

# Electrical Power System Instability and Chaos

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**ABSTRACT:** The principal objective of this paper was deeply described, discussed and studied an approach to chaotic oscillation in power systems and relationships between chaos and various power system instability modes. The basics of the mathematical theory were introduced and then specific applications to power system engineering problems were discussed. The applications were encompassed modelling and simulation, control, and measurements where the Lyapunov exponents for the strange attractors were calculated. We have presented, illustrated and discussed that by using a three-bus simple system, three routes i.e. route of cascading period doubling bifurcation, torus bifurcation route and directly initiated by large disturbance route may cause chaos in power systems. This paper has showed us the true that chaos in power systems is in fact caused by the injected energy introduced by some kind of disturbances. By also using a simple system, we have illustrated that chaos lead power system to voltage collapse, angle divergence, or voltage collapse with angle divergence oscillation. This paper has strongly showed us that chaos in power system is very likely to be an intergrade existing in the transient stage after a large disturbance. In order to prevent the happening of power system instability incidents effectively, it was necessary to keep up on the studying of chaotic phenomena in power systems. This paper has been helpful to understand the mechanisms of various instability modes and to find effective anti-chaos strategies in power systems.

**KEYWORDS:** Chaos, Instability mode, torus bifurcation, cascading period doubling bifurcation.

## I. INTRODUCTION

Chaos theory studies the behavior of dynamical systems that are highly sensitive to initial conditions, an effect which is popularly referred to as the butterfly effect. The phenomenon of chaos has attracted widespread attention amongst mathematicians, physicists, and engineers and has been also extensively studied in many fields, such as chemical reactions, biological systems, information processing and secure communications [1].

Study on power system, chaotic phenomena is one important part of power system stability studies. This paper focuses on the cause of chaos in power system and the relationship of chaos and different instability modes. Since three routes may cause the power system to chaos are enumerated, we will be able to discuss in details the effects of those routes in this paper. They are route of cascading period doubling bifurcation (PDB), route of torus bifurcation (TB), and route of directly initiated by a large disturbance of energy [11]. Another topic of this paper is concerning the relationship between power system instability modes and chaos. This study reveal that chaos

can lead power system to voltage collapse, angle divergence or voltage collapse with angle divergence simultaneously when their stability conditions of chaos are broken [18].

## II. THREE BUS POWER SYSTEM MODEL AND ASSUMPTIONS

Let us consider a three-bus power system model which is illustrated in Fig. 1. This model consists of two generators feeding a load, which is represented by an induction motor in parallel with a capacitor and a PQ load. One generator is an infinite bus and the other generator has a constant voltage magnitude  $V_m$  [3] [4].

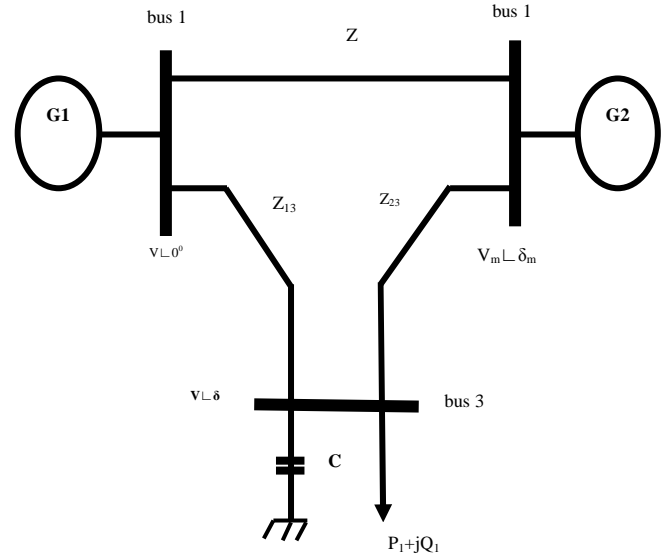


Fig. 1 A three-bus power system

The load model is described by:

$$\begin{aligned} P_d &= P_0 + P_1 + K_{pw}\delta' + K_{pv}(V + TV') \\ Q_d &= Q_0 + Q_1 + K_{qw}\delta' + K_{qv} + K_{qv2}V^2, \quad (1) \end{aligned}$$

