A Novel Placement of Phasor Measurement Units Using Binary Bat Algorithm (BBA)

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Abstract: *The current state estimators need to estimate the power system by means of the state variables. Phasor measurement unit (PMU) measures the system state variables accurately by synchronizing the time of measurement. So, for controlling, monitoring and protecting the power system accurately, it is necessary to install PMUs. This paper suggests a multi-constrained optimal PMU placement using Binary Bat Algorithm (BBA) for system complete Observability line/PMU contingencies and channel limitations. Initially it considers system observability under normal operating conditions and then, contingencies like line/PMU outages and, channel limitations are added. The suggested scheme has been tested and then, it has been practiced for different practical power grids in India. A new index, System Observability Index (SOI), is also introduced to demonstrate the effectiveness of the proposed placement technique.*

Key words: *Binary Bat Algorithm (BBA), Channel limits, Line/PMU outage, Optimal PMU Placement (OPP), Power System Observability, System Observability Index (SOI).*

1. Introduction

The Phasor Measurement Unit (PMU) is capable of measuring voltage and current phasors which are synchronized. This has lead PMU in finding wide range of applications such as risk assessment during power system dynamics [1], islanding detection [2] and post-event analysis [3] after wide-area disturbances like Blackouts. Once the PMU optimization problem was introduced , many algorithms like later genetic algorithm, particle swarm optimization have been applied. Later, integer linear programming (ILP) in [4,5], participation factor-based method, binary search scheme, were suggested. Later, to make the wide-area monitoring system (WAMS) robust, paper [6] has been implemented considering line/PMU outages and/or channel limits. Recently, authors have proposed Artificial Bee Colony [7], multistage placement [8], and Binary Particle Swarm Optimization [9], Binary Integer Programming [10], Firefly Algorithm [11], Binary Cat Swarm Optimization [12], but they haven't considered Line/PMU contingencies. Later, a complete and incomplete observability based PMU placement [13] was presented. But, none of the above methods have considered all possible line/PMU outages.

Recently, Constrained based PMU placement strategy [14] considering line/PMU outages, System observability and bad data detection based PMU location [15], Multi-objective based PMU placement [16], Incremental PMU placement [17] considering critical contingencies, Controlled islanding based optimal PMU placement [18], a quadratic optimization problem [19] with non-linear observability constraints and a topological observability constraint based OPP [20] have been presented which have failed in considering all possible line/PMU outages along with channel limitations. The objective of the proposed OPP problem using Binary Bat Algorithm (BBA) is to find least number of PMUs for system complete observability by considering all possible (line/PMU) contingencies with only one contingency (either one line outage or one PMU failure) at a time, and channel limits. Also, to maximize the observability redundancy, we introduced a performance index called System Observability Index (SOI) with the help of n_p which represents the number of buses which would be observed for more than one time. The proposed method is tested on few of the IEEE test systems, and then applied to Southern Region of Indian power Grid(SRIG) of Indian Power Grid for finding PMU locations optimally.

2. Proposed OPP problem formulation

The PMU insalled at one bus not only observes the bus to which it is connected but also its connected buses. The objective of the proposed OPP problem is to find least number of PMUs for system complete observability. The optimization problem is formulated as below.

Minimize

$$\sum_{q\in N} x_q \tag{1}$$

Subjected to

$$s_p(X) \geq 1, \quad \forall p \in N \tag{2}$$

Where

$$s_p = \sum_{q\in N} c_{pq} x_q, \quad \forall p \in N \tag{3}$$

From equation (2), the Binary decision variable (x_q) is equal to 0 if q is not a PMU installed bus and 1otherwise, S_p is the p^{th} bus observability function and its valvue should be greater than or equal to 1 for all the buses incident to p^{th} bus to be observable. Here, N represents set of buses, c_{pq} is binary connectivity parameter and is described as,

$$c_{pq} = \begin{cases} 1, \text{if } p = q \\ 1, \text{if buses p, q are connected} \\ 0, \text{otherwise} \end{cases} \tag{4}$$

Equation (1) means the objective function for our OPP problem. This OPP problem will works if and only if there are no abnormal conditions like line outage (fault) or PMU failure (communication channel failure). So, in-order to consider these situations which have high probability to occur in power system, this paper formulates a new approach for observing the system completely even under the situations that are mentioned above.

