

Analytical Modeling of a Cable fled in Homogenous Soil

Mabrouki. H¹, Derbel. N², Hadj Abdallah .H³

Advanced Command and Energy Management Laboratory, in Engineering National School of Sfax

Sfax University, Tunisia.

hichemmabrouki@gmail.fr, dernizar@yahoo.fr, hsan.haj@enis.rnu.tn

Abstract: *In this paper we try to simplify the modeling of cable by using a new approach. In fact, we start by using the Maxwell's equations in order to settle on the electric field within every element of the cable. Then, and from the electric field expression, we extract the analytical expression of impedance for each element of the cable. After that, we simulate the procedure "impedance versus frequency" by using specific software "MATLAB" with the aim of taking accounts the transient phenomena).*

Keywords: *Modeling cable, high frequency performance, homogenous soil, and Maxwell's equations.*

1. Introduction

The electric modeling of the transmission lines of power generally consists in establishing the relations between the tension and the current which control this system [1]. These relations are known as telegrapher's equations. They are deduced mainly, from the circuits theory (the law of ohm, law of Kirchhoff...), also from the electromagnetic field propagation along the line, so the Maxwell's equations can be used. In fact, the drivers of the line can be assimilate like a waveguides [2][3].

Shelkunoff published in 1936 [4] a significant work concerning the coaxial transmission line. It was developed in a rigorous way, starting from the Maxwell's equations, to the study of the electromagnetic field propagation.

Moreover, the electromagnetic field propagation in the ground was well studied by Carson and Pollaczec (1926) [6] [7]. This paper is devoted to the electric modeling of a buried cable fled in a homogeneous ground.

Work in this paper is presented as follows:

We, initially, determined the general shape of the electromagnetic field in all studied space, by using the cylindrical co-ordinates and Laplace transformation. Then, we studied the case of a cable only presumed fled in homogeneous soil and extended to the infinite.

Precisely, we determine the impedance evolution (in function of frequency) of the main element of cable (core end screen). In addition; this study can be exploited in analysis of transient phenomena [5] (by determination of the current circulating in every element).

2. General shape of the electromagnetic field

In what follows, we will use the basic electromagnetism equations to determine the general shape of the magnetic field in a cylindrical transmission system.

2.1. The basic electromagnetism equations

- Maxwell's Equations :

Maxwell's equations have since been found to govern all classical electromagnetic phenomena.

$$\nabla \cdot \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial T} \quad \text{Modified Ampere's Law}$$

$$\nabla \cdot \bar{E} = -\frac{\partial \bar{B}}{\partial T} \quad \text{Faraday's Law}$$

$$\nabla \cdot \bar{B} = 0 \quad \text{No magnetic charges}$$

$$\nabla \cdot \bar{D} = \rho \quad \text{Gauss' Law}$$

Where:

$$\bar{E} = \text{Electric Field Intensity} \quad [V/m]$$

$$\bar{H} = \text{Magnetic Field Intensity} \quad [A/m]$$

$$\bar{J} = \text{Electric Current Density} \quad [A/m^2]$$

$$\bar{D} = \text{Electric Flux Density} \quad [C/m^2]$$

$$\bar{B} = \text{Magnetic Flux Density} \quad [T]$$

$$\rho = \text{Electric Charge Density} \quad [C/m^3]$$

- Constitutive Relations:

The two divergence equations can be derived from the curl equations and the continuity equation.

Hence, Maxwell's equations represent six scalar equations with twelve unknowns. The remaining six scalar equations are required for a unique field solution which found by using the constitutive equations, relating the fields in a certain material as:

$$\left. \begin{aligned} \vec{D} &= \varepsilon \vec{E} \\ \vec{B} &= \mu \vec{H} \\ \vec{j} &= g \vec{E} \end{aligned} \right\} \quad (1)$$

Where ε , is the permittivity and μ is the permeability of the material, and both are tensors in general.

- Use of symbolic system calculation:

By using the Laplace transformation and by considering the two following assumptions:

- The free loads are null everywhere except on the surface of the conductor.
- The line is at rest at the initial moment.