Where  $P_0$  and  $Q_0$  represent the constant real and the reactive powers of the motor, respectively, and  $P_1$  and  $Q_1$  are the P-Q loads [4]. Since we know that  $\delta_m$ ,  $\omega$ ,  $\delta$ , and  $v$  are the state variables;  $\delta_m$  and  $\delta$  are the power angles of the machine and the load, respectively;  $\omega$  is the radian frequency of the machine;  $v$  is the load voltage [6];  $Q_1$  plays an important role in the dynamical behaviour of the entire power system and is usually taken as the bifurcation parameter of the power system [8]. It may be easily found that when the  $Q_1$  set as different values, power systems experience complicated dynamic bifurcations. This three bus power system is approximately described by the following nonlinear differential equations [5] [7];

$$\dot{\delta}_m = \omega$$

$$\dot{\omega} = 16.6667 \sin(\delta - \delta_m + 0.0873) V - 0.1667 \omega + 1.8807$$

$$\dot{\delta} = 496.8718 V^2 - 166.6667 \cos(\delta - \delta_m - 0.0873) V - 666.6667 \cos(\delta - 0.2094) V - 93.3333 V + 33.3333 Q_1 + 43.333,$$

$$\dot{V} = -78.7638 V^2 + 26.2172 \cos(\delta - \delta_m - 0.0124) V + 104.8689 \cos(\delta - 0.1346) V + 14.5229 V - 5.2288 Q_1 - 7.0327$$

The initial conditions assumed to be  $(\delta_m, \delta, V)_{t=0} = (0.3, 0.2, 0.97)$ . The state variables of the system are  $\delta_m, \omega, \delta$  and  $V$ . This system equations has four equilibrium points [9]. From our knowledge of engineering mathematics, the Jacobean matrix of the system has to be written as follows:

$$J = \begin{bmatrix} 0 & 1 & 0 & 0; \\ -16.667V \cos(\delta - \delta_m + 0.0873) & -0.1667 & 16.667V \cos(\delta - \delta_m + 0.0873) & 16.667 \sin(\delta - \delta_m + 0.0873); \\ -166.6667V \sin(\delta - \delta_m - 0.0873) & 0 & 166.6667V \sin(\delta - \delta_m - 0.0873) + 666.6667V \sin(\delta - 0.2094) & 2496.871V - 166.6667 \cos(\delta - \delta_m - 0.0873) - 666.6667 \cos(\delta - 0.2094) - 93.3333; \\ 26.2172V \sin(\delta - \delta_m - 0.0124) & 0 & -26.2172V \sin(\delta - \delta_m - 0.0124) - 104.8689V \sin(\delta - 0.1346) & -157.5276V + 26.2172 \sin(\delta - \delta_m - 0.0124) + 104.8689 \sin(\delta - 0.1346) + 14.5229 \end{bmatrix};$$

While discussing the stability properties of the attractors as briefly as possible, let us now denote the state variables together by  $E^* = (\delta_m, \omega, \delta, V)^T$ . If the nominal equilibrium points [13] of attractors are  $E^* = (0.3366, 0, 0.1330, 0.9727)^T$ .

The Jacobean matrix will become:

$$J = \begin{bmatrix} 0 & 1.0000 & 0 & 0; \\ -16.1025 & -0.1667 & 16.1025 & -1.9340; \\ 46.4974 & 0 & -95.9921 & 48.8932; \\ -5.4656 & 0 & 5.6288 & -144.4910 \end{bmatrix};$$

The eigenvalues are found by solving the characteristic equation,  $|J - \lambda I| = 0$ , which is  $0\lambda^4 + 240\lambda^3 + 1.365 \times 10^4 \lambda^2 - 5380\lambda - 1145200 = 0$ . The eigenvalues for attractors are -149.6200, -90.7900, and  $-0.1200 + 2.9000i$  hence the equilibrium points of  $E^*$  are seen to be locally stable.