2.1. Considering line outage/PMU failure

This part will some consider additional constraints like line/PMU outage, and are given below,

$$S_p^l + S_p \geq 2, \ \forall p \in N, \quad \forall l \in L \tag{5}$$

Where, S^l_p is system observability function for l^{th} line outage at bus p. It is given by

$$S_p^l = \sum_{q\in N} C_{pq}^l x_q \tag{6}$$

And, C^l_{pq} is a binary parameter when line l^{th} outage between buses p and q. This tells that both PMU outage and line outage will have the connectivity parameter. This will reduce the complexity of optimizing OPP problem under line outage.

2.2. Considering channel limitations

To communicate the measured data from PMUs to PDCs an exclusive communication channel. Hence this paper considers the effect of communication channel limitations just by replacing $\sum C_{pq}\, x_q$ with $\sum C_{pq} m_{pq} x_q$. So, the observability function will be given as

$$s_p = \sum_{q\in N} c_{pq}\, m_{pq}\, x_q \tag{7}$$

In addition with

$$s_p = \sum_{q\in N} c_{pq} m_{pq} \leq m_q^{max} \tag{8}$$

And

$$m_{pq} \leq x_q \quad \forall p, q \in N \tag{9}$$

The equation (8) is an additional constraint which will limit the number of communication channels. Here, each PMU has been limited to measure four measurements. And, the measurement at bus p with PMU at bus q is m_{pq}.

2.3. System Observability Index (SOI)

It is a parameter that tells the level of system observability. It can be defined as the ratio of the sum of number of buses observed for at least one time and the number of buses observed for more than one time to the total number buses in the system. It is given by,

SOI=

$$= 1 + \frac{\text{No. of buses observed for more than one time } (n_p)}{N} \tag{10}$$

Its value is typically greater than or equal to (≥) unity. If SOI is unity then the system is said to be completely observable with all the buses having zero measurement redundancy. Otherwise, if SOI is greater than unity, then the system is completely observable with the percent of measurement redundancy equal to an amount by which SOI exceeds unity. High is the SOI, maximum will be the system redundancy level. So, the solution corresponds to high SOI can be considered as the best solution to OPP problem. The above formula will holds good for normal operating conditions but under line/PMU outage conditions No. of buses observed for more than two times will be considered instead of No. of buses observed for more than one time (n_p).

2.3.1. Calculation of n_p

The observability of buses can be computede effectively using the equation given below.

$$[O]_{k\times 1} = A.x \tag{11}$$

Where, *A* represents the system connectivity matrix. *x* is column matrix which gives PMU locations. Means, its k^{th} entry will be 1 if PMU presents at bus *k*, otherwise zero. *k* gives the number of buses in the system. For the system to be observable completely, none of the element of matrix *O* should be zero. Similarly, consider,

$$[N_P]_{K\times 1} = [O]_{k\times 1} - u_{k\times 1} \tag{12}$$

where, *u* is unity matrix. And, the number of non-zero elements of *[Np]* gives the number of buses observed more than one time (n_p).

2.4. OPP formulation for maximizing observability

In order to achieve maximum redundancy, the objective function can be defined as follows,

Maximize,

$$u^T c_{pq} x_q \tag{13}$$

Subjected to,

$$\sum_{q=1}^{N} x_q = n_0 \tag{14}$$

$$c_{pq} x_q \geq u \tag{15}$$

Where, u is vector of length N and is given by $u=[1,1...1]^T$ for observability under normal operating conditions, and is $u=[2,2...2]^T$ for observability under line/PMU outage conditions. And n_0 is the optimal number of PMUs obtained from the main optimal PMU placement problem for system observability.