The Maxwell's equations, taking account of the relations (1) become:

$$\left\{ \begin{aligned} \text{rotH} &= (g + \varepsilon p). E \\ \text{rotE} &= -\mu p. H \end{aligned} \right. \quad (2)$$

$$\left\{ \begin{aligned} \text{divH} &= 0 \\ \text{divE} &= 0 \end{aligned} \right. \quad (3)$$

Where : $p = j\omega$ is the Laplace Operator

- General differential equation of the electric field:

From the previous equations, we can establish the general differential equation which governed the electric field while using the following equation (4):

$$\text{rot}(\text{rot}\vec{E}) = \text{grad}(\text{div}\vec{E}) - \text{Lap}\vec{E} \quad (4)$$

In fact we obtain:

$$\text{LapE} - (\mu g p + \mu \varepsilon p^2). E = 0 \quad (5)$$

3. The longitudinal electric field expression

This study takes in particular, the cylindrical conductors, so we use the cylindrical coordinates (r, θ, z) , where the (oz) axis merges with the axis of a cable.

By projecting the relation (5) on the axis (oz) , we obtain the following differential equation which governed the longitudinal electric field.

$$\frac{\partial E_z^2}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \theta^2} + \frac{\partial^2 E_z}{\partial z^2} - K^2 E_z = 0 \quad (6)$$

With: $K^2 = \mu g p + \mu \varepsilon p^2$

The determination of the longitudinal electric field expression consists in solving the equation (6). The traditional method of resolution for this kind of equations is the Laplace producing method (separation of variables) [9].

So we have,

$$E(r, \theta, z) = R(r). \Theta(\theta). Z(z) \quad (7)$$

$R(r)$: Function which regroups the variables r

$\Theta(\theta)$: Function which regroups the variables θ

$Z(z)$: Function which regroups the variables z

By substitution of those three expressions in equation (6) we obtain:

$$\frac{1}{R} \left[\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} \right] + \frac{1}{r^2} \frac{\partial^2 \Theta}{\partial \theta^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} - K^2 = 0 \quad (8)$$

It is clear that the first two terms of (8) are independent of Z ; consequently the third one must be like the two proceeding terms, so we can presume that:

$$\frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = \gamma^2$$

We note that the general solution is a linear combination of the functions $e^{-\gamma z}$ and $e^{+\gamma z}$, those functions translates the exponential character of the electromagnetic wave propagation along the (oz) axis, with γ is called propagation constant.

So that the equation (8) becomes:

$$\frac{r^2}{R} \left[\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} \right] + \frac{1}{\Theta} \frac{\partial^2 \Theta}{\partial \theta^2} - (K^2 - \gamma^2) r^2 = 0 \quad (9)$$

If we assume that:

$$\frac{1}{\Theta} \frac{\partial^2 \Theta}{\partial \theta^2} = \chi^2; \text{ as an arbitrary constant.}$$

Sight the periodicity of the phenomenon following θ , we take $\chi^2 = n^2$, where n is an integer.

In this case, the general solution is a linear combination of the functions $\cos n\theta$ and $\sin n\theta$.

By making a good choice of the reference mark corresponding to $\theta = 0$, (axial symmetry of the field), only the terms $\cos n\theta$ appear in the solutions.

We thus obtain:

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} - \left[m^2 + \frac{n^2}{r^2} \right]. R = 0 \quad (10)$$

Where:

$$m^2 = K^2 - \gamma^2 \quad (11)$$

Based on the equation (11), two cases are distinguished:

- **Case 1:** when $m = 0$

The general solution of the equation (10) is given by:

$$R = A \cdot r^n + B \cdot r^{-n}$$

In this case the constant of propagation is defined by:

$$\gamma = K.$$

This corresponds to the physical conditions of an ideal line; it is about a purely theoretical case towards.

- **Case 2 :** when $m \neq 0$

The equation (10) admits as general solution, a combination of modified Bessel function of the first and second kind of argument(mr), noted for the order(n): $I_n(mr)$ and $K_n(mr)$.

N.B: We can either use, like solution, a combination of a Bessel function of first and second kind $J_n(mr)$ and $Y_n(mr)$. [8]

The combination of the partial solutions R , θ and Z makes it possible to obtain a solution for the differential equation (6).

If we consider only the waves which move in the increasing Z direction, the longitudinal component of the electric field has the following general form:

$$E_z(r, \theta, z) = \sum_{n=0}^{\infty} [A_n I_n(mr) + B_n K_n(mr)] \cos(n\theta) e^{-\gamma z} \quad (12)$$

With: A_n and B_n are the constants of integration which is determined by starting from the boundary conditions of the system.

We present by the following flow chart the general method to follow to find the electric field expression.