For the case when the fixed point are  $E^* = (-0.3366, 0, 0.1330, 0.9727)^T$  the jacobian matrix will change the value signs due to -0.3366 and 0 but the eigenvalues are found by solving the characteristic equation,  $|J - \lambda I| = 0$ , which will remain the same as before [1],

$$0\lambda^4 + 240\lambda^3 + 1.365 \times 10^4 \lambda^2 - 5380\lambda - 1145200 = 0$$

The eigenvalues for the same attractors were again found to be -149.6200, -90.7900, and  $-0.1200 + 2.9000i$  which means that the equilibrium points of  $E^*$  are again seen to be locally stable. Using a Matlab-Simulink model, as shown in Fig 2; the  $\delta_m V$  and  $\omega V$  phase portraits of the system achieved are shown in Fig 3 and Fig 4.

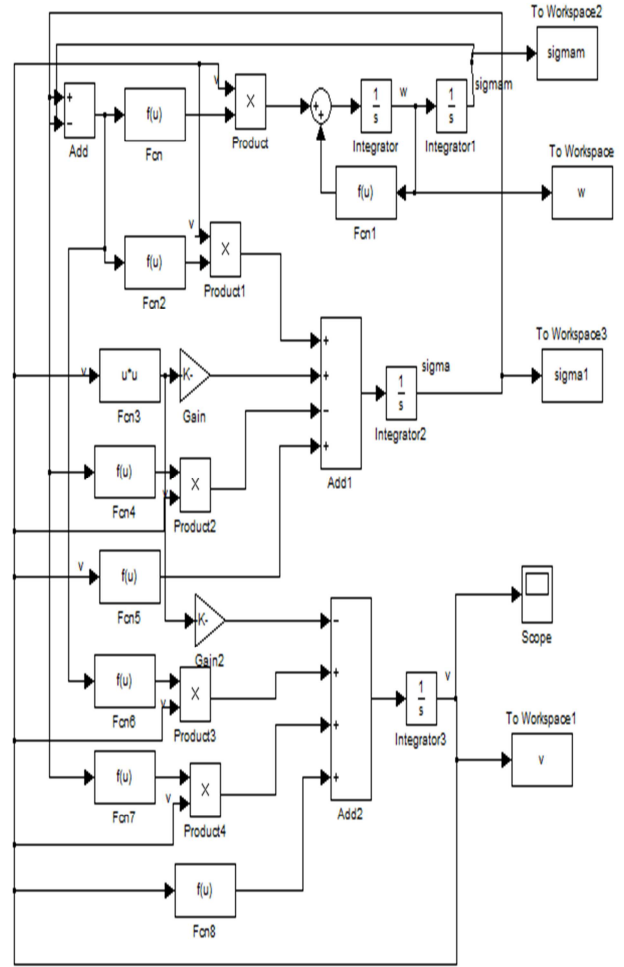


Fig. 2 The Matlab-Simulink model of the system for  $Q_1=11.3776$  and the initial conditions  $(\delta_m, \delta, V)_{t=0} = (0.3, 0.2, 0.97)$ .