This optimal solution will be considered whenever there exist multiple solutions in actual OPP problem. From this solution, n_p will be calculated and, then SOI can be calculated as,

$$SOI = \sum_{p=1}^{N} O_p \tag{16}$$

Where, O_p is bus observability index and is given by,

$$O_p \leq n_p + 1 \tag{17}$$

The function (16) will be optimized until it produces optimal n_p that could produce maximum SOI. Maximum SOI provides measurement redundancy even when there would be some abnormalities like line/PMU failure. Once the main optimal PMU placement problem has been solved, the SOI function will be maximized subjecting it to main OPP constraints and additional constraint that the number of PMUs should be restricted to n_0.

3. Proposed Binary Bat Algorithm

3.1. Bat Algorithm

Bat algorithm is a heuristic algorithm, which is proposed recently. It was inspired by the bats with their echolocation behavior for finding the optimal point. In this algorithm, artificial bats are considered as particles (or agents). When the bats chase prey, they will try to reduce the loudness and increase the rate of ultrasonic sound emitted. These two characteristics were adopted in designing the Bat Algorithm (BA). In BA, each bat is provided with a position vector, velocity vector and a frequency vector. Each of these parameters is updated after each-iteration using the following equations.

$$v_i(t+1) = v_i(t) + (x_i(t) + g_{best}) f_i \tag{18}$$

$$x_i(t+1) = x_i(t) + x_i(t+1) \tag{19}$$

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{20}$$

Where f_i is the frequency of the i^{th} bat, which is updated in each-iteration. As per the literature, we have set $f_{min}=0$ and $f_{max}=2$. g_{best} is the best solution attained so far and β is the random number of a uniform distribution.

If we observe all the above equations it could be infer that artificial bats will have diverse tendency to the best solution even though they are with different frequencies. The random walk of each bat to perform the exploitation is as follows.

$$x_{new} = x_{old} + \gamma L^t \tag{21}$$

Here γ represents random number, and it is random value between 0 and 1. L represents the loudness of the sound emitted from bat.

The advantages of this method over earlier optimization techniques are it improves local minima avoidance and convergence speed [21]. These advantages are just because of the parameters such as pulse emission rate (r) and loudness (L), which are the controlled parameters used to balance the optimization with intensive local search. They are given by,

$$L_i(t+1) = \varepsilon L_i(t) \tag{22}$$

$$r_i(t+1) = r_i(0)[1 - \exp(-\alpha t)] \tag{23}$$

Where ϵ and α are the constants. Typically, ε should be in between (0, 1) and α should be greater than zero. Here, in the simulation, we have considered the range for α and ε in between 0.9 to 0.98.These L and r will be updated until the new solutions are converging towards the best solution.

3.2. Binary Bat Algorithm (BBA)

In BA updating positions can be easily done by adding current velocities to the previous positions using the equations given below. And in BA, the bats are moving around the continuous search space. But the present OPP problem is binary optimization problem whose solution is a set of zeros (0s) and ones (1s). Here binary one represents presence of PMU and binary zero represents absence of PMU. But the search space for the standard BA is continuous, means its solution is a set of real number. Hence, it cannot be applied for the problems like binary optimization problems (example, OPP problem) whose solution should be in terms of bits. So, a new version of BA algorithm, called Binary Bat Algorithm (BBA), is introduced here, which can handle binary optimization problems.

The population represents the bits equal to the number of buses in a particular system. If any of the bits is 1 then it means there would be a PMU at that bus, otherwise no PMU. In Binary Bat Algorithm (BBA), the agents will move in a discrete search space.This movement can only be done by flipping the various numbers of bits [22]. So, while designing BBA, as it uses only two bits (0's and 1's), the standard position and velocity updating processes have to be modified. It is just because of the fact that it is difficult to add a real valued velocity to a position vector in a binary search space. Here, this problem has been answered by changing the position of bat with respect to the probability of its velocity [23, 24]. This procedure needs a transfer function for mapping velocity with its probability to update the position. Also, the chosen transfer function has to obey the rules listed below.