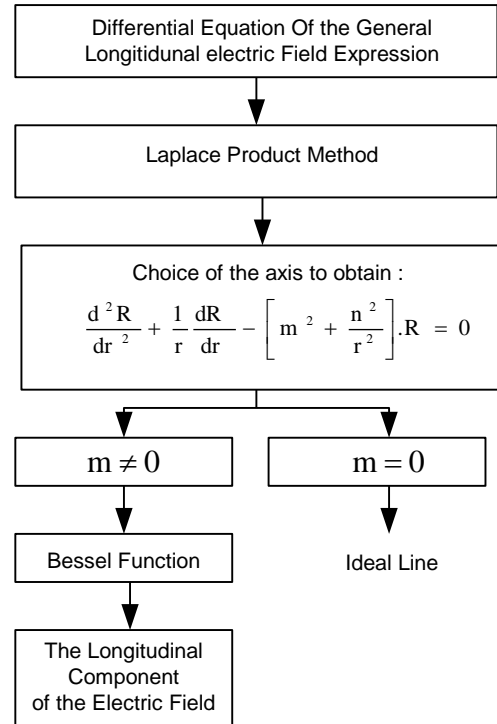


Fig. 1. Flow chart of the Electric Field Expression.

4. Modeling of a cable fled in a ground, presumed homogeneous and infinitum extended

The cable is considered composed of four coaxial components: a conductive core of ray (r_1), an insulating envelope, a metal screen with interior ray (r_2) and external ray (r_3), a sheath (insulating) with external ray (r_4). In the core circulates a current (I_c) and in the shield a current (I_{sc}), the return is ensured by the ground (figure 2) [10].

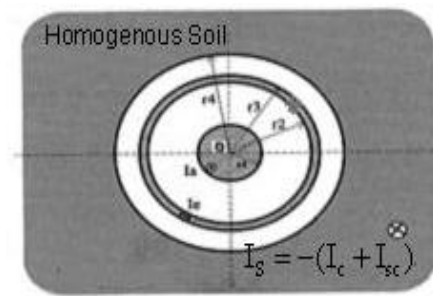


Fig.2. Cross section of a cable fled in an infinite ground.

By considering a cable alone we simplify the equations. Besides, the determination of the field distribution in a cross-section is made easier.

4.1. The electromagnetic field form

The ground assumption extended ad infinitum around the cable gives to the system a symmetry revolution around the (oz) axis. Therefore, all the sizes are independent of θ and the proximity effects are null.

Thus, from relation (12), we can draw the longitudinal component from the electric field, while limiting oneself has the order ($n = 0$).

$$E_z(r, z) = [A_0 \cdot I_0(mr) + B_0 \cdot K_0(mr)] \cdot e^{-\gamma z} \quad (13)$$

The other components can be deduced by means of the Maxwell's equations from E_z . Indeed if we pose that:

$$E_z(r, z) = E_z(r) \cdot e^{-\gamma z} \quad (14)$$

The Maxwell's equations (2) and (3) in cylindrical coordination and under the conditions of a field with symmetry of revolution ($\frac{\partial}{\partial \theta} = 0$) are reduced, respectively as following:

$$\begin{cases} \gamma H_\theta = (g + \epsilon p) \cdot E_r \\ \frac{H_\theta}{r} + \frac{\partial H_z}{\partial r} = (g + \epsilon p) \cdot E_z \end{cases} \quad (15)$$

And;

$$\gamma E_r + \frac{\partial E_z}{\partial r} = -\mu p H_\theta \quad (16)$$

The components H_r , H_z and E_θ are null (symmetry revolution around the (oz) axis).

Utilizing the relation (11), with $m \neq 0$ we can express E_r and H_θ according r and the derivative of E_z .

$$E_r = \frac{\gamma}{m^2} \cdot \frac{\partial E_z}{\partial r} \quad (17)$$

$$H_\theta = \frac{g + \epsilon p}{m^2} \cdot \frac{\partial E_z}{\partial r} \quad (18)$$

If we exclude the factor $e^{-\gamma z}$, which translates the propagation phenomenon, into the expressions of the fields, we generally have.

$$E_z = A_0 \cdot I_0(mr) + B_0 \cdot K_0(mr) \quad (19)$$

$$E_r = \frac{\gamma}{m} [A \cdot I_1(mr) - B \cdot K_1(mr)] \quad (20)$$

$$H_\theta = \frac{g + \epsilon p}{m} [A \cdot I_1(mr) - B \cdot K_1(mr)] \quad (21)$$

With: $m = \sqrt{\mu g p + \mu \epsilon p^2 - \gamma^2}$; not null.