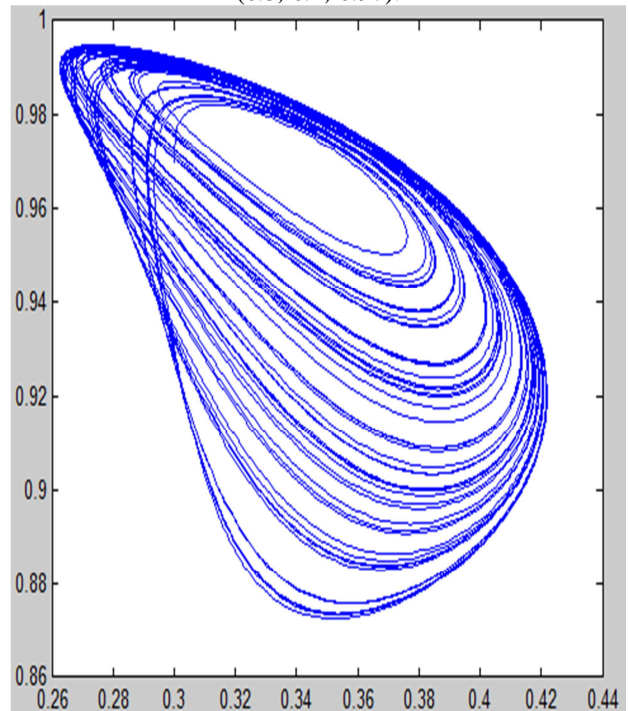


Fig. 3  $\delta_m V$  phase portrait of the system when  $Q_1=11.3776$  and the initial conditions  $(\delta_m, \delta, V)_{t=0} = (0.3, 0.2, 0.97)$

From Fig. 3 and below Fig. 4, we can see that the value of parameter  $Q1$  depends on the type of bifurcation, which satisfy the necessary and sufficient conditions for the existence of a chaotic attractor. Therefore, chaos exists in power system<sup>[15]</sup>. Taking the above mentioned analyses into consideration, the power system may undergo the chaotic state and the typical chaotic attractor is shown again in Fig. 4.

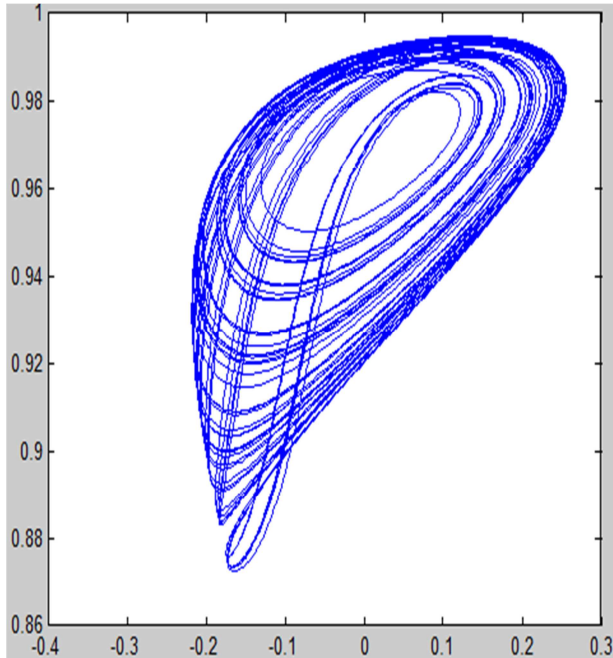


Fig. 4  $\omega V$  phase portrait of the system when  $Q1=11.3776$  and the initial conditions  $(\delta_m, \delta, V)_{t=0} = (0.3, 0.2, 0.97)$

From the above results, we can strongly conclude that when the power system falls into the chaotic behaviour, the stable operation of power system is severely hampered<sup>[10]</sup>. Therefore, it is necessary to control the chaos in the power system by using the finite-time stability theory to design two controllers<sup>[2]</sup> and make the power system asymptotically stable.

### III. TIME SERIES RESULTS OF THE SYSTEM

In this section, we give numerical simulations to show the effectiveness of the proposed two control methods. The controlled system of the ordinary differential equations is integrated by using the fourth-order Runge–Kutta in time steps of 0.001 s. In the numerical simulations, we set  $Q1=11.3776$ , with which the unstable equilibrium point of the system is  $X_{eq} = (0.3366, 0, 0.1330, 0.9727)$ . The initial conditions  $(\delta_m, \delta, V)_{t=0} = (0.3, 0.2, 0.97)$  are chosen in all simulations<sup>[9]</sup>, and the control parameters of the second controller are taken to be  $H1 = 0.0667$  and  $H2 = 15$ . Power system has experienced chaotic behaviours before the two proposed controllers are carried out. To demonstrate why the presented controllers are so effective in controlling the chaos in a power system, we turn on the controller at  $t=100$  s<sup>[12]</sup>.