- The transfer function range must be within the limits [0, 1].
- The transfer function should be enough sensitive so that it could provide all the probabilities of changing the position for all the ranges of absolute values of velocities.
- The return values of transfer function should vary proportionally for every change in the velocity.

The transfer function thus obtained from the above guidelines will makes sure that it can map accurately the search space in continuous domain to that of in discrete domain. The transfer function that we considered here is a v-shaped transfer function, and is as given below.

$$s\left(v_i^k(t)\right)=\left|\frac{2}{\pi}\arctan\left(\frac{\pi}{2}v_i^k(t)\right)\right| \tag{24}$$

$$x_i^k(t+1)=\begin{cases} x_i^k(t)^{-1}, \text{ rand} < s\left(v_i^k(t+1)\right) \\ x_i^k(t), \text{ rand} \geq s\left(v_i^k(t+1)\right) \end{cases} \tag{25}$$

Where, x_i^k and v_i^k indicate position and velocity of i^{th} particle respectively at iteration t in k^{th} dimension. Here equation (24) is used as a transfer function for mapping velocities to the probabilities of flipping their positions. Equation (25) is employed to update the position of the bat.

The algorithm of this Binary Bat technique is given below.

Algorithm:

step1: Initialize bat population, pulse emission rate (r) and loudness (l).

step2: Define the pulse frequency (f_i).

step3: while (t < max. number of iterations), go to step4, else go to stop.

step4: Modify the frequency and update the velocity.

step5: Calculate the transfer function using equation (1) and update the positions using equation (2).

step6: If (rand >r_i), go to step7, else go to step8.

step7: Select g_{best} from the best solution set randomly, and change some of the dimensions of position vector with some of the dimensions of the g_{best}. End if.

step8: Create new solution by flying arbitrarily.

step9: If (rand < l and f(x_i) <f (g_{best})) go to step 10, else go to step11.

step10: Adopt the new solutions. Increase r_i and $L_{i.}$ End if.

step11: Obtain the g_{best} by ranking the bats.

step12: Stop.

4. Results and Discussions

The results that have been obtained on applying the proposed BBA technique to standard IEEE test systems under normal and line/PMU outage conditions are given in Tables 1, 2, 4 and 5. The results are also compared with some of the recent methods applied for OPP problem, as shown in Table 3 and Table 6.

4.1. Regional Indian power Grids

As we know, Indian power Grid (IG) consists of five Inter Regional power Grids (IRGs), like, Northern Region of Indian power Grid (NRIG), Southern Region of Indian power Grid(SRIG), Eastern Region of Indian power Grid (ERIG), Western Region of Indian power Grid and North-Eastern Region of Indian power Grid (NERIG) [Power Grid Corporation Ltd., Govt. of India]. This paper studies the PMU placement for Southern Regional Indian power Grid.

Table 1: Complete Observability under normal conditions

system	Location of PMUs	No. of PMUs
IEEE 9	4,6,8	3
IEEE 14	2,6,8,9	4
IEEE 30	2,4,6,10,11,12,19,24,26,29	10
IEEE 57	1,4,9,20,24,27,29,30,32,36,38,39,41,45,46,51,54	17
IEEE118	2,5,9,12,13,17,21,23,26,29,34,37,42,45,49,53,56,62,64,71,75,77,80,85,86,90,94,101,105,110,115,116	32
IEEE 300	2,3,5,10,13,15,23,24,26,33,36,42,43,51,55,59,60,63,70,71,73,74,77,84,86,97,102,103,104,107,108,109,114,119,122,124,130,132,133,134,137,139,140,144,145,152,154,159,161,162,166,173,178,181,184,189,193,194,200,204,206,211,214,215,219,221,225,230,231,232,234,237,238,240,244,245,249,9012,9002,9022,9023,9003,9004,9005,9533,9007,531.	87

Table 2: Complete observability under normal operating conditions, considering channel limits.