4.2. Propagation Constant - approximation

In this section, we consider that the line is without losses (perfect conductors and insulators).

So we can know the values that the constant γ can take.

- Line without losses

Shelkunoff has to clarify well this point in his articles on the electromagnetic theory of the coaxial transmission lines [4]. In his opinion, the principal modes of propagation waves are carried out without attenuation at the speed $v = \frac{1}{\sqrt{\mu \epsilon}}$.

With the conditions of ideal materials, we can summarize the results like hereafter:

At the interior of the conductors, the electromagnetic field is null (surface phenomenon).

In insulators, the magnetic field H_θ is equal by simple application of the theorem of Ampere, with $\frac{I}{2\pi r}$; the current I being the total current crossing the cross-section limited by the circumference of ray r .

The propagation constant is, under these conditions, purely imaginary.

$$\gamma = \sqrt{\mu \epsilon p^2} = j\omega \sqrt{\mu \epsilon} \text{ for } p = j\omega$$

In a perfect dielectric, the electric conductivity g is null, by consequence $m = 0$.

- Real line

In the real case, we can accept the following approximations:

- ✓ The displacement currents can be neglected in front of the conduction currents, in the conductors.
- ✓ The longitudinal currents in insulators are negligible in front of the currents into the conductors.
- ✓ Therefore the magnetic field in insulators is equal, as in the perfect case of materials, with $\frac{I}{2\pi r}$ (consequence of the ampere theorem).

4.3. Electric and magnetic fields in the conductors

In the conductors, the quantity $\mu \epsilon p$ takes large values in front of γ^2 which are generally small (see reference [4]). Generally, the displacement currents are neglected in front of the current of conduction ($g \gg \epsilon p$), we take:

$$m = \sqrt{\mu g p} \quad (22)$$

- Expression of the field in the conductors:

- In the core:

$$E_z^{(c)} = a_0 \cdot I_0(m_1 r) \quad (23)$$

$$H_\theta^{(c)} = \frac{g_1}{m_1} [a_0 \cdot I_1(m_1 r)] \quad (24)$$

With, $m_1 = \sqrt{\mu g_1 p}$

- In the screen

$$E_z^{(sc)} = e_{10} \cdot I_0(m_3 r) + e_{20} \cdot K_0(m_3 r)$$

$$H_\theta^{(sc)} = \frac{g_3}{m_3} [e_{10} \cdot I_1(m_3 r) - e_{20} K_1(m_3 r)]$$

$$\text{With : } m_3 = \sqrt{\mu g_3 \rho}$$

- In the soil:

$$E_z^{(s)} = s_0 \cdot K_0(m_s r) \quad (25)$$

$$H_\theta^{(s)} = -\frac{g_s}{m_s} [s_0 \cdot K_1(m_s r)] \quad (26)$$

$$\text{With } m_s = \sqrt{\mu g_s \rho}$$

g_1 , g_3 and g_s , being respectively, the conductivity of the core, the screen and the ground. a_0 , e_{10} , e_{20} and s_0 are the integration constants, determined by the physical conditions and the boundary conditions of the problem.

Determining integration constants:

Constants are determined by applying the continuity conditions at the conductor-insulation interfaces. Relations continuity of tangential magnetic field at the corresponding surfaces, $r = r_1$, $r = r_2$, $r = r_3$ and $r = r_4$ are clarified using the following relations:

$$r = r_1 \quad \frac{g_1}{m_1} [a_0 \cdot I_1(m_1 r)] = \frac{I_c}{2\pi r_1}$$

$$r = r_2 \quad \frac{g_3}{m_3} [e_{10} I_1(m_3 r_2) - e_{20} \cdot K_1(m_3 r_2)] = \frac{I_c}{2\pi r_2}$$

$$r = r_3 \quad \frac{g_3}{m_3} [e_{10} \cdot I_1(m_3 r_3) - e_{20} \cdot K_1(m_3 r_3)] = \frac{I_c + I_{sc}}{2\pi r_3}$$