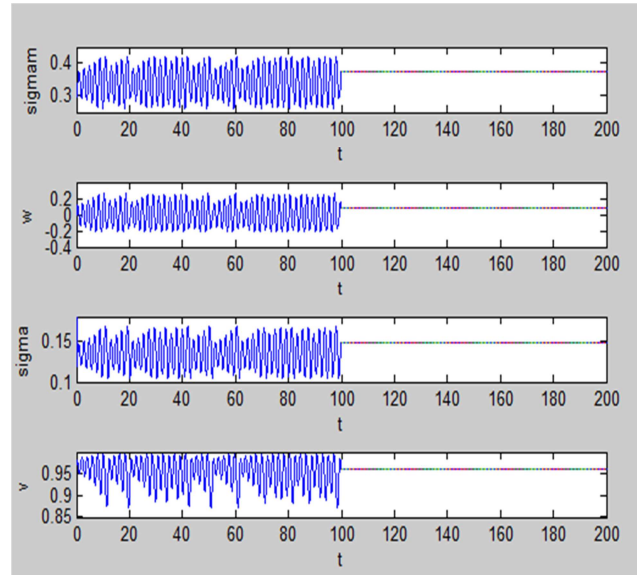


Fig. 5 Time series simulation result of the system for the attractor  $\delta_m, \omega, \delta$  and  $V$

This Time series simulation above confirm the Control chaotic power system to the equilibrium point  $X_{eq} = (0.3366, 0, 0.1330, 0.9727)$ , with uncontrolled chaotic evolution  $t$  includes between  $[0, 100]$  and controlled dynamics  $t$  between  $[100, 200]$ . Panel (a) shows the dynamic responding curve of  $\delta_m$ <sup>[12]</sup>, panel (b) the dynamic responding curve of  $\omega$ , panel (c) the dynamic responding curve of  $\delta$ , and panel (d) the dynamic responding curve of  $V$ . Since the time series includes powerful functions for the statistical analysis of time series, from this theory we can easily find the attractor's spectral plots, correlations and histograms of the system.

### IV. THREE ROUTES TO CHAOS IN POWER SYSTEMS

This paper prove that by using a three-bus simple system, three routes which may cause chaos in power systems must be taken into account and discussed. They are the route of cascading period doubling bifurcation (PDB)<sup>[11]</sup><sup>[14]</sup>, torus bifurcation route and directly initiated by large disturbance route. PDB is caused by a real Floquet multiplier (FM) moving counter to the real axis and going out of the unit circle from a point  $(-1,0)$  in the complex plane. The route of cascading PDB is a typical route to chaos and has to be studied deeply in many nonlinear systems. Torus bifurcation (TB) is also a typical route to chaos. TB is caused by a couple of conjugate Floquet multipliers (FM) going out of the unit circle with a nonzero imaginary part in the complex plane<sup>[11]</sup>. Chaos caused by TB has some interesting features, such as self-organizing phenomenon, coexistence of divergent subspace and chaotic subspace. These features are helpful to deeply understand various modes of power system instability. All studies told us that chaos in power systems is in fact caused by some kind of external disturbances. This paper reports that when the  $Q1$  set as different values, power systems experience complicated dynamic bifurcations, as shown in fig. 6.

