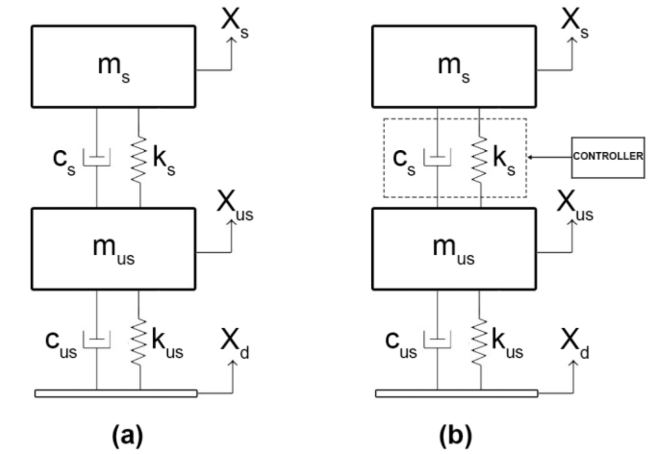

Fig.1 – 2DOF Quarter-car model: (a) Passive (b) Adaptive

Table 1. Quarter-car parameters

	Mass m (kg)	Stiffness k (N/m)	Damping Coefficient c
Sprung Mass	221	48873	657.275
Unsprung Mass	32	125000	1447.213

The quarter car is modelled in Matlab environment considering the passenger mass and vehicle body mass combined as sprung mass where the vehicle body mass is constant and passenger mass varies. The axle on which suspension is mounted is taken as unsprung mass. The suspension spring rate and damping co-efficient and tire stiffness and damping co-efficient are taken as given in Table 1. A step input of 0.1m amplitude is given for studying the response of the passive system.

Before evaluating, necessary assumptions are to be made. Pressure gradient in air spring is neglected. Air flow resistance in the orifice of air spring is neglected. For ease of analysis, pitch and role angles experienced by the system are assumed to be negligible. The input can be experienced from two sources; the road undulations and the passenger mass along with the vehicle mass. It is assumed that the system experiences a complete longitudinal acceleration only.

2.1 System Description

The quarter car model consists of a sprung mass of 221kg. The stiffness of the air spring is directly proportional to the air pressure inside the bellow. Thus the pressure of the system is adjusted based on the displacement due to load for static load and dynamic loading conditions. Initially the pressure of the system should be maintained at operating pressure.

The system can be represented as,

$$m_s \ddot{x}_s + c_s(\dot{x}_s - \dot{x}_{us}) + k_s(x_s - x_{us}) = 0 \quad (1)$$

$$m_{us} \ddot{x}_{us} + c_s(\dot{x}_{us} - \dot{x}_s) + k_s(x_{us} - x_s) + c_{us}(\dot{x}_s - \dot{x}_{us}) + k_{us}(x_s - x_{us}) = 0 \quad (2)$$

The adaptive air spring consists of four parts: rubber bellows, on/off (solenoid) valve, an auxiliary reservoir and a compressor. The air spring is connected to a compressor through a solenoid valve which is controlled by the controller. No external reservoir is considered for this study.

2.2 Non-Linear System

The airspring is a non-linear system by nature. The behaviour of the system can be defined by its volume, effective area, pressure inside the bellow, height of the bellow and the mass acting on the bellow. The other influencing factors are temperature and environmental conditions. The temperature inside the bellow is assumed to be constant.

The mass flow rate of air into airspring can be written as:

$$\dot{m} = -\frac{d(\rho_s V_s)}{dt} \quad (3)$$

Since density and volume are time dependent, differentiating (3) we get:

$$\dot{m} = -\dot{\rho}_s V_s - \rho_s \dot{V}_s \quad (4)$$

Assuming an isentropic process, the relationship between density and pressure at two different states are given as:

$$\frac{\rho_s}{\rho_{se}} = \left(\frac{P_s}{P_{se}}\right)^n \quad (5)$$

The gas is assumed as an ideal gas and ideal gas law for the density at equilibrium is applied. Differentiating the equation with respect to time and rearranging the terms, we get:

$$\dot{P}_S = \frac{nRT}{V_s}\left(\frac{P_s}{P_{se}}\right)^{\frac{1-n}{n}} \dot{P}_S \quad (6)$$