$$r = r_4 \quad -\frac{g_s}{m_s} [s_0 \cdot K_1(m_s r_4)] = \frac{I}{2\pi r_4}$$

$$\text{With: } I = I_c + I_{sc}$$

I_c : is the core current

I_{sc} : is the screen current

The first equation relating to the core gives:

$$a_0 = \frac{m_1}{2\pi r_1 g_1 I_1(m_1 r_1)} I_c \quad (27)$$

The resolution of the system consisting of the second and the third relation (screen) gives us:

$$e_{10} = \frac{m_3(I_c + I_{sc})}{2\pi g_3 D_0} \cdot \frac{K_1(m_3 r_2)}{r_3} - \frac{m_3 I_c}{2\pi g_3 D_0} \cdot \frac{K_1(m_3 r_3)}{r_2}$$

$$e_{20} = \frac{m_3(I_c + I_{sc})}{2\pi g_3 D_0} \cdot \frac{I_1(m_3 r_2)}{r_3} - \frac{m_3 I_c}{2\pi g_3 D_0} \cdot \frac{I_1(m_3 r_3)}{r_2}$$

With:

$$D_0 = K_1(m_3 r_2) \cdot I_1(m_3 r_3) - I_1(m_3 r_2) \cdot K_1(m_3 r_3)$$

Finally the last equation (soil) gives:

$$s_0 = -\frac{m_s I}{2\pi g_s r_4} \cdot \frac{1}{K_1(m_s r_4)} \quad (28)$$

In fact, the flow chart of the impedance expression is as follow.

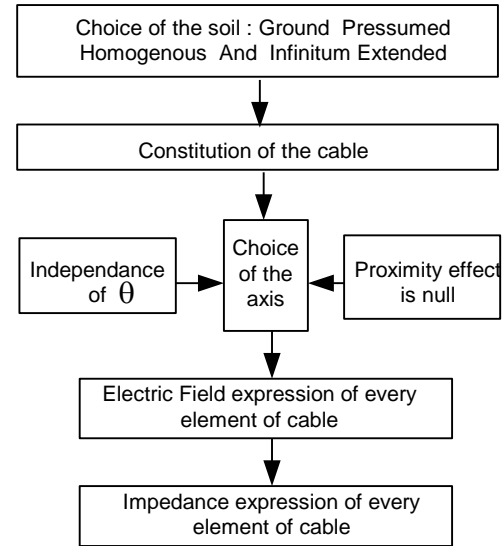


Fig.3. Flow chart of the impedance calculating.

4.4 Linear impedance of the conductor surface:

4.4.1 Linear impedance of the core:

While replacing a_0 given by the equation (27) in the relation (23) we obtain the electric longitudinal field in the core:

$$E_z^{(c)}(r) = \frac{m_1 I_c}{2\pi r_1 g_1} \cdot \frac{I_0(m_1 r)}{I_1(m_1 r_1)}$$

The impedance surface of a full cylindrical conductor is defined by the ratio of the surface electric field to the total current that crosses the surface. If we consider that Z_1 is the linear impedance of the surface of the conducting core so:

$$E_z^{(c)}(r_1) = Z_1 \cdot I_c$$

Consequently, we have:

$$Z_1 = \frac{E_z^{(c)}(r_1)}{I_c} = \frac{m_1}{2\pi r_1 g_1} \cdot \frac{I_0(m_1 r_1)}{I_1(m_1 r_1)}$$

For modeling impedance previous work have a complex approach [11], but in this paper the expression of the core impedance is simple and can be straightforwardly simulated.

Simulation results:

We represent on the following figures the variation of the module of the core impedance Z_1 and his phase according to the frequency (high frequencies are taking account).

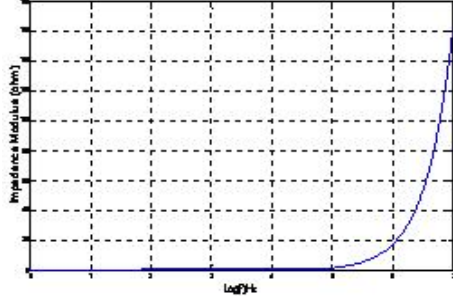


Fig.4. The core impedance modulus variation versus frequency.

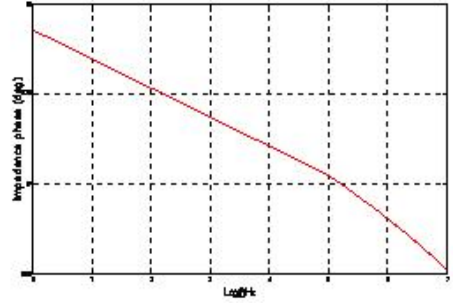


Fig.5. The core impedance phase variation versus frequency.

In high frequency, resistance believes exponentially with the frequency. Here, we find one of the aspects of these high frequency effects, namely the skin effect.

The basic problem within skin effect is that it attenuates the higher frequency components of a signal more than the lower frequency components. This frequency dependent behavior is fairly easy to compute in the frequency domain [12, 13, and 14]. It is of same with the imaginaries part (inductive reactance of the core).

To validate the model we found that the shape of the impedance module and the argument is almost the same presented in the paper [15].

4.4.2 Linear impedance of the screen:

About the screen, the electric field in any points of the conductor is obtained by replacing the constant data e_{10} and e_{20} given by the relations (23) and (24) in (18).

$$E_z^{(sc)}(r) = \frac{m_3 I_c}{2\pi g_3 r_2} \cdot \frac{K_1(m_3 r_3) I_0(m_3 r) + I_1(m_3 r_3) K_0(m_3 r)}{D_0} + \frac{m_3 (I_c + I_{sc})}{2\pi g_3 r_3} \cdot \frac{K_1(m_3 r_2) I_0(m_3 r) + I_1(m_3 r_2) K_0(m_3 r)}{D_0}$$

For hollow cylindrical conductors, Shelkunoff [4] was defined three impedances. When the screen is scanned by a total current I_{sc} , the return coaxial path can be found whether in interior (in the core), or, in exterior (under soil). In external partly (I_{ext}) and in interior partly (I_{int}) which applied in the following figure.

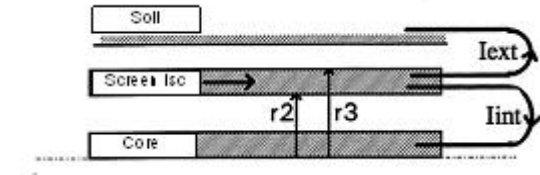


Fig.6. Half longitudinal section of a part of the cable - current of loop.

On surfaces $r = r_2$ and $r = r_3$ we write:

$$E_z^{(sc)}(r_2) = Z_3 I_{int} + Z_4 I_{ext}$$

$$E_z^{(sc)}(r_3) = Z_4 I_{int} + Z_5 I_{ext}$$

If we substitute the currents of loop I_{int} and I_{ext} currents I_c and I_{sc} which are indicated on figure 6.

$$I_{int} = -I_c \text{ And } I_{ext} = I_c + I_{sc}$$

Finally, we easily deduce from it the expressions of the impedances of interior surface Z_3 and external surface Z_5 and the mutual impedance Z_4 .

$$Z_3 = m_3 \frac{K_1(m_3 r_3) I_0(m_3 r_2) + I_1(m_3 r_3) K_0(m_3 r_2)}{2\pi g_3 r_2 D_0}$$

For determining Z_4 expression, we use the following property:

$$K_1(z) I_0(z) + I_1(z) K_0(z) = \frac{1}{z}$$

So we have:

$$Z_4 = \frac{1}{2\pi g_3 r_2 r_3 D_0}$$

And:

$$Z_5 = m_2 \frac{K_1(m_3 r_2) I_0(m_3 r_3) + I_1(m_3 r_2) K_0(m_3 r_3)}{2\pi g_3 r_3 D_0}$$

Simulation results:

The following figures show the variation according to the frequency of the report/ratio of the impedance of Z_5 to the resistance of the screen in D.C. current.

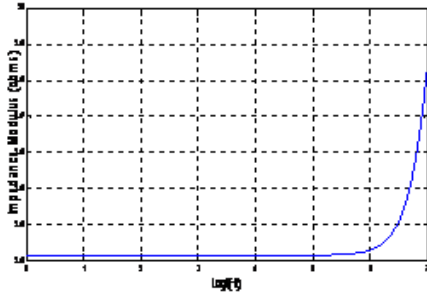


Fig. 7. The impedance modulus of Z_5 variation versus frequency.

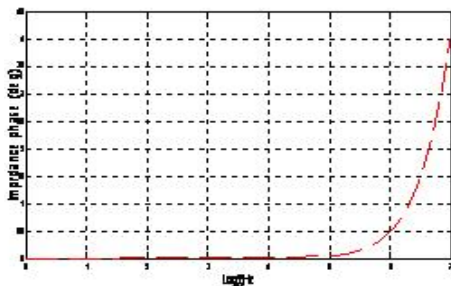


Fig.8. The impedance phase of Z_5 variation versus frequency.