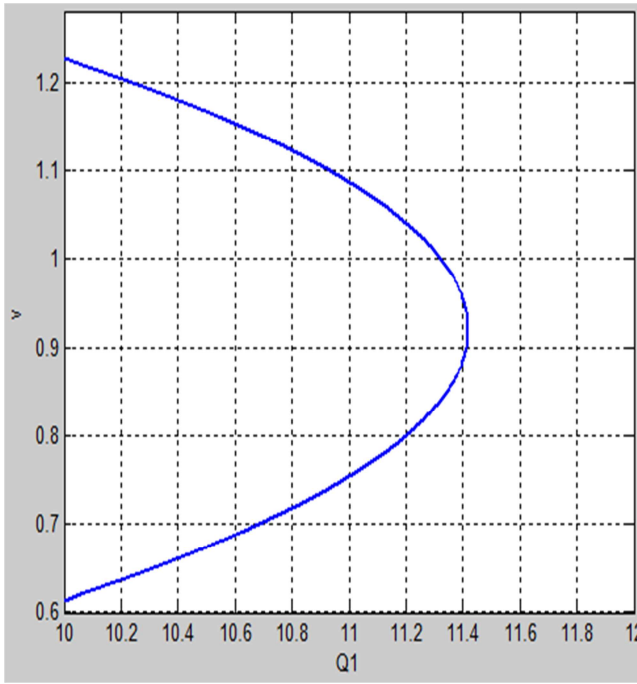


Fig. 5 Bifurcation diagram

From Fig. 5, we can see that the value of parameter  $Q1$  depends on the type of bifurcation, which is summarized as follows:

- (i) Periodic doubling bifurcation at  $Q1 = 10.8859$ ;
- (ii) Subcritical Hopf bifurcation at  $Q1 = 10.9461$ ;
- (iii) Periodic doubling bifurcation at  $Q1 = 11.3776$ ;
- (iv) Subcritical Hopf bifurcation at  $Q1 = 11.4066$ ;
- (v) saddle-node bifurcation at  $Q1 = 11.4106$ .

We calculate the Lyapunov exponents by using the method of singular-value decomposition with setting  $Q1 = 11.3776$  and choosing initial condition  $(\delta_m, \delta, V)t=0 = (0.3, 0.2, 0.97)$ . The four Lyapunov exponents of power system are obtained to be  $\lambda_1 = 0.3136$ ,  $\lambda_2 = 0$ ,  $\lambda_3 = -3.7496$ , and  $\lambda_4 = -93.0108$ , which satisfy the necessary and sufficient conditions for the existence of a chaotic attractor. Therefore, chaos exists in power system<sup>[17]</sup>. Taking the above mentioned analyses into consideration, the power system undergoes the chaotic state and the typical chaotic attractor is again shown in figures above. We confirm that when the power system falls into the chaotic behaviour, the stable operation of power system is severely hampered<sup>[16]</sup>. Therefore, it is necessary to control the chaos in the power system.