Substituting (6) in (4), we get a first order differential equation for pressure in the airspring:

$$\dot{P}_S = \frac{nRT}{V_s}\left(\frac{P_s}{P_{se}}\right)^{\frac{1-n}{n}} \dot{m} - \frac{nP_s}{V_s}\dot{V}_s \quad (7)$$

2.3 Linearized Dynamics

The above section proves that change in pressure inside the airspring can be expressed as a function of change in volume and the mass flow rate of the air. Thus to design an adaptive suspension system, we need to control the volume of air inside the airspring and its mass flow rate. The non-linear equations are linearized to gain useful insights into system dynamics and to develop and perform simple simulations. The equation (7) can be expanded using Taylor's series.

Applying Taylor's series to (7) to obtain linearized equation, we get,

$$\dot{P}_S = \dot{P}_{Se} + \frac{d\,\dot{P}_S}{dP_s}|_e\,(P_s - P_{se}) + \frac{d\,\dot{P}_S}{dV_s}|_e\,(V_s - V_{se}) + \frac{d\,\dot{P}_S}{d\,\dot{V}_s}|_e\left(\dot{V}_s - \dot{V}_{se}\right) + \frac{d\,\dot{P}_S}{d\,\dot{m}}|_e\left(\dot{m} - \dot{m}_e\right) \quad (8)$$

The subscript e indicates evaluation at equilibrium conditions where,

$$P_{se} = \dot{m}_e = \dot{V}_{se} = 0 \quad (9)$$

Thus, applying the condition,

$$\dot{P}_s = -\frac{nRT}{V_{se}}\dot{m} - \frac{nP_{se}}{V_{se}}\dot{V}_s \quad (10)$$

The height of the airspring at a particular period can be expressed as a sum of relative displacement and height of the airspring at equilibrium.

$$h = (z_S - z_r) + h_e \quad (11)$$

Therefore,

$$\dot{h} = \dot{z}_S - \dot{z}_r \quad (12)$$

Computing change in volume with respect to height using chain rule we get,

$$\dot{V}_s = \left(\frac{dV_s}{dh}\right)\left(\frac{dh}{dt}\right) = \vartheta\,\dot{h} \quad (13)$$

Where ϑ is specific volume of the system.

Substituting (13) in (10),

$$\dot{P}_s = -\frac{nRT}{V_{se}}\dot{m} - \frac{n\vartheta P_{se}}{V_{se}}\dot{h} \quad (14)$$

We know that, pressure is expressed as force per unit area. The total internal force is differentiated to produce,

$$\dot{F} = \dot{P}_s\,A_{eff} \quad (15)$$

Substituting (14) in (15),

$$\dot{F} = -\frac{nRTA_{eff}}{V_{se}}\dot{m} - \frac{n\vartheta P_{se}A_{eff}}{V_{se}}\dot{h} \quad (16)$$

At static conditions, there is no flow of air inside of outside the bellow. Thus mass flow rate approaches zero. That is,

$$\dot{m} \to 0$$

Therefore (16) can be rewritten as

$$\dot{F} = -\frac{n\vartheta P_{se}A_{eff}}{V_{se}}\dot{h} \quad (17)$$

The above equation relates the change in force to change in height which is force verses deflection relationship. Therefore we can conclude that,

$$\dot{F} = K_S\,\dot{h} \quad (18)$$

where, K_s is defined as the static stiffness of airspring.

The other additional assumptions include, the gas inside the bellow is perfect gas. The base area of the air bellow is assumed to be the same as the effective area of the spring. The simulation performed is for the system under closed valve condition. The other frictional characteristics are neglected. The air spring is considered as a hollow cylindrical structure.

The generalized state space representation is given by two equations, a state equation and the output equation to observe the output (19). The advantage in the state space approach is, it can be analyzed for different initial conditions. The state and the output equation is given by,

$$\left.\begin{aligned} \dot{x}(t) &= Ax + Bu \\ y &= Cx \end{aligned}\right\} \quad (19)$$

where,
x is State vector (order of equation n x 1)
A is State matrix (n x n)
B is Input matrix (n x p)
u is Input vector (q x 1)
C is Output matrix (1 x n)

The state, input and output matrices are shown in (20). The assumed states are displacement and velocity of sprung mass, the matrices, and to observe the displacement of sprung mass,