As shows it figure 8 at the low frequencies Z_5 modulus is equal to the resistance of the D.C. current of the screen.

The current in the screen is on has through all the section in a homogeneous way; there is no skin effect. To the approximately 10^4 Hz (according to the thickness of the screen) the skin effect appear. In fact the impedance believes in an exponential way with the frequency.

These results are very important in the direction of simplifying the identification of the current in each component examined of the cable. Moreover, this identification will be successful in the event of default (lightning current), characterized by a sudden rise in frequency (transient phenomena).

5. CONCLUSION

The objective of this work is to give a simple analytical model of a complex system (cable). So, the impedance expression of each element of the cable is determined. Then we do the simulation in high frequency to take account the transient phenomena. Finally those results can be exploited in order to perfectly study the behavior of the return current especially, at high-frequency analysis of the ground [16] and also in the study of the grounding response to lightning currents [17].

REFERENCES

- [1] L.M WEDEPOHL and D.J WICOX « Transient analysis of underground power-transmission systems» Proc. IEEE, Vol 120 ,n°2 February 1973.
- [2] L. Grcev, F. Dawalibi, « An electromagnetic model for transients in grounding systems » IEEE Trans. Power Delivery, Vol. 5, pp. 1773- 1781, Oct. 1990.
- [3] IEEE Guide for safety in AC Substation Grounding.2000.
- [4] S.A Shelkunoff «The electromagnetic theory of coaxial transmission lines and cylindrical shields» Bell System Technical .Journal, 1934, 13 pp 532-579.
- [5] Moreira, A.F. Lipo, T.A. ; Venkataramanan, G. ; Bernet, S.« High frequency modeling for cable and induction motor overvoltage studies in long cable drives » Industry Applications Conference, 2001. Thirty-Sixth IAS Annual Meeting. Conference Record of the 2001 IEEE (Volume: 3), page 1787 - 1794 vol.3.
- [6] F .POLLACZEK « Sur le champ produit par un conducteur simple infiniment long parcouru par un courant alternatif», Rev.Gen .d'Electr, 1931,29 pp 851 – 867.
- [7] J.R Carson « Wave propagation of overhead with ground return », Bell sys. Tech. Journal, October 1926, n° 5, pp .539-554.
- [8] A .ANGOT « Complément de Mathématique» Ed Masson 1972.
- [9] W. Strauss, « Partial differential equations, an introduction » John Wiley, 1992.
- [10] IEEE, « IEEE Guide for the Application of Sheath Bonding Methods for Single-conductor Cables and Calculation of induced Voltage and Current in Cable Sheaths », ANSI/IEEE Std. 575-1998.
- [11] Wilcox, D.J. and Lawler, K.J «Transient phenomena in cross bonded cable systems: analytical results», Electrical Engineers, Proceedings of the Institution of (Volume: 125, Issue: 10) October 1978.
- [12] Leonid Grcev and Vesna Arnautovski « Comparison between simulation and measurement of frequency dependent and transient characteristics of power transmission line grounding » Proc. of 24-th International Conference on Lightning protection (ICLP'98), Birmingham,U.K, 14-18Sept.1998, Vol. 1,pp.524-529.

[13] T.C. Yu and J.R. Martí, «zCable model for frequency dependent modelling of cable transmission systems, » IPST01 in Rio De Janeiro, Brazil, paper no. IPST01-146, June 2001.

[14] Liu.Y «Response of grounding Systems Caused by Lightning :Modeling and Experiments» PhD in Acta Universilatis Upsaliens Comprehensive Summaries of Uppsala Dessertation from the Faculty of Science and Technology.1015. 89 pp Uppsala.

[15] A. Elices, L. Rouco, J. Barquín «Frequency and Time Domain Analysis of Grounding Systems Components».

[16] L. Grcev, and M. Popov, « On High-Frequency Circuit Equivalentents of a Vertical Ground Rod» IEEE Transactions on Power Delivery, VOL. 20, NO. 2, April 2005.

[17] S. Visacro, « A comprehensive approach to the grounding response to lightning currents», IEEE Trans. Power Delivery, vol. 22, pp.381-386, Jan. 2007.