Stability Analysis of Network Controlled Micro-Grid Systems with Communication Delays and Nonlinear Perturbations

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Abstract—In this paper, the problem of delay-dependent stability of micro-grid load frequency control systems under networked environment with time-invariant delays and bounded nonlinear perturbations has been addressed using the Lyapunov-Krasovskii functional approach. In the networked control environment, it is observed that transfer of feedback variable from the sensor to centralized controller, and the control effort from controller back to the actuator through communication links introduces time-delays in the feedback path. The time-delays adversely affect the overall performance of the closed-loop system paving way to system instability. In addition, in distributed generation scenario, the uncertainties in the time-delayed microgrid system brought about by the penetration of fluctuating power generators, viz., solar and wind power combined with perturbations in the system load also affect the performance of the overall system. To assess the impact of these time-varying uncertainties to the closed-loop stability of the micro-grid system, they are included in the mathematical model of the system as a norm-bounded nonlinear perturbation term. Subsequently, the stated problem is solved in a less conservative manner by employing the classical Lyapunov-Krasovskii functional approach combined with Wirtinger inequality. The analysis results in a delay-dependent stability criterion in linear matrix inequality (LMI) framework. In the sequel, the presented stability criterion is validated on a standard benchmark system and supported with extensive simulation results.

Index Terms—Delay-dependent stability, Time-delays, Communication network, Nonlinear perturbations, Lyapunov-Krasovskii functional, Linear Matrix Inequality (LMI).

I. INTRODUCTION

IN the last decade, a paradigm shift in the topology of conventional power system has taken place, and the modern power system has evolved in the form of small pockets of independent entities called micro-grids. A typical microgrid system connected to utility grid encompasses several distributed and interconnected generator units, energy storage units, and loads. Different types of the distributed generation sources include micro-turbine, wind-turbine, solar PV panel, fuel cell with electrolyzer system etc. The load essentially consists of domestic and small/medium-scale industrial loads. The energy storage unit, an optional entity, usually includes batteries, flywheels, super-capacitors, super conducting magnetic energy storage, plug-in-electrical vehicles etc., refer [1],

[2]. From the grid perspective of the modern power system, a micro-grid system can be considered as a group of controllable generator units and associated loads; refer [3], and the references cited therein. With increasing penetration of renewable energy sources into the utility grid, given their environment dependent power generation characteristics, design and control of distributed generation in micro-grid environment have become quite complicated; nevertheless, owing to the challenges involved in the process, it has invariably emerged as an active area of research in recent times. The advantages of the microgrid system include increase in reliability and security of the power system, decrease in the cost of operation by effectively utilizing the micro-sources during peak load condition, base load support and reduction in green house effects.

In this paper, a networked micro-grid closed-loop load frequency control system with time-invariant approximation for delay is investigated for its delay-dependent stability. In this system, wind and PV generators serve as primary sources of electric power. As power from these renewable sources are intermittent in nature, a gas based micro-turbine unit is employed cater the base load. In addition, when unexpected real power imbalance occurs in power system, regulation of grid frequency may not be possible with micro-turbine itself; in such a scenario, for frequency compensation, a fuel cell and electrolyzer system are appropriately integrated into the micro-grid system. This system model is taken from [3].

In micro-grid systems, to compensate for continuous change in load demand, real power generation control is required to keep the frequency constant since incremental frequency variable $(\Delta f(t) = f(t) - f_e)$ is an indication of the in-balance that exists between the generation and the demand [4]. To accomplish this objective, load frequency control (LFC) control strategy is employed so that all distributed generation units are operated and controlled cohesively to ensure a stable operation with desirable frequency and voltage profile in the system [5]. This task is facilitated in a micro-grid system through a remotely operated micro-grid central controller (MGCC). The main functions of this centralized control are to obtain measurement from the power system and control information from local controllers, and to decide and implement necessary