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{c_s c_{us}}{m_s m_{us}} & 0 & \left(\left(\frac{c_s}{m_s}\right)*\left(\left(\frac{c_s}{m_s}\right)+\left(\frac{c_s}{m_{us}}\right)+\left(\frac{c_{us}}{m_{us}}\right)\right)\right)-\left(\frac{k_s}{m_s}\right) & -\left(\frac{c_s}{m_s}\right) \\ \frac{c_{us}}{m_{us}} & 0 & -\left(\left(\frac{c_s}{m_s}\right)+\left(\frac{c_s}{m_{us}}\right)+\left(\frac{c_{us}}{m_{us}}\right)\right) & 1 \\ \frac{k_{us}}{m_{us}} & 0 & -\left(\left(\frac{k_s}{m_s}\right)+\left(\frac{k_s}{m_{us}}\right)+\left(\frac{k_{us}}{m_{us}}\right)\right) & 0 \end{bmatrix}$$

$$B=\begin{bmatrix} 0 & 0 \\ \frac{1}{m_s} & \frac{c_s/c_{us}}{m_s m_{us}} \\ 0 & -\frac{c_{us}}{m_{us}} \\ \frac{1}{m_s}+\frac{1}{m_{us}} & -\frac{k_{us}}{m_{us}} \end{bmatrix} \tag{20}$$

Thus, to control the stiffness of the air spring effectively, we require effective and reliable control strategies.

3. DESIGN OF CONTROLLERS

The air spring suspension system is practically a nonlinear system. For handling the non-linearity and non-uniformity of the system, complex fuzzy adaptive sliding mode controllers were proposed in the past [7]. By tracking the sprung mass motion and obtaining a tracking error, a sliding surface can be defined to reduce the sprung mass acceleration. The authors of this article propose an equivalent linear system which is reduced from the nonlinear system by assuming the air bellow as a uniform cylinder and the air inside the bellow is a perfect gas. The system is simulated in MATLAB. For comparative study, two widely used control methods are considered – PID and LQR. Both the controllers are tuned manually to obtain optimum results.

Two types of controller design approaches are commonly followed by designers. The conventional method (CM) does not take passenger acceleration into consideration but only considers suspension travel and performance index. Acceleration Dependent method (ADM) prioritizes passenger acceleration to performance index. It is observed that the whole body vibration exposure RMS acceleration values reduced to 50% by CM and 90% by ADM [8]. For our system, we follow ADM since it provides better results. A lot of other strategies such as back-stepping control, genetic algorithms were proposed by researchers in the past. At the same time, they are also time consuming and highly complex to implement in practical cases.

3.1 PID Controller

The Proportional Integral Derivative (PID) controller works on the Control Loop Feedback principle. Although it is practically feasible and effective, tuning a PID controller is the critical part of the process. Rajagopal et al have effectively tuned the PID parameters and have claimed that their controller efficiently reduces the vehicle body acceleration [9]. The most common Ziegler-Nichols tuning is effective for random road conditions. But the iterative learning algorithm works best for random road, sine input, bump-hole conditions [10]. On the other hand, a Particle Swarm Optimized (PSO) PID controller is effective while it provides user simplicity [11].

For an electronically controlled air suspension system, the system is considered as a spring and damper combination. Researches show that a linear PID controller is efficient in providing better ride performance and reduces the vertical acceleration [12]. For a nonlinear system, the LQ based semi-active system simulation proves that it is effective by reducing the braking distance and improved comfort index and can be used in practical cases [13].

The schematic of the PID controller is given Figure 2. The control parameter can be defined as in (21). The sensitivity of the PID controller is tuned manually by differing the proportionality gain, integral gain and the differential gains.

$$u(t)=K_p[e(t)+K_i \int e(t)dt+\frac{K_d\, de(t)}{dt}] \tag{21}$$

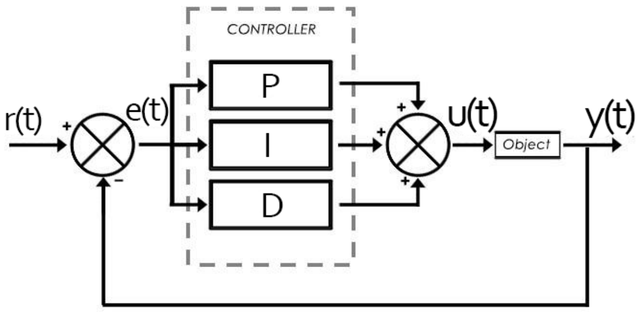

Fig.2 - Principle of PID control system

3.2 LQR Controller

For much complex systems, the nonlinear model is linearized and reduced order for ease of analysis. LQ based control reduces unwanted acceleration and pitch [14]. The LQR control technique performs better for step and random inputs comparatively [15].