## V. INSTABILITY MODES AND CHAOS

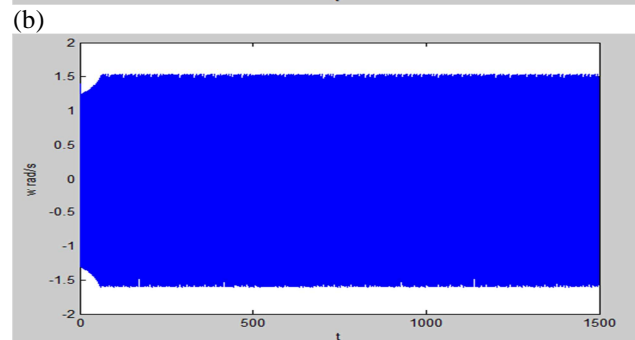
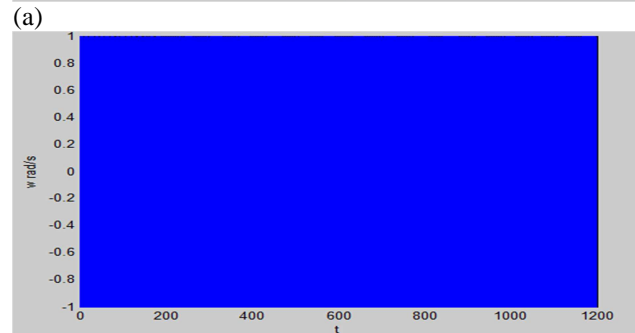
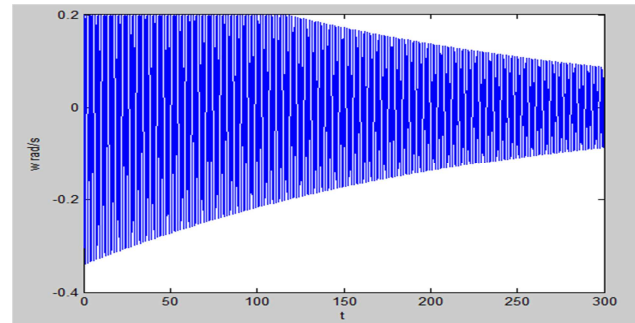
Chaos is very sensitive to initial condition and system parameters. Any small changing to them strongly break their stable oscillations. Previous studies have reported that the broken of chaos can lead power systems to voltage collapse<sup>[18]</sup>. This paper report that chaos can lead power systems to voltage collapse, angle divergence, or voltage collapse with angle divergence simultaneously. Based on these researches, it is derived that chaos possibly exists in power system as an intermediate stage of the instability

incident after large disturbance<sup>[16]</sup>. When disturbance happens, power system comes into transient stage. If the disturbances are small, Hopf Bifurcation (HB) may happen and continuous oscillation follows. If the disturbance is large, system may come into chaos. And, when the disturbance becomes larger, the chaos may be broken. Voltage collapse, angle instability or voltage collapse and angle divergence simultaneously may happen<sup>[10][16]</sup>. If the disturbance is very large, system may directly come into the above three instability conditions over the stages of HB, chaos and chaos breaking.

wo (rad/s)	Simulation Time	Final state	Phase diagram
0.50	300	A equilibrium point	Fig.6a
1.3024478	1200	A equilibrium point	Fig.6b
1.3024479	15000	Chaos	Fig.6c
1.40	15000	Chaos	Fig.6d
1.5980378	15000	Chaos	Fig.6e
1.5980379	100	Monotonic divergence	Fig.6f

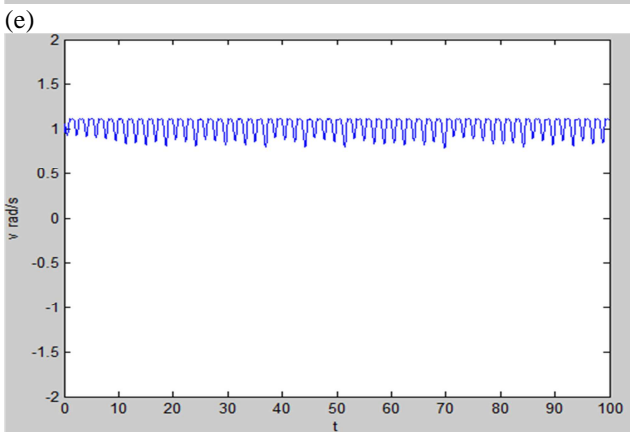
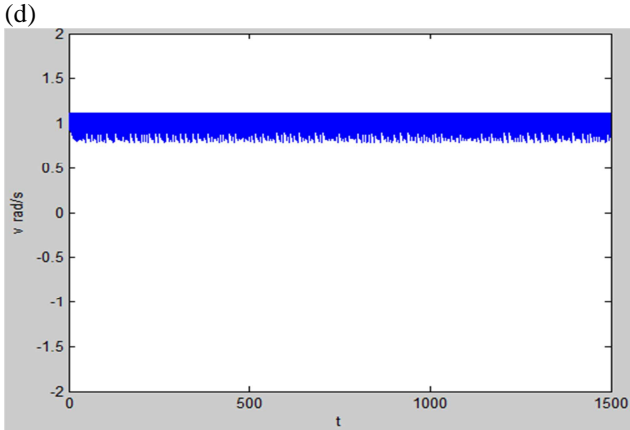
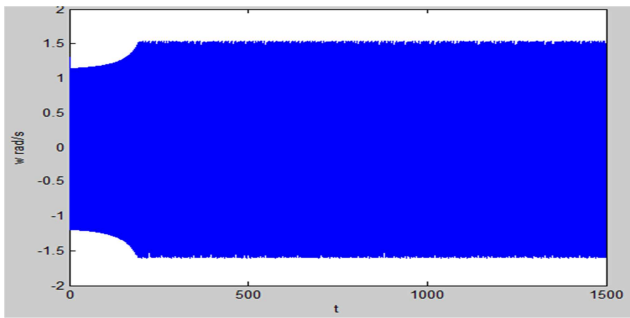
Table1: Different system conditions with different initial angle speeds

There also exists another type of route to chaos in power system. It is the route of initiated directly by a large disturbance of energy. By selecting initial values:  $Q1=10.894$ ,  $\delta_m(0)=0.3$ ,  $\delta(0)=0.2$ ,  $V(0.97)$ , slowly increasing the initial angle speed  $\omega_0$  in the range of 0 to 1.7 rad/s, we can yield the following results:



(c) due to the different initial disturbed energies. That is the





(f)

Fig. 6 Simulation results with different initial angle speed

Since except the initial angle speed  $\omega_0$ , all the other parameters are same for the six cases shown in Fig.6, hence we define the disturbed energies (DEs) of the above initial points as the disturbance of kinetic energy which are only related to the associate  $\omega_0$ <sup>[15] [16]</sup>. The larger  $\omega_0$  is, the larger DE will be. When DE is smaller ( $\omega_0=1.3024478$  rad/s), power system can converge to a stable equilibrium point just as shown in Fig.6a and Fig.6b.

When DE increases, the convergence becomes more and more difficult. At about  $\omega_0=1.3024479$  rad/s, power system converges to a chaos after a long time transient oscillation (Fig.6c). At the range of 1.3024479 and 1.5980378 (rad/s), the final states are controlled by a chaotic condition. While at  $\omega_0=1.5980378$  (rad/s), the system appears a monotonic divergence<sup>[18]</sup>. Comparing Fig.6b with Fig.6c (or Fig.6e with Fig.6f), it is found that the  $\omega_0$  value of Fig.6c (or Fig.6f) is only  $10^{-7}$  larger than Fig.6b (Fig.6e). Between them, neither period doubling bifurcation nor torus bifurcation exists.

The appearance (or dissolve) of the chaotic state is just

reason why we classified the initial disturbances of energy into a new type route to chaos. It tells us that chaos of power system is in fact caused by injected energy introduced by unexpected disturbances<sup>[16] [18]</sup>.

## VI. CONCLUSION

Chaos is the science of surprises, of the nonlinear and the unpredictable. It teaches us to expect the unexpected. While most traditional science deals with supposedly predictable phenomena like gravity, electricity, or chemical reactions, Chaos Theory deals with nonlinear things that are effectively impossible to predict or control, like turbulence, weather, the stock market, our brain states, and so on. These phenomena are often described by fractal mathematics, which captures the infinite complexity of nature. Many natural objects exhibit fractal properties, including landscapes, clouds, trees, organs, rivers etc, and many of the systems in which we live exhibit complex, chaotic behaviour. In the field of electrical power systems, this paper has proved that chaos can induce voltage collapse, angle divergence or voltage collapse with angle instability simultaneously when its stable condition is broken. It shows us that chaos in power systems is very likely to be an intermediate stage in the transient after a large disturbance. All studies are helpful to understand the various instability modes and to find effective anti-chaos strategies in power systems.

## VII. ACKNOWLEDGEMENT

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