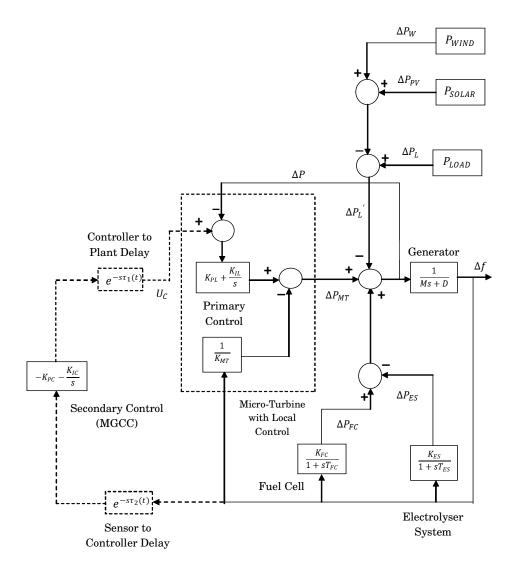


Fig. 1. Block Diagram of Micro-Grid System with Communication Delays.

control actions for regulating the system frequency to the desired value. Since solar and wind power sources constantly generate shortfalls and excesses, MGCC, from the measurement of $\Delta f(t)$, sends appropriate control signal automatically to local controllers in micro-grid to ensure that total amount of power demand at grid is consistently equal the total power supply (i.e. $\Delta f(t) \rightarrow 0, \ \forall \ t$). This centralized automatic control loop is configured in networked control framework with information exchange carried out through communication channels; this orientation, in turn, introduces time-delays in the feedback path as shown in Fig. 1.

The feedback loop delays are inevitable in such a distributed system scenario where the system to be controlled and the controller that executes the control action are connected through a communication channel in which the measured/controlled data that are realized in the form of discrete pockets of information experience buffering, processing and propagation at various internodes. These network-induced feedback loop

delays pose serious limitations to achievable performance of the closed-loop system; in dire situations, when the delay margin exceeds a critical value, the micro-grid system will be driven to instability, and subsequently, it automatically trips from the main grid. The loss of synchronism may sometimes generate low frequency high amplitude power swings in the power system paving way for catastrophic tripping of various generation units connected to the grid leading to islanding of regional power grids, and sometimes, blackouts [6], [7].

Hence, stable delay margin (i.e. maximum allowable delay bound that the networked controlled system can accommodate without losing asymptotic stability) for networked micro-grid systems must be computed for various subsets of controller parameters through a less conservative delay-dependent stability analysis procedure so that they will serve as a practical guideline for fine tuning of controller parameters at the implementation stage even with partial knowledge about network delay size [8]. This, in turn, will enable the operating personnel

to achieve optimal performance from the closed-loop LFC system under delayed control inputs. In this direction, from the perspective of fluctuating power generators, the undertaken study is a capacity management problem; hence, it is significant. Delay-dependent stability criteria are basically sufficient conditions that are employed to compute the stable margin for the network delays within which the power system controlled through a remote MGCC remains asymptotically stable in the sense of Lyapunov.

In this paper, using the classical Lyapunov-Krasovskii functional approach [9] combined with Wirtinger inequality, a less conservative stability criterion is presented for ascertaining delay-dependent stability of the networked micro-grid load frequency control system with time-invariant delays. In the presented analysis, the controller to actuator delay $\tau_1(t)$ and sensor to controller delay $\tau_2(t)$ (see, Fig. 1) are assumed to be time-invariant (i.e., $\tau_i(t) = \tau_i$, $i = 1, 2 \ \forall t$); hence, they are treated as a single feedback loop delay $\tau = \sum_{i=1}^2 \tau_i$. Furthermore, the environment dependent power generation from non-conventional distributed sources solar and wind introduces unmodelled time-varying dynamics into the system. In this paper, we have considered these uncertainties by modelling them as bounded nonlinear perturbations with respect to state variables of the system. This is the novelty of the paper.

In literature, there are two results reported in recent times for ascertaining delay-dependent stability of micro-grid systems that involve communication delays [3], [10]. However, in both these results, the impact of load perturbation on system stability is not considered in the stability analysis. The stability analysis presented in [3] and [10] are derived only for nominal systems - systems without any uncertainties. This has motivated the authors to explore the networked microgrid system for assessing impact of load perturbations on the system performance and stability. In this aspect, to the best of authors' knowledge, we wish to record that the problem of delay-dependent stability of networked micro-grid system with time-invariant delays and time-varying nonlinear perturbations has not been addressed using Lyapunov-Krasovskii functional approach so far.

II. MATHEMATICAL MODELLING

The mathematical model of various components of the micro-grid control system are presented herewith.

A. Gas-Based Micro-Turbine

The transfer function model the micro-turbine is developed taking into account the linear speed drop characteristics between the power and frequency. This transfer function is given as follows:

$$G_{MT}(s) = \frac{\Delta P_{MT}}{\Delta f} = -\frac{1}{K_{MT}} \tag{1}$$

where Δf , ΔP_{MT} and K_{MT} represent frequency deviation, change in output power and drop characteristics of the microturbine, respectively.

B. Fuel Cell and Electrolyzer

A fuel cell with an electrolyzer system is utilized to compensate for real power imbalance when the local controller of the micro turbine becomes less effective for substantial variations in load. A part of the wind power is utilized by the aqua electrolyzer to produce hydrogen for fuel cell. The transfer function model of fuel cell and electrolyzer are given as follows:

$$G_{FC}(s) = \frac{\Delta P_{FC}}{\Delta f} = \frac{K_{FC}}{1 + sT_{FC}},$$
 (2)

$$G_{ES}(s) = \frac{\Delta P_{ES}}{\Delta f} = \frac{K_{ES}}{1 + sT_{ES}},$$
 (3)

where ΔP_{FC} , K_{FC} and T_{FC} represent the change in output power, the gain, time constant of the fuel cell, respectively, while ΔP_{ES} , K_{ES} and T_{ES} denote similar variables of the electrolyzer.

C. Extended Load

The extended load demand $\Delta P_L^{'}$ consists of housing load, wind power and PV generation; it is expressed as follows:

$$\Delta P_{L}^{'} = \Delta P_{L} - \Delta P_{PV} - \Delta P_{W}. \tag{4}$$

The dynamics of PV and wind power are considered as nonlinear perturbations in the test system employed for study. These uncertainties do substantially affect delay margins results.

D. Local and Central Controller

The local and micro-grid central controller $(G_{LC}(s))$ and $G_{CC}(s)$ respectively) are configured with PI control law. The controller transfer functions are given below:

$$G_{LC}(s) = K_{PL} + \frac{K_{IL}}{s}, (5)$$

$$G_{CC}(s) = K_{PC} + \frac{K_{IC}}{s}, \tag{6}$$

where K_{PX} and K_{IX} represent proportional and integral gains of the controller.

E. Power System

The power system with its connected conventional (high inertia) generators is modelled as a first order transfer function given by

$$G_P(s) = \frac{1}{sM + D},\tag{7}$$

where M and D are moment of inertia and damping constant respectively.

F. State-Space Model

The overall system including central and local controller encompassing network-induced delay is shown in Fig. 1. The state-space model of the closed-loop system shown in Fig. 1 can be derived in the following autonomous framework:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau) + \omega(t), \tag{8}$$

$$x(t) = \phi(t), t \in [-\max(\tau), 0], \tag{9}$$

$$\int_{b}^{a} \dot{\omega}^{T}(u) R \dot{\omega}(u) du \geq \frac{1}{b-a} \begin{bmatrix} \omega(b) \\ \omega(a) \\ \frac{1}{b-a} \int_{a}^{b} \omega(u) du \end{bmatrix}^{T} \begin{bmatrix} 4R & 2R & -6R \\ \star & 4R & -6R \\ \star & \star & 12R \end{bmatrix} \begin{bmatrix} \omega(b) \\ \omega(a) \\ \frac{1}{b-a} \int_{a}^{b} \omega(u) du \end{bmatrix}.$$

where the system matrices $A \in \mathbb{R}^{5 \times 5}$ and $A_d \in \mathbb{R}^{5 \times 5}$ are as follows:

with state vector $x(t) \in \mathbb{R}^{5 \times 1}$ being $x(t) = [K_{IC} \int \Delta f(t) dt \Delta P_{MT}(t) \ \Delta P_{FC}(t) \ \Delta P_{ES}(t) \ \Delta f(t)]^T$. The elements of the matrices (in terms of system parameters) are given below:

$$\begin{array}{rcl} a_{21} & = & 0, \\ a_{22} & = & \frac{1}{1+K_{PL}} \bigg[-K_{IL} - \frac{1}{MK_{MT}} \bigg], \\ a_{23} & = & \frac{1}{1+K_{PL}} \bigg[\frac{K_{PL}}{T_{FC}} - K_{IL} - \frac{1}{MK_{MT}} \bigg], \\ a_{24} & = & \frac{1}{1+K_{PL}} \bigg[-\frac{K_{PL}}{T_{ES}} + K_{IL} + \frac{1}{MK_{MT}} \bigg], \\ a_{25} & = & \frac{1}{1+K_{PL}} \bigg[-\frac{K_{PL}K_{FC}}{T_{FC}} + \frac{K_{PL}K_{ES}}{T_{ES}} + \frac{D}{MK_{MT}} \bigg], \\ d_{21} & = & -\frac{K_{IL}}{1+K_{PL}}, \\ d_{22} & = & -\frac{K_{PL}K_{PC}}{M(1+K_{PL})}, \\ d_{23} & = & -\frac{K_{PL}K_{PC}}{M(1+K_{PL})}, \\ d_{24} & = & \frac{K_{PL}K_{PC}}{M(1+K_{PL})}, \\ d_{25} & = & \frac{1}{1+K_{PL}} \bigg[-K_{IL}K_{PC} + \frac{K_{PL}K_{PC}D}{M} - K_{PL}K_{IC} \bigg]. \end{array}$$

The time-delay satisfies the following condition:

$$0 \le \tau \le \bar{\tau}; \tag{12}$$

where $\bar{\tau}$ is the upper bound of the time-delay.

III. MAIN RESULT

The proposed result of this paper is derived by modelling unknown extended power system load disturbance as time-varying nonlinear perturbation in terms of current and delayed state vector. The term $\omega(t)$ represents the uncertainties caused due to the intermittent penetration of solar and wind power to the micro-grid and load perturbation as well. It is treated as a function of state-variables as follows:

$$\omega(t) = \omega(t, x(t), x(t - \tau)), \tag{13}$$

satisfying the following norm-bounded condition:

$$||\omega(\cdot)|| \le \alpha ||x(t)|| + \beta ||x(t-\tau)||, \tag{14}$$

where $\alpha \geq 0$ and $\beta \geq 0$ are known scalars. A somewhat restricted version of the condition (14), which is used in this paper, is given by

$$\omega^{T}(\cdot)\omega(\cdot) \leq \alpha^{2}x^{T}(t)G^{T}Gx(t) + \beta^{2}x^{T}(t-\tau)F^{T}Fx(t-\tau), \quad (15)$$

where G and F are known constant matrices of appropriate dimensions. The problem addressed in this paper is stated below:

Problem: To develop a less conservative robust stability criterion in LMI framework to ascertain delay-dependent stability of the networked micro-grid system described by the state-space model (8) with the load disturbance satisfying the condition (14), and network-induced time-delay (12) using Lyapunov-Krasovskii functional approach.

For deriving the delay-dependent stability criterion, following lemma is required:

Lemma 1. For given symmetric positive definite matrix R, and for any differentiable signal ω in $[a, b] \to \mathbb{R}^n$, the Wirtinger inequality stated at the top of this page holds.

The proposed delay-dependent stability criterion is stated in the form of a theorem.

Theorem 1. For a given delay bound τ , networked micro-grid frequency control system (8) with time-delay (12) and nonlinear perturbation (14) is asymptotically stable in the sense of Lyapunov, if there exist real, symmetric, positive definite matrices P, S and R; symmetric matrix Z and free matrix Q of appropriate dimensions and scalar $\epsilon > 0$ such that following LMIs hold:

$$\Pi(\tau) > 0, \tag{16}$$

$$\Phi(\tau) \quad < \quad 0, \tag{17}$$

where $\Phi(\tau) = \sum_{i=1}^{4} \Phi_i(\tau)$. The elements of $\Phi_i(\tau)$, i=1 to 4 are given at the top of next page, where \star represents transposed terms in the symmetric matrix.

Proof: The proof of this stability criterion employs the following LK functional (positive definite energy functional):

$$V(t) = \begin{bmatrix} x(t) \\ \int_{t-\tau}^{t} x(s)ds \end{bmatrix}^{T} \begin{bmatrix} P & Q \\ \star & Z \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t-\tau}^{t} x(s)ds \end{bmatrix} + \int_{t-\tau}^{t} x^{T}(s)Sx(s)ds + \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^{T}(s)R\dot{x}(s)dsd\theta.$$
(18)

$$\begin{split} \Pi(\tau) &= \begin{bmatrix} P & Q \\ \star & Z + \frac{1}{\tau}S \end{bmatrix}, \\ \Phi_1(\tau) &= \begin{bmatrix} A^TP + PA + Q + Q^T + S & -Q + PA_d & \tau A^TQ + \tau Z & P \\ & \star & -S & \tau A_d^TQ - \tau Z & 0 \\ & \star & \star & 0 & \tau Q^T \end{bmatrix}, \\ \Phi_2(\tau) &= \begin{bmatrix} A^T \\ A_d^T \\ 0 \\ I \end{bmatrix} (\tau R) \begin{bmatrix} A^T \\ A_d^T \\ 0 \\ I \end{bmatrix}^T, \\ \Phi_3(\tau) &= -\frac{1}{\tau} \begin{bmatrix} R & -R & 0 & 0 \\ \star & R & 0 & 0 \\ \star & \star & \star & 0 \end{bmatrix} - \frac{1}{\tau} \begin{bmatrix} 3R & 3R & -6R & 0 \\ \star & 3R & -6R & 0 \\ \star & \star & 12R & 0 \\ \star & \star & \star & \star & 0 \end{bmatrix}, \\ \Phi_3(\tau) &= \begin{bmatrix} \epsilon \alpha^2 F^T F & 0 & 0 & 0 & 0 \\ \star & \epsilon \beta^2 G^T G & 0 & 0 \\ \star & \star & \star & -\epsilon I \end{bmatrix}. \end{split}$$

By bounding the time-derivative of the LK functional with Wirtinger inequality stated in Lemma 1, and including the constraint imposed on non-linear perturbation term (14), the delay-dependent stability criterion is derived. For detailed derivation, [11] and [12] may be referred simultaneously.

IV. BENCHMARK MICRO-GRID SYSTEM

The parameters of the standard benchmark system taken from [3] are given in Table I.

TABLE I NOTATIONS

Parameter	Value	
M	10	
D	1	
K_{MT}	0.04	
K_{FC}	1	
T_{FC}	4	
K_{ES}	1	
T_{ES}	1	
K_{PL}	1	
K_{IL}	1	

For this system, the maximum delay bound $\bar{\tau}$ of the time-varying delay for different sub-set of the centralized controller parameters (K_{PC},K_{IC}) obtained by the delay-dependent stability criterion stated in Theorem 1 is given in Table II. The analytical results are validated using simulation based study. The simulation based study assumes white noise model for intermittent power output from renewable sources viz., wind and solar. Since power electronics based converters are invariably employed for interface with power system grid, this assumption is appropriate. The micro-grid central controller parameters are set at $K_{PC}=1$ and $K_{IC}=0.8$. The uncertain extended load demand through out the study is kept

TABLE II $\mbox{Maximum upper delay bound $\bar{\tau}$ for $F=G=0.1$ $I_{5\times 5}$}$

K_{PC}	$\alpha = 0, \beta = 0$		$\alpha = 0, \beta = 0.05$		$\alpha = 0.05, \beta = 0.05$	
	K_{IC}	$ar{ au}$	K_{IC}	$\bar{ au}$	K_{IC}	$ar{ au}$
1.0	0.2	9.943	0.2	6.753	0.2	5.477
	0.4	5.264	0.4	3.240	0.4	2.408
	0.6	3.664	0.6	2.076	0.6	1.415
	0.8	2.857	0.8	1.511	0.8	0.947
	1.0	2.371	1.0	1.184	1.0	0.684
2.0	0.2	12.688	0.2	10.097	0.2	9.103
	0.4	7.155	0.4	5.347	0.4	4.606
	0.6	5.053	0.6	3.574	0.6	2.957
	0.8	3.949	0.8	2.667	0.8	2.127
	1.0	3.270	1.0	2.122	1.0	1.637

at $P_L^{\prime} \in [0.4, 0.6]$, and time-delay is kept initially at $\tau = 0$; at this condition, the system is assumed to be at the equilibrium state with $\Delta f(t) = 0$. The closed-loop system is introduced with a sudden step change in time-delay at t = 200secs. For validating the stable closed-loop operation, the time-delay τ is increased from 0 secs to 2.6 secs. The maximum allowable delay bound, in accordance to Table II, for the chosen MGCC parameters, is $\bar{\tau} = 2.857 secs$. Since $\tau = 2.6 < \bar{\tau}$, the incremental frequency variable $\Delta f(t)$ evolves asymptotically towards equilibrium point $\Delta f(t) = 0$ as shown in Fig. 2. If the delay τ is increased to $\bar{\tau} = 2.857$, the system is marginally stable as shown in Fig. 3, and if the delay is further increased to $\tau = 3 > \bar{\tau}$, the micro-grid system becomes unstable with $\Delta f(t)$ variable evolving unboundedly with time t as shown in Fig. 4. These results clearly substantiate the impact of time-invariant delay on the performance and stability of the networked micro-grid load frequency control system, thereby validating the presented stability analysis.

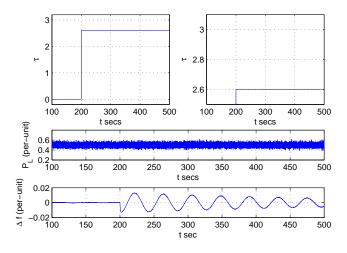


Fig. 2. Time-domain Simulation for $\tau = 2.6$ secs.

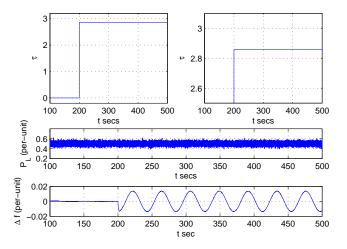


Fig. 3. Time-domain Simulation for $\tau = 2.857$ secs.

V. Conclusions

In this paper, a less conservative stability analysis has been presented for ascertaining delay-dependent stability of networked micro-grid load frequency system with time-delays that emerges due to the communication channels employed in the feedback path, and nonlinear perturbations due to fluctuating power generators connected to the micro-grid system. The presented analysis is based on Lyapunov-Krasovskii functional approach combined with Wirtinger inequality, and the resulting stability criterion is expressed in LMI framework. The nonlinear perturbation term in the mathematical model depicting the uncertainties in the fluctuating power generators is norm-bounded, and they are associated with appropriate scaling factors to quantify the magnitude of the disturbance in the micro-grid system. The proposed stability criterion is illustrated on a standard bench mark system and substantiated

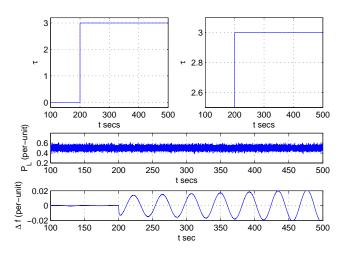


Fig. 4. Time-domain Simulation for $\tau = 3$ secs.

with appropriate simulation results. The possibility of alleviating the limitations of the presented results may be explored as a future work.

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