The LQR controller on the other hand is tuned by selecting an appropriate weighting function that would provide the optimum result. The system can be best stabilized using LQR controller by changing the poles of the system to an optimal value for the given time response, steady state and overshoot.

3.2.1 Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a population based optimization technique coined by [16]. It works on defining the dimensional search space, number of steps and number of particles (birds) in the swarm. The number of birds denotes the size of the swarm. Each bird moves with its own velocity and has its own position. The position and velocity are set by themselves by their own moving experience and with that of the other birds in the swarm. Each bird carries the information of the best solution obtained at a particular position and they are termed as local best solutions and positions. All birds finally move towards a global best solution, contributing to an optimal solution.

This method is inspired by the social behavior of birds. The current velocity and position of each bird can be updated or measured using (22).

$$v_i^{k+1} = wv_i^k + c_1 rand_1(pbest_i - x_i) + c_2 rand_2(gbest_i - x_i)$$
$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (22)$$

The process is continued till the satisfying result is obtained. The fitness function is given by (23). The fitness value and number of iterations decide the stopping condition. The local best positions and global best positions of each bird are evaluated and additionally considering their velocities of the birds, a new best position is obtained.

$$F = \frac{1000}{1+J} \quad (23)$$

where J is the RMS of prediction error.

3.2.2 LQR tuned by PSO and Manual Tuning

The population for this evaluation is set as 100 since it is a simple quarter car model. Kothandaraman et al have effectively tuned their PID controller using PSO [17]. Even though it is effective, the PSO is a time consuming process. The size of the population decides the accuracy of the solution. A larger population while providing a better solution also consumes a lot of time in the process. A combination of PSO and manual tuning is preferred so that it saves time while we obtain best solution. The solution obtained from PSO is considered as the base from which manual tuning starts and it allows the users to arrive at a best solution much sooner by manual tuning.

The Linear Quadratic Regulator (LQR) is another feedback loop based controller which works on Riccati equation (24) depending on a weighting function and a design function normally represented in matrix forms. The uniform stabilizing ability and detectability of finite systems were easier under strong conditions. These results can be used to analyze complex multiple input multiple output systems [18]. The performance of the LQR controller can be verified with a cost function J as given in (25). It can be minimized by designing the state feedback control K to attain stability of the system. The cost function consists of two parts. The first part represents the transient energy and the next represents control energy. Bryson's Rule of tuning is used to tune the LQR controller as given in (26). The state weighing matrix Q and control weighting matrix R values are obtained by a combination of PSO and manual tuning techniques.

$$PA + A^T - PBR^{-1}P + Q = 0 \quad (24)$$

$$J = \int_0^{\alpha} \left(x^T Q_{ii} x + u^T R_{jj} u\right) dt \quad (25)$$

$$\left.\begin{aligned} Q_{ii} &= \frac{1}{\max x_i^2} \\ R_{jj} &= \frac{1}{\max u_j^2} \end{aligned}\right\} \quad (26)$$

where,

$i = \varepsilon(1, 2, \ldots l)$

$j = \varepsilon(1, 2, \ldots l)$

Since the air spring is kept in closed valve condition, the pressure of air inside the chamber is directly proportional to the stiffness of the air spring suspension system. Compared with the conventional suspension systems, the controllability of the pneumatic systems is simpler since variable stiffness can be achieved by varying the gas pressure inside the container and thus, the choice of air springs will provide us with preferable results.

4. RESULTS AND DISCUSSION

The air spring is nothing but a deformable container which consists of high pressure air inside. The application and withdrawal of load on the airspring not only influences the pressure but also the height, volume and area of the bellow. As discussed earlier, the stiffness equation for the bellow is derived by considering all these necessary parameters. For simulation, a step input of 0.1m amplitude is given. The response of the system, with and without the controller is plotted in MATLAB.

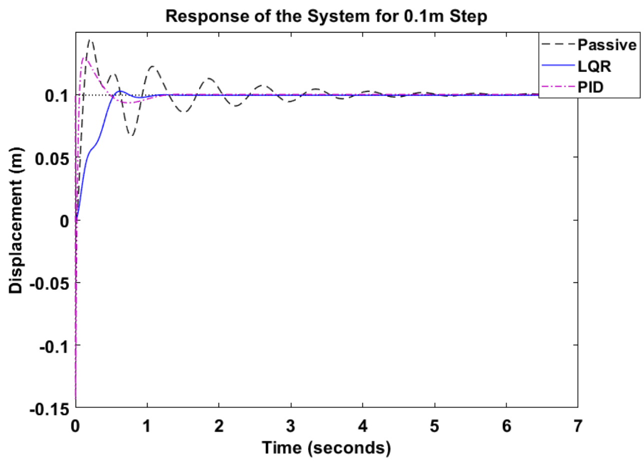

Fig.3 - Step Response of the system

Table 2. Performance overview on 0.1m step input

Load kg	Displacement m			Settling Time s		
	Without Control	PID	LQR	Without Control	PID	LQR
221	0.145	0.125	0.103	4.48	1.315	1.169

On observing the output from Figure 3, we can see an undesirable overshoot of the system. This is due to the constant stiffness of the system which is incapable of adjusting itself to the input conditions. On the other hand, a controlled suspension system shows better performance than a passive system. From Table 2, it is observed that, settling time of the system have decreased considerably after the application of the controller ensuring the stability of the system. Even though PID controller settles the system earlier, the LQR is also competitively efficient.

From Table 3, comparing the peak amplitudes, no overshoot is observed with the controllers. It can be clearly seen that the displacement of the system is efficiently controlled by LQR thereby assuring maximum ride comfort to the passengers. Comparing the overall performance, the LQR provides commendable ride comfort compared with PID controller and ensures effective vehicle stability.

4.1 Response on Bumps and Potholes

For evaluation, the system is simulated for bumpy road conditions and random road conditions. 2 bumps and 2 pot holes are generated in MATLAB with peak amplitude of 0.25m and the behaviour of the system with PID and LQR controllers on a bumps and pot holes as seen in Figures 4 and 5 are also checked and plotted.

Sinusoidal bumps and pot holes were generated for 10 seconds. The performance evaluation of the passive and adaptive systems on bumps and pot holes will help us identify the control strategy that attenuates vibrations better.

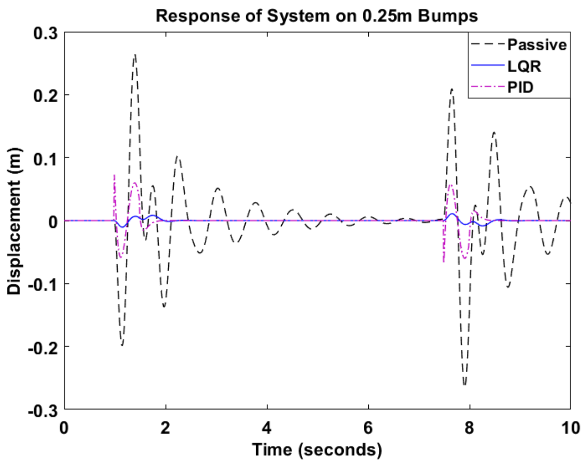

Fig.4 - Sprung mass displacement on 0.25m bumps

Table 3. Performance of the system on Bumps

Load kg	Displacement on Bumps m			Improvement	
	Without Control	With PID Control	With LQR Control	With PID Control	With LQR Control
221	0.0896	0.0269	0.00466	69%	95%

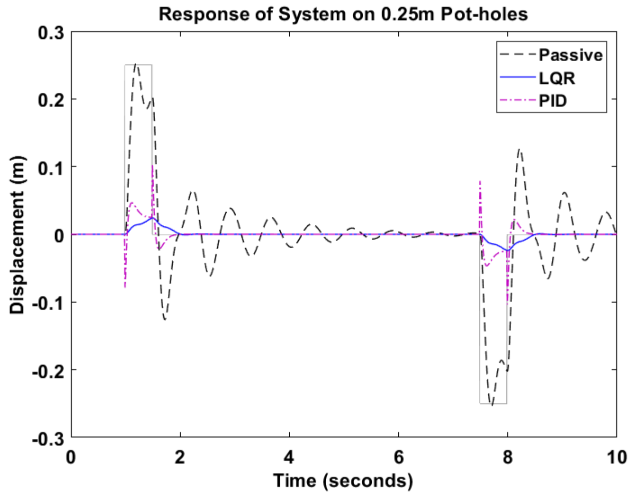

Fig.5 - Sprung mass displacement on 0.25m Pot holes

Table 4. Performance of the system on Pot Hole

Load kg	Displacement on Pot hole m			Improvement	
	Passive System	With PID Control	With LQR Control	With PID Control	With LQR Control
221	0.254	0.104	0.0243	59%	90%

On bumpy roads, comparatively the system controlled by LQ Control strategy performs better for the same load and road conditions than the PID control. The results as seen in Table 4 show an improvement in reduction of peak amplitude upto 95% using LQR. Thus, we can infer that, low frequency vibrations can be attenuated efficiently by the adaptive system using LQR.

4.2 Response on Random Road Conditions

The ISO 8608 road standards as shown in Table 5 are considered to model the random roads. The random roads considered for evaluation are generated in MATLAB as shown in Figure 6. These studies can be used to develop strategies and techniques useful to avoid the dynamic overloading consequences [19].

Table 5. ISO 8608 road classes

Road class	$G_d(n_0)$ $(10^{-6}\ m^3)$		$G_d(\Omega_0)$ $(10^{-6}\ m^3)$	
	Lower limit	Upper limit	Lower limit	Upper limit
A	–	32	–	2
B	32	128	2	8
C	128	512	8	32
D	512	2048	32	128
E	2048	8192	128	512
F	8192	32768	512	2048
G	32768	131072	2048	8192

n_0=0.1 cycles/m	Ω_0 =1 rad/m

Fig.6 - Sprung mass displacement on 0.25m bumps

The responses are obtained for the considered ISO road conditions before and after the application of control strategies are shown in Figures 7, 8 and 9.

Table 6. Performance of system on ISO Standard Roads

Road Standard	Input Peak Amplitude mm	Peak Amplitude Without Control mm	Controlled Peak Amplitude mm	
			PID	LQR
A-B	4.75	4.95	4.5	3.4
D-E	38.7	39.6	36.4	27.5
E-F	78.86	79.7	72.8	55.2

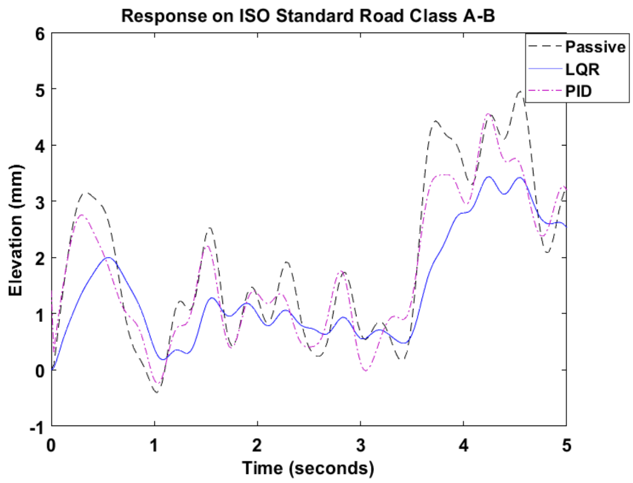

Fig.7 - Sprung mass displacement on 0.25m bumps

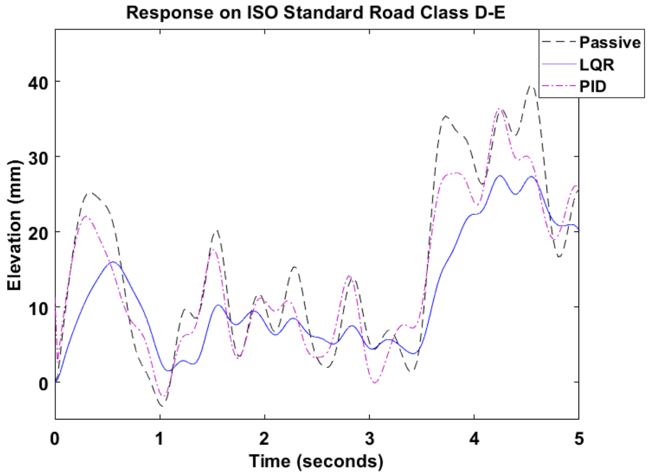

Fig.8 - Sprung mass displacement on 0.25m bumps

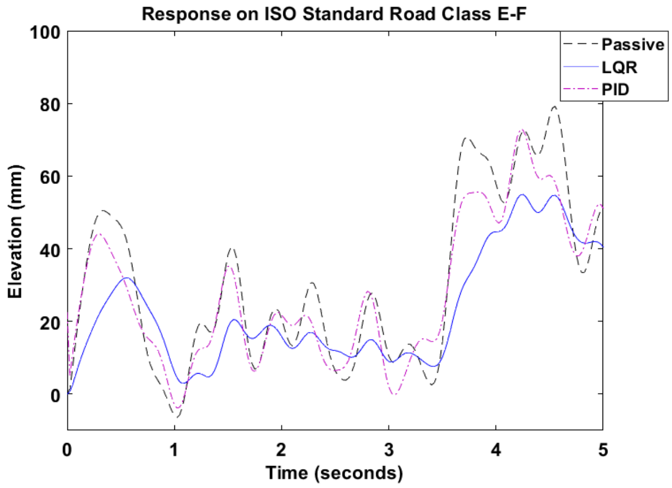

Fig.9 - Sprung mass displacement on 0.25m bumps

The Class A-B roads are considered to be good roads and Class E-F are considered to be poor roads. Since, class F-G and above are highly rare to occur, they are neglected and only the practical road classes are considered for evaluation. The responses of the system are shown in Table 6. The results clearly show enormous control in vehicle peak amplitude on the random roads with the adaptive system using LQR.

These results show that, for any load and road conditions, the system adapts itself to provide the same level of comfort to the passengers and the driver. Thus, from these results, we can infer that, even in extreme conditions, an adaptive system can perform efficiently and provide better ride comfort and creditable handling using LQ control strategy.

5. CONCLUSIONS

A quarter car model was considered to analyse the performance of a suspension system under user defined bumps, pot holes and random road conditions with and without control strategies. Unlike the previous air suspension systems that solely ensure the level of the vehicle with respect to ground, with no explicit concentration on stiffness of the system, our system solely concentrates on providing necessary dynamic stiffness based on the load experienced. A static stiffness equation was derived for the air suspension system which is a function of pressure, relative height and volume. These considerations improve the accuracy of the results. The simulation results show that, the system experiences undesirable overshoot and undergoes transient vibration due to incapability of adapting itself to the road. When the control strategies are introduced, the system adapts itself to the road conditions and performs efficiently providing better ride comfort and handling characteristics. The system controlled with LQR control strategy effectively tuned by a combination of PSO and manual tuning reduces the overshoot from 44.6% to a negligible 3.17% and thereby provides better ride comfort. The system settling time is also reduced by 85%. For better handling the settling time of the system should be maintained as low as possible.

The effectively tuned LQR controller provides excellent ride comfort while not compromising the handling. The LQR settles the system 4 times as fast as the passive system. Overall performance shows that the LQR controller performs commendably while reducing the complexity in control.

Thus the proposed system bridges the gap between the soft and hard springs providing the precise required stiffness for the suspension system ensuring both ride comfort and vehicle-road contact.

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APPENDIX

m_s : Sprung mass
m_c : Mass of Car
tm_p : Total mass of passengers
C_s : Damping Coefficient of suspension

C_{us} : Damping Coefficient of tire
K_s : Stiffness of the spring
K_{us} : Stiffness of the tire
F_D : Controller Force
ζ : Damping Ratio
X_s : Sprung mass displacement
X_d : Road disturbance
F_s : Force of the spring
n : Quantity of matter
R : Gas constant
T_s : Temperature of the spring
P_a : Atmospheric pressure
A : Area of cross-section
z_B : Length of spring
F_{RS} : Friction force
K_p : Proportionality coefficient
K_i : Integral constant
K_d : Differential constant
e(t) : Error
x : State Vector
u : Input Vector
Q_{ii}, R_{jj} : Positive definite real symmetric matrices
A_{eff} : Effective area
P_s : Pressure inside airspring
V_s : Volume of airspring