System	Locations of PMUs	Number of PMUs
IEEE-9	1,6,8	3
IEEE-14	2,6,8,9	4
IEEE-30	1,2,6,9,10,12,19,23,26,27	10
IEEE-57	1,6,9,15,19,22,25,27,29,32,36,38,39,41,46,50,54	17
IEEE-118	3,5,10,12,15,17,21,23,25,29,34,37,42,45,49,53,56,62,64,68,71,75,77,80,85,87,90,92,96,100,105,110,115	33
IEEE-300	3,7,11,15,20,24,26,33,37,42,43,44,52,58,62,71,77,78,88,92,94,98,99,108,109,110,115,118,120,123,126,133,135,137,140,143,151,154,158,161,162,166,169,172,175,178,181,185,189,192,193,197,200,204,206,211,214,215,219,220,224,230,231,232,236,237,238,243,247,250,281,9012,9022,9024,9025,9026,9003,9004,9005,9053,9007,526,528,531,562,323,322.	87

Table 3: Results comparison of complete observability under normal conditions

Method	IEEE-9	IEEE-14	IEEE-30	IEEE-57	IEEE-118	IEEE-300
BBA	3	4	10	17	32	87
[4]	n/a	4	10	17	32	n/a
[7]	n/a	4	10	17	32	n/a
[8]	n/a	4	n/a	17	32	n/a
[9]	n/a	4	n/a	n/a	32	n/a
[10]	n/a	n/a	n/a	n/a	n/a	87

Table 4: Complete observability under Line/PMU failure, without considering channel limits

System	Locations of PMUs	Number of PMUs
IEEE-9	1,2,3,4,6,8	6
IEEE-14	1,2,3,6,7,8,9,11,13	9
IEEE-30	1,2,4,6,7,8,9,10,11,12,13,15,17,18,19,21,24,25,26,29,30	21
IEEE-57	1,2,4,6,9,11,12,15,19,20,22,24,25,26,28,29,30,32,33,35,36,38,39,41,45,46,47,50,51,53,54,56,57	33
IEEE-118	1,3,5,7,9,10,11,12,15,17,19,21,22,24,26,27,28,30,31,32,34,36,37,40,42,44,45,46,49,50,52,53,56,58,59,62,63,64,66,68,71,72,73,74,75,77,78,80,84,85,86,87,89,90,92,94,96,100,101,105,107,108,110,111,112,114,116,117,118	69
IEEE-300	2,3,5,7,11,12,13,14,15,16,17,20,23,24,26,33,35,36,39,41,42,43,44,48,51,52,54,58,59,60,62,63,69,70,71,72,74,77,78,81,84,85,86,87,89,90,94,98,100,103,104,105,108,109,110,113,114,116,117,118,119,121,122,124,126,130,132,133,135,136,137,138,140,142,143,145,147,153,155,156,157,160,162,164,166,167,168,170,171,172,174,175,177,178,182,183,184,185,187,188,191,194,197,198,200,201,205,206,208,209,210,211,213,214,215,217,219,221,222,223,225,226,227,229,230,231,232,233,234,236,237,238,239,240,241,244,245,246,249,250,281,9012,9002,9021,9022,9023,9024,9025,9026,9003,9031,9032,9033,9034,9035,9036,9037,9038,9004,9041,9042,9043,9005,9051,9052,9053,9054,9055,9533,9007,9071,9072,9121,319,320,526,528,7071,531,562,552,609,323,322,324.	185

Table 5: complete observability under Line/PMU failure, considering channel limits.

System	Locations of PMUs	Number of PMUs
IEEE-9	1,2,3,4,6,8	6
IEEE-14	2,4,5,6,7,8,9,10,13	9
IEEE-30	1,3,5,6,7,8,9,10,11,12,13,15,17,18,19,21,24,25,26,29,30	21
IEEE-57	1,3,4,6,9,11,12,15,19,20,22,24,26,28,29,30,31,32,33,35,36,38,39,41,45,46,47,50,51,53,54,56,57	33
IEEE-118	2,3,5,6,9,10,11,12,15,17,19,21,22,24,25,27,29,30,31,32,34,35,37,40,42,44,45,46,49,50,51,52,54,56,59,62,64,65,66,68,70,71,72,73,75,76,77,78,80,84,85,86,87,89,90,92,94,96,100,102,105,107,109,110,111,112,115,116,117	69
IEEE-300	2,3,5,7,11,12,13,14,15,16,17,20,23,24,26,33,35,36,39,41,42,43,4	185

4,48,51,52,54,58,59,60,62,63,69,70,71,72,74,77,78,81,84,85,86,87,89,90,94,98,100,103,104,105,108,109,110,113,114,116,117,118,119,121,122,124,126,130,132,133,135,136,137,138,140,142,143,145,147,153,155,156,157,160,162,164,166,167,168,170,171,172,174,175,177,178,182,183,184,185,187,188,191,194,197,198,200,201,205,206,208,209,210,211,213,214,215,217,219,221,222,223,225,226,227,229,230,231,232,233,234,236,237,238,239,240,241,244,245,246,249,250,281,9012,9002,9021,9022,9023,9024,9025,9026,9003,9031,9032,9033,9034,9035,9036,9037,9038,9004,9041,9042,9043,9005,9051,9052,9053,9054,9055,9533,9007,9071,9072,9121,319,320,526,528,7071,531,562,552,609,323,322,324.

Table 6: Results comparison of complete observability under Line/PMU failure conditions

method	IEEE-9	IEEE-14	IEEE-30	IEEE-57	IEEE-118	IEEE-300
BBA	6	9	21	33	68	185
[4]	n/a	n/a	n/a	n/a	n/a	n/a
[7]	n/a	n/a	n/a	n/a	n/a	n/a
[8]	n/a	n/a	n/a	n/a	n/a	n/a
[9]	n/a	n/a	n/a	n/a	n/a	n/a
[10]	n/a	n/a	n/a	n/a	n/a	n/a

Table 7: PMU locations of SRIG without considering channel limitations

System	PMU locations under normal operating conditions	PMU locations under line/PMU outage conditions
SRIG	5,7,8,10,12,19,22,27,28,30,33,42,48,49,54,58,60,63,67,73,75,76,88,89,92,95,99,102,105,107,110,111,115,117,118,121,122,127,128,132,136,138,145,147,152,154,157,159,161,165,181,184,188,190,191,196,201.	1,3,4,6,9,10,12,13,14,16,19,22,23,25,26,27,28,30,31,32,33,35,36,37,42,43,45,47,48,49,54,56,57,58,59,60,62,63,64,66,67,69,70,71,73,75,76,78,79,83,85,86,87,88,89,92,93,95,96,98,99,100,103,104,106,107,108,110,111,112,113,115,116,117,118,119,120,121,122,123,124,127,128,130,132,133,134,136,138,141,142,145,146,147,148,150,152,154,157,158,159,160,161,163,164,165,166,167,169,170,174,175,177,179,180,181,184,185,188,190,191,192,194,196,197,198,201,202,203,205,207,208.

Table 8: PMU locations of SRIG considering channel limitations

System	PMU locations under normal operating conditions	PMU locations under line/PMU outage conditions
SRIG	5,7,8,10,12,19,22,27,28,30,33,42,48,49,54,58,60,63,67,73,75,76,88,89,92,95,99,102,105,107,110,111,115,117,118,121,122,127,128,132,136,138,145,147,152,154,157,159,161,165,181,184,188,190,191,196,201.	1,5,6,7,9,10,12,13,14,16,19,22,23,25,27,28,30,31,32,33,35,36,37,40,41,42,43,44,45,48,52,53,54,56,58,60,61,63,64,66,67,69,73,75,76,78,79,83,84,85,86,88,89,92,93,95,96,98,99,100,102,104,105,106,107,108,110,111,112,113,115,116,117,118,119,120,121,122,123,124,127,128,130,132,133,134,136,137,138,141,142,145,146,147,148,150,152,154,156,157,158,159,160,161,163,165,166,167,169,170,171,174,175,176,179,180,181,183,184,187,188,190,191,192,194,196,197,198,201,202,203,205,207,208.

4. 2. Southern Regional Indian power Grid (SRIG)

As shown in Fig. 1, Southern Regional Indian power Grid (SRIG) consist 208 buses. This is the only IRG that was connected asynchronously to the remaining parts of the Indian power grid till December 31, 2012, after which it was synchronized to the rest of Indian power grid to form a single Indian grid. The results are given in Tables 7and 8.

4.3. System Observability Index Results

From the Table 9, it is clear that the proposed method yields the best solution for the OPP problem as it provides approximately 25 percent of measurement redundancy to the system.

Table 9: System observability indices

	Under normal conditions		**Under line/PMU outage conditions**	
System	**No.of buses observed more than one time(n_p)**	**SOI**	**No.of buses observed more than two times(n_p)**	**SOI**
IEEE-14	3	1.22	5	1.36
IEEE-30	9	1.3	11	1.37
IEEE-57	8	1.14	9	1.16
IEEE-118	25	1.22	37	1.31
SRIG	59	1.29	66	1.32

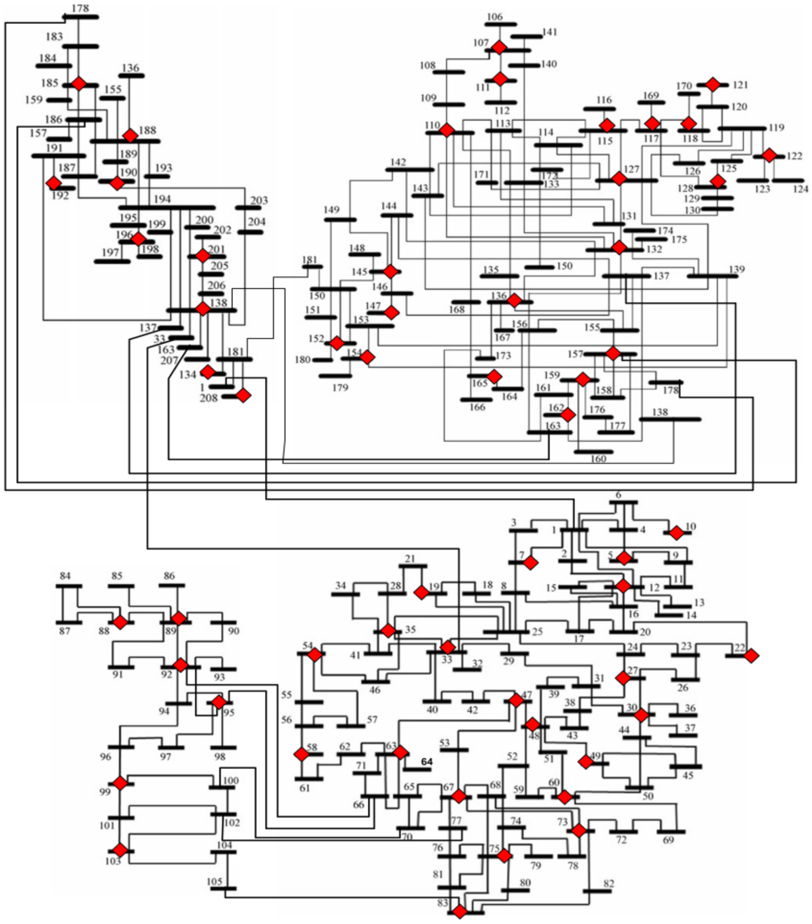

Fig 1: PMU locations for SRIG under normal operating conditions

5. Conclusion

The faults that would occur on the power system could cause line/PMU outage. This paper has considered even these conditions also for finding PMU placements. The proposed BBA method has solved the OPP problem under both normal and abnormal conditions in power system accurately. The proposed method has been tested on few IEEE standard test systems, and then applied for Southern Region of Indian power Grid (SRIG). From the results and discussions, it is verified that the proposed method has successfully optimized the OPP problem.

6. References

1. Jie Yan., Manimaran Govindarasu., Chenching., Umesh Vaidya., *Risk assessment framework for power control systems with PMU-based intrusion response system*, Journal of Modern Power Systems and Clean Energy, vol. 3, no. 3, pp: 321-331 (2015).

2 Pardeep, Anil Kumar, Naresh Kumar, *Islanding detection by phasor measurement unit in distribution network*, Journal of Electrical Engineering, vol. 17, no. 30(2), pp: 241–246 (2017).

3. Yongjie FANG., *Reflections on stability technology for reducing risk of system collapse due to cascading outages*, Journal of Modern Power Systems and Clean Energy, vol. 2, no. 3, pp: 264–271 (2014).

4. R. Kavasseri., and S. K. Srinivasan., *Joint placement of phasor and power flow measurements for observability of power systems*, IEEE Transactions on Power Systems, vol. 26, pp: 1929-1936 (2011).

5 Divyadeep verma," optimal repair and placement of phasor measurement units on power systems", Journal of Electrical Engineering.29(2), vol 13, 206-213, (2013).

6. C. Rakpenthai., S. Premrudeepreechacharn., S. Uatrongjit., N. R. Watson., *An optimal PMU placement method against measurement loss and branch outage*, IEEE Transactions on Power Del., vol. 22, no. 1, pp:101–107 (2007).

7. Aruljeyaraj kulanthaisamy., rajasekaran vairamani., nandha kumar karunamurthi., chandrasekaran koodalsamy., *Multi-objective PMU Placement method Considering Observability and measurement Redundancy using ABC Algorithm*, Advances in Electrical and Computer Engineering, vol. 14, no. 2, pp:117-128 (2014).
8. B.K. Saha Roy , A.K. Sinha, A.K. Pradhan, *An optimal PMU placement technique for power system observability*, Electrical Power and Energy Systems, vol. 42, pp: 71–77 (2012).
9. A. Ahmadi, Y. Alinejad-Beromi., M. Moradi., *Optimal PMU placement for power system observability using binary pa.rticle swarm optimization and considering measurement redundancy*, Expert System Applications, vol.38, pp: 7263-7269 (2011).
10. Anamitra Pal, Gerardo A. Sanchez-Ayala, Virgilio A. Centeno, James S. Thorp,, *A PMU Placement Scheme Ensuring Real-Time Monitoring of Critical Buses of the Network*, IEEE transactions on power delivery, vol. 29, no. 2, pp: 510-517 (2014).
11. K. Arul jeyaraj., V. Rajasekaran., S. K. Nandha kumar., K. Chandrasekaran., *A Multi-objective Placement of Phasor Measurement Units Considering Observability and Measurement Redundancy Using Firefly Algorithm*, Journal of Electrical Engineering and Technology, vol. 10, no. 2: 474-486 (2015).
12. Ankur Srivastava., Sydulu Maheswarapu., *Optimal PMU Placement for Complete Power System Observability using Binary Cat Swarm Optimization*, International Conference on Energy Economics and Environment, Noida, India, pp: 1-6 (2015).
13. Li Q., Cui T., Weng Y., *An information-theoretic approach to PMU placement in electric power systems*, IEEE Transactions on Smart Grid, vol. 4, no. 1, pp: 446–456 (2013).
14. Gou B., Kavasseri R.G., *Unified PMU placement for observability and bad data detection in state estimation*, IEEE Transactions on Power Systems, vol. 29, no. 6, pp: 2573–2580 (2014).
15. Mazhari S M., Monsef H., Lesani H., *A multiobjective PMU placement method considering measurement redundancy and observability value under contingencies*, IEEE Transactions on Power Systems, vol. 28, no. 3, pp: 2136–2146 (2013).
16. Wang Y., Li C W W., Li J., *Unified PMU placement for observability and bad data detection in state estimation*, IEEE Transactions on Power Systems, vol. 29, no. 6, pp: 2744–2752 (2014).
17. Huang L., Sun Y., Xu J., *Optimal PMU placement considering controlled islanding of power system*, IEEE Transactