# Flux and Speed Control of an Induction Motor Powered by an Inverter Voltage: Moment Approach

J. KHEDRI<sup>1</sup> M. CHAABANE<sup>1</sup>, M.SOUISSI<sup>1</sup>

<sup>1</sup>Laboratory of Sciences and Techniques of Automatic Control and Computer Engineering (Lab-STA), National School of Engineering of Sfax, University of Sfax, Postal Box 1173, 3038 Sfax, Tunisia khedrijamel@yahoo.fr

**Abstract:** This paper discusses the method of temporal moments in order to develop a controller for flux and speed of an induction motor (IM). Controller dynamics is set a priori through time domain constraints which in their turn are set in terms of equality between the first (n+1) moments of the transfer function of the closed-loop system and those of an adopted reference model. This reference model, which best defines the dynamics of the closed-loop system, is easy to use. Closed-loop stability and time specifications (overshoot, response time...) are ensured using the linear matrix inequality formalism LMI. A simulation study was carried out on an induction motor to prove the effectiveness of the proposed method.

**Keywords:** Moments, times specifications, LMI, reference model, induction motor.

# **1. INTRODUCTION**

Since the 70s, the method of temporal moments has been the basis of several works [1], but it has mostly been studied since the 90s [2] where works have been devoted to the study of temporal moments in control by highliting the link between this technique and the internal model control. Later, this method has a decisive progress by tackling the frequency moments [3] and [4] where a combination of both technical frequency and time moments is done in order to ensure the stability of the closedloop system and satisfy temporal performance targets. Based on these works, our approach consists in proposing a controller synthesis method ensuring the performance of the closed-loop system through an approximation between this latter and a suitably selected reference model. Since the strict equality between this reference model and the concerned closed-loop system is not practically possible, this equality is replaced by equality of the first (n+1) temporal moments of the two model (the closed-loop system and those of the reference model) [5], [6] and [7]. The choice of the reference model depends on the studied system. It is composed of a set of dominant poles often characterized by second order system through which its dynamics is fixed using its damping coefficient and own pulsation. As previously mentioned the linear matrix inequality formalism LMI is used to ensure closed-loop stability and specify time constraints by minimizing the norm 2 of time moments cost. This method is applied to control the flux and the speed of an induction motor.

The paper is organized as follows: in section 2, an extended state space representation is presented which includes the state model of the system and that of a dynamic controller. Besides closed-loop stability and transient performance which are characterized by a reference model with the help of temporal moments are given as LMIs. In section 3, the dynamic controller is computed by the demonstration of a special lemma and theorem. The last section is dedicated to simulation results applied to an induction motor.

#### Notations

\*: mean the transposed  $Sym\{P\} = P^T + P$  $U^{\perp}$ : the orthogonal complement of U so that

# $U^{\perp}U=0$

#### 2. PROBLEM FORMULATION

Let consider a square invertible LTI system with n inputs and n outputs G(s):

$$\dot{x}_G(t) = A_G x_G(t) + B_G u(t)$$

$$y(t) = C_G x_G(t)$$
(1)

And the state space representation of a controller C(s) defined by:

$$\dot{x}_{c}(t) = A_{c}x_{c}(t) + B_{c}e(t)$$

$$u(t) = C_{c}x_{c}(t) + D_{c}e(t)$$
With
(2)

$$e(t) = r(t) - y(t)$$
(3)  
Where  $x_G(t) \in \Re^l$ ,  $u(t) \in \Re^n$ ,  $y(t) \in \Re^n$ 

 $x_c(t) \in \Re^n$  and  $r(t) \in \Re^n$  are respectively the state vector of the system, the control variable, the

output, the state vector of the controller and the reference of the closed-loop system.

Matrices include in the state space representation of controller:  $A_c$ ,  $B_c$ ,  $C_c$  and  $D_c$  are to be computed.

Let consider now an extended variable

z(t) gathering the state vector of the system and the controller

$$z(t) = \begin{bmatrix} x_G(t) \\ x_c(t) \end{bmatrix}$$
(4)

Consequently, the closed-loop system due to the application of the controller C(s) to the system G(s) is given as follows:

$$\dot{z}(t) = A_{bf} z(t) + B_{bf} r(t)$$

$$y(t) = C_{bf} z(t)$$
With
(5)

$$A_{bf} = \begin{bmatrix} A_G - B_G D_c C_G & B_G C_c \\ -B_c C_G & A_c \end{bmatrix}, B_{bf} = \begin{bmatrix} B_G D_c \\ B_c \end{bmatrix} \quad (6)$$
$$C_{bf} = \begin{bmatrix} C_G & 0 \end{bmatrix}$$

Among the objectives of this work is to ensure closed-loop stability. This is satisfied if there exists a positive definite matrix X

 $X = X^T > 0$  such that:

$$Sym\left\{XA_{bf}\right\} < 0 \tag{7}$$

**Remark:** to facilitate the study, matrices

 $A_c$  and  $B_c$  in the controller state representation are defined in advance. Thus it remains to determine  $C_c$  and  $D_c$  and the controller C(s) is then interpreted as a static feedback gain K such that:

$$K = \begin{bmatrix} D_c & C_c \end{bmatrix}$$
(8)

According to this assumption, the extended system will be characterized by the triplet  $(\hat{A}, \hat{B}, \hat{C})$  and

matrix  $A_{bf}$  can then written:

$$A_{bf} = \hat{A} + \hat{B}K\hat{C} \tag{9}$$

Where

$$\hat{A} = \begin{bmatrix} A_G & 0\\ -B_c C_G & A_c \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B_G\\ 0 \end{bmatrix}, \quad \hat{C} = \begin{bmatrix} -C_G & 0\\ 0 & I \end{bmatrix}$$
(10)

As previously alluded, the purpose of this work, in addition to the closed-loop stability, is to ensure some transient performance characterized by a reference model  $M_{ref}(s)$  whose dynamics are

fixed with the help of temporal moments. First of all let's recall the representation of a given transfer function G(s) represented in state space

by  $(A_G, B_G, C_G, D_G)$ . According to its temporal moments G(s) can be

written [8] and [9]:

$$\mathcal{M}_{n}(G) = (-1)^{n+1} C_{G} A_{G}^{-(n+1)} B_{G} + D_{G}$$
(11)  
for  $n = (1....+\infty)$ 

Where *n* refers to the moment order. Thus we can define the  $0^{th}$  order moment  $\mathcal{M}_0$ , the first  $\mathcal{M}_1$ , the second order  $\mathcal{M}_2 \dots$ 

This three first temporal moments  $\mathcal{M}_0$ ,  $\mathcal{M}_1$  and  $\mathcal{M}_2$  provide sufficient information to characterize the essential elements of an impulse response such as static gain, response time, variance.

Indeed for an impulse response h(t) of a system as shown in Fig. 1



Fig. 1. Characterization of an impulse response

- *M*<sub>0</sub> represents the area of the impulse response of the system and is equal to its static gain.
- $\mathcal{M}_{l}$  describes the average time tm. Indeed for a first order system  $\mathcal{M}_{l} = \tau \mathcal{M}_{0}$ , where  $\tau$ refers to the constant time of the system. We have also  $tm = \frac{\mathcal{M}_{l}(h)}{\mathcal{M}_{0}(h)}$
- $\mathcal{M}_2$  represents the dispersion  $\Delta \tau$  of the impulse response around tm.

The controller synthesis technique by temporal moments is to achieve equality according to Fig. 2

between the closed-loop transfer T(s) and the reference model  $M_{ref}(s)$ 

$$T(s) = G(s)C(s)(I + G(s)C(s))^{-1} = M_{ref}(s)$$
 (12)  
Or equivalently

$$C(s)(I - M_{ref}(s)) = G^{-1}(s)M_{ref}(s)$$
(13)



Fig. 2. Closed-loop control configuration

For a control law u(t) = Ke(t) and using the state space representation (2) of C(s), u(t) will be written

$$u(t) = \left[ C_c (sI - A_c)^{-1} B_c + D_c \right] e(t)$$
(14)

And according to (8), equation (13) becomes:

$$K \begin{bmatrix} I \\ (sI - A_c)^{-1} B_c \end{bmatrix} (I - M_{ref}(s)) = G^{-1}(s) M_{ref}(s)$$
(15)

Or in a compact form, the time constraints can be represented by a linear expression where the formalism LMI can be applied

$$KH(s) = F(s) \tag{16}$$

Where

$$H(s) = \begin{bmatrix} I \\ (sI - A_c)^{-1} B_c \end{bmatrix} (I - M_{ref}(s))$$

$$F(s) = G^{-1}(s)M_{ref}(s)$$
(17)

**Remark:** that the ideal equality (16) is not possible. Thus an approximation between T(s) and  $M_{ref}(s)$  via temporal moments is necessary.

Indeed when characterizing the response time this last equality (16) is substituted by the

minimization of the quadratic distance between the first (n+1) time moments of the reference model and the closed-loop system as follow:

$$\left(K\mathcal{M}_{j}(G) - \mathcal{M}_{j}(F)\right)^{T} \left(K\mathcal{M}_{j}(G) - \mathcal{M}_{j}(F)\right) < \gamma I$$
(18)

The resolution of this LMI gives transitory time

responses approximatively equal for T and  $M_{ref}$  (same response time, same overshoot...).

Notice that the main goal is reached if inequalities (7) and (18) are fulfilled. In the next section a solution of this problem is presented.

### **3. PROBLEM SOLUTION**

The solution of the previous problem is given by the next theorem **Theorem:** [7] for the LTI system (1) defined by the triplet  $(A_G, B_G, C_G)$  and its corresponding extended system (10)  $(\hat{A}, \hat{B}, \hat{C})$  and for a static state feedback  $K_s$  ensuring closed-loop stability of the pair  $(\hat{A}, \hat{B})$ , if the optimization problem below:

$$\min \gamma \\ (L, N, X)$$

With the constraints 
$$X > 0$$

$$\begin{bmatrix} Sym\left\{X\left(\hat{A}+\hat{B}K_{s}\right)\right\} & \left(L\hat{C}-NK_{s}\right)^{T}+X\hat{B}\\ * & -\left(N+N^{T}\right) \end{bmatrix} < 0$$
(19)

$$\begin{bmatrix} -I & \left( L\mathcal{M}_{j}(G) - N\mathcal{M}_{j}(F) \right) \\ * & -\gamma I \end{bmatrix} < 0$$
 (20)

has a solution, thus the controller C(s) with

$$K = N_{opt}^{-1} L_{opt} \tag{21}$$

guarantees closed-loop stability and ensures the time constraints fixed by the reference model  $M_{ref}(s)$ 

Notice that  $N_{opt}$  and  $L_{opt}$  correspond to  $\gamma$  minimal renowned  $\gamma_{opt}$ 

# Proof of the theorem:

The demonstration of theorem requires the recall of the following lemma:

**Lemma:** Consider the extended system (5) where  $A_{bf}$  is given by (9), the matrices inequalities

below included in item 1) are equivalent to that included in 2):

1) there exist two matrices  $K_s$  and K and a

positive definite matrix  $X = X^T > 0$  such that:  $S_{\text{sum}} \left\{ X \left( \hat{A} + \hat{B} X \right) \right\} < 0$ (22)

$$Sym\left\{X\left(\hat{A}+BK_{s}\right)\right\}<0$$

$$Sym\left\{X\left(\hat{A}+\hat{B}K\hat{C}\right)\right\}<0$$
(22)
(23)

2) it exists a non singular matrix N, two matrices K and  $K_s$  and a positive definite matrix X such that:

$$\begin{bmatrix} Sym\left\{ \left( \hat{A} + \hat{B}K_{s} \right)^{T} X \right\} & X\hat{B} \\ * & 0 \end{bmatrix}^{+}$$

$$Sym\left\{ \begin{bmatrix} 0 \\ I \end{bmatrix} N \left[ \left( K\hat{C} - K_{s} \right) & -I \end{bmatrix} \right\} < 0$$
(24)

#### **Proof of lemma:**

The proof of this lemma in turn requires the statement of the elimination lemma [10], [11] and [12]:

Elimination lemma: For given matrices  $M = M^T$ , U and V of appropriates dimensions, the following statements are equivalent:

i) there exists a matrix X satisfying  

$$M + U^T XV + V^T X^T U < 0$$
 (25)

ii) 
$$\frac{U^{\perp}MU^{\perp T} < 0, \text{ or } UU^{T} > 0}{V^{\perp T}MV^{\perp} < 0, \text{ or } V^{T}V > 0}$$
(26)

Item ii) of the elimination lemma can be used with

$$V^{\perp} = \begin{bmatrix} I & 0 \end{bmatrix}, \ V = \begin{bmatrix} 0 \\ I \end{bmatrix}$$
  
and 
$$M = \begin{bmatrix} Sym\{(\hat{A} + \hat{B}K_s)^T X\} & X\hat{B} \\ * & 0 \end{bmatrix}$$

this leads to inequality (22).

With

$$U^{\perp} = \begin{bmatrix} I & \left(K\hat{C} - K_{s}\right)^{T} \end{bmatrix}, \ U = \begin{bmatrix} \left(K\hat{C} - K_{s}\right)^{T} \\ -I \end{bmatrix}$$
$$M = \begin{bmatrix} Sym\left\{\left(\hat{A} + \hat{B}K_{s}\right)^{T}X\right\} & X\hat{B} \\ * & 0 \end{bmatrix}$$

leads us to inequality (23).

# **Proof of theorem:**

Using Schur complement, (20) becomes:

$$(L\mathcal{M}_{j}(G) - N\mathcal{M}_{j}(F)) (L\mathcal{M}_{j}(G) - N\mathcal{M}_{j}(F))^{T} < \gamma I$$

$$j = 0, 1....n$$

$$(27)$$

In addition, the multiplication of (16) by N leads to:

$$LH(s) = NF(s) \tag{28}$$

Thus (18) becomes:

$$\left(L\mathcal{M}_{j}(G) - N\mathcal{M}_{j}(F)\right)\left(L\mathcal{M}_{j}(G) - N\mathcal{M}_{j}(F)\right)^{T} < \gamma I$$
  

$$j = 0, 1....n$$
(29)

Then it is evidence that (27) and (20) refers to the minimization of the norm 2 of the error between the first (n+1) moments of the chosen reference model and those of the closed-loop system. More than  $\gamma$  is optimal more than the two transfers T(s)

and  $M_{ref}(s)$  are very close more than the desired transients are satisfied.

Condition (10) which implies (

Condition (19) which implies (23) equivalent to (7) ensure closed-loop stability.

# 4. METHOD APPLICATION TO AN INDUCTION MOTOR

The electromagnetic dynamic model of the induction motor in the synchronously rotating reference frame d-q axis can be expressed by the set of differential equations below [13], [14], [15] and [16]

$$\frac{dI_d}{dt} = -\alpha I_d + w_s I_q + \frac{\beta}{\tau_r} \Phi_{dr} + \beta p w_m \Phi_{qr} + \frac{1}{\sigma L_s} v_d$$

$$\frac{dI_q}{dt} = -w_s I_d - \alpha I_q - \beta p w_m \Phi_{dr} + \frac{\beta}{\tau_r} \Phi_{qr} + \frac{1}{\sigma L_s} v_q$$

$$\frac{d\Phi_{dr}}{dt} = \frac{M}{\tau_r} I_d - \frac{1}{\tau_r} \Phi_{dr} + (w_s - p w_m) \Phi_{qr}$$

$$\frac{d\Phi_{qr}}{dt} = \frac{M}{\tau_r} I_q - (w_s - p w_m) \Phi_{dr} - \frac{1}{\tau_r} \Phi_{qr}$$

$$\frac{dw_m}{dt} = \frac{pM}{JL_r} (\Phi_{dr} I_q - \Phi_{qr} I_d) - \frac{f}{J} w_m - \frac{c_r}{J}$$
(30)
$$c_e = \frac{pM}{L_r} (\Phi_{dr} I_q - \Phi_{qr} I_d)$$
(31)

$$\alpha = \frac{R_s + R_r \frac{M^2}{L_r^2}}{\sigma L_s}, \ \beta = \frac{M}{\sigma L_s L_r}, \ \tau_r = \frac{L_r}{R_r},$$
$$\sigma = 1 - \frac{M^2}{L_s L_r}$$

Where  $w_m$  is the rotor speed,  $w_s$  is the stator electrical speed,  $(\Phi_{dr}, \Phi_{qr})$  are the d-q rotor flux,  $(I_d, I_q, v_d, v_a)$  are the stator currents and

voltages, p is the number of pole pair,

 $(L_s, L_r, R_s, R_r)$  are the stator and rotor inductances and resistances respectively, J is the moment of inertia, f is the friction coefficient , M is the mutual inductance between stator and rotor,  $c_e$  is the electromagnetic torque and  $c_r$  is the load torque.

In this paper, to facilitate our study, we will adopt an orientation of the rotating reference such that the axis d coincides with the direction of  $\Phi_{dr}$ . On this basis, from (30) we can extract the expression of the open-loop control  $v_d$  and  $v_q$ , the electrical

reference speed and the flux  $\Phi_{dr}$ 

$$v_{d} = \sigma L_{s} \frac{dI_{d}}{dt} + \left(R_{s} + R_{r} \frac{M^{2}}{L_{r}^{2}}\right)I_{d} - w_{s}\sigma L_{s}I_{q}$$
$$-\frac{M}{L_{r}^{2}}R_{r}\Phi_{dr}$$
$$v_{q} = \sigma L_{s} \frac{dI_{q}}{dt} + \left(R_{s} + R_{r} \frac{M^{2}}{L_{r}^{2}}\right)I_{q} + w_{s}\sigma L_{s}I_{d}$$
$$+\frac{M}{L_{r}^{2}}pw_{m}\Phi_{dr}$$
(32)

$$\tau_r \frac{d\Phi_{dr}}{dt} + \Phi_{dr} = MI_d \tag{33}$$

$$w_s = pw_m + \frac{M}{\tau_r \Phi_{dr}} I_q$$

$$c_e = \frac{pM}{L_r} \Phi_{dr} I_q \tag{34}$$

Let's define two other control variables  $v_{d1}$  and  $v_{q1}$  such that:

 $v_d = v_{d1} - e_d$  and  $v_q = v_{q1} - e_q$ Where

$$e_d = w_s \sigma L_s I_q + \frac{M}{L_r^2} R_r \Phi_{dr}$$
$$e_q = -w_s \sigma L_s I_d - \frac{M}{L_r^2} p w_m \Phi_d$$

The two new commands  $v_{d1}$  and  $v_{q1}$  are defined as follows:

$$\begin{cases} v_{d1} = \sigma L_s \frac{dI_d}{dt} + \left(R_s + R_r \frac{M^2}{L_r^2}\right) I_d \\ v_{q1} = \sigma L_s \frac{dI_q}{dt} + \left(R_s + R_r \frac{M^2}{L_r^2}\right) I_q \end{cases}$$
(35)

As it was mentioned in previous sections, our study focuses on the comparison of the first (n+1) moments of transfers function representation of a given reference model and that corresponding to a state model of the system.

In this work, we want to control the flux and the speed, then referring to (33) and (34), (35) becomes:

$$\begin{cases} v_{d1} = \frac{\sigma L_s(s+\alpha)(\tau_r s+1)}{M} \Phi_{dr} \\ v_{q1} = \frac{\sigma L_s L_r(s+\alpha)}{p M \Phi_{dr}} c_e \end{cases}$$
(36)

*s* is Laplace operator.

The open-loop transfer function of the flux  $\Phi_{dr}$ and  $c_e$  are given then by:

$$H_{\Phi_{dr}} = \frac{M}{\sigma L_s (s+\alpha)(\tau_r s+1)}$$

$$H_{c_e} = \frac{pM\Phi_{dr}}{\sigma L_s L_r (s+\alpha)}$$
(37)

We have too

$$w_m = \frac{c_e}{Js+f} - \frac{c_r}{Js+f}$$
(38)

Thus

$$H_{w_m} = \frac{H_{bfce} \frac{1}{f}}{1 + \tau s}$$

$$\tau = \frac{J}{f}, \ H_{bfce} = \frac{1}{1 + \tau_{bfce} s}$$
(39)

 $H_{bfce}$ : is the closed-loop transfer function of the electromagnetic torque.

 $\tau_{bfce}$ : is the closed-loop system constant time of

the electromagnetic torque. It was chosen equal to 79e-5 in order to obtain a time response of the closed-loop system ten times faster than open-loop

one.

On these bases, the transfer function of the flux  $H_{\Phi_{dr}}$  has a second order dynamics, the same for the speed  $w_m$ . We assume that its transfer function denoted  $H_{w_m}$  is of the form:

$$H_{wm} = \frac{1}{1 + \frac{2\xi}{w_n}s + \frac{1}{w_n^2}s^2}$$
(40)

Let's now recall a general transfer function H. This transfer function contains the open-loop transfer of variables that we want to control ie the flux and the speed:

$$H = \begin{bmatrix} H_{\Phi_{dr}} & 0\\ 0 & H_{w_m} \end{bmatrix}$$
(41)

Notice that *H* is diagonal to ensure decoupling between the two variables  $\Phi_{dr}$  and  $w_m$ . The objective is to ensure some transients performances characterized by a reference model such that the reduction of the time response of the system, keeping the overshoot of the closed-loop inferior to 5% whereas decoupling the two outputs. These transient performances can be performed by a diagonal second order reference model with unity static gain:

$$M_{ref} = \begin{bmatrix} \frac{1}{1 + \frac{2\xi}{w_{n1}}s + \frac{1}{w_{n1}^2}s^2} & 0\\ 0 & \frac{1}{1 + \frac{2\xi}{w_n}s + \frac{1}{w_n^2}s^2} \end{bmatrix} (42)$$

Such a choice corresponds to settling time of 0.12s for the flux  $\Phi_{dr}$  and of 0.3s for the speed,

 $w_{n1} = 39.5833 r ds^{-1}$ ,  $w_n = 15.833 r ds^{-1}$ . The static state feedback gain  $K_s$  defined in (19) stabilizing the pair  $(\hat{A}, \hat{B})$  has been computed:

$$K_s = \begin{bmatrix} -0.1061 & 0 & 1.6977 & 0 \\ 0 & -0.4937 & 0 & 0.004 \end{bmatrix}$$

The matrices X, N and L verifying (19) and (20) are given as follow:

$$N = 10^{8} \begin{bmatrix} 2.1182 & 0\\ 0 & 5.7960 \end{bmatrix}$$
  
$$\gamma_{opt} = 8.8585e - 14$$

The obtained static gain is:

$$K = \begin{bmatrix} 0.0128 & 0 & 1.6977 & 0 \\ 0 & 0.0592 & 0 & 0.004 \end{bmatrix}$$

# 5. SINULATION RESULTS AND INTERPRETATIONS

To test the effectiveness of the proposed method, simulation results are accomplished on an induction motor powered by a voltage inverter as illustrated in Fig. 3



Fig. 3. Overall scheme of the time moments based controller

Notice that (33) allows estimating the flux and the electrical stator speed  $w_s$  so that [14] and [17]:

$$\hat{\Phi}_{dr} = \frac{M}{1 + \tau_r s} I_d \; .$$

For  $w_s$ , which is equal to  $w_s = pw_m + \frac{M}{\tau_r \hat{\Phi}_{dr}} I_q$ ,

cannot be used as it is since the flux  $\hat{\Phi}_{dr}$  is null when starting the motor. We use then the following equation:

$$w_{s} = pw_{m} + \frac{M}{\tau_{r}\hat{\Phi}_{dr} + \varepsilon}I_{q}$$
  
Where  
 $\varepsilon = 0.01$ .  
 $\theta_{s} = \frac{1}{s}w_{s}$ .

The designed control algorithm was fulfilled by the software Matlab/SIMULINK. The specifications of the induction motor are listed in Table1.

Table 1: Parameters of the motor

Motor rated power (kW)	1.2
Rated current (A)	6
Pole pair number (p)	2
Stator inductance $L_s$ (H)	0.261
Rotor inductance $L_r(H)$	0.261
Stator resistance $R_s(\Omega)$	2.3
Rotor resistance $R_r(\Omega)$	1.83
Mutual inductance $M$ (H)	0.245
Motor inertia $J$ (Kg.m <sup>2</sup> )	0.03
Viscous coefficient	0.002
f (N.m.s/rd)	

Fig. 4 represents the step responses of the flux  $\Phi_{dr}$  and the speed  $w_m$  and their corresponding reference model. We notice that the transient performances, fixed by the reference model, are almost guaranteed with a slight gap. Fig. 5 represents respectively evolutions of the electromagnetic torque, d,q-axis current, d,q-axis rotor flux and the rotor speed. The reference flux is taken equal to  $\hat{\Phi}_{dref} = 0.8$ Wb.

The speed reference goes from 0 to 100 rd/s and remains to this value up to t=6s beyond which it changes its sign in going to -100 rd/s and keep this values until t=11s. Le cycle is repeated going

through a zero value between t=11s and t=12s. An external load torque is introduced at t = 4s and cancelled at t = 5s.

Fig. 6 represents the evolutions of the previously cited variables without load torque. The results obtained show robustness against external disturbances and show the convergence of the measured states around the desired trajectory. These results show also that when the reference speed changes, all shapes of existing variables undergo small fluctuations but remain close to their reference values. This is clearly visible in the shape of the d-axis flux which tends to 0.8Wb whereas the q-axis flux remains null.



a) Step response of the flux and the reference model



b) Step response of the speed and the reference model

Fig. 4 Step response of the speed and the flux and their corresponding reference models





Fig. 5. Simulation results of a temporal moment based controller for a trapezoidal command signal with load torque



a) electromagnatic torque



b) q-axis current







# 6. CONCLUSIONS

In this paper, we have applied the method of temporal moment to synthesize a controller whose various stages of synthesis are performed using LMI formalism. The closed-loop stability that must verify a system and transient performances have been analyzed with the help of a reference model. The effectiveness of the proposed method is demonstrated through numerical simulation applied to the flux and the speed regulation of an induction motor. This yielded adequate results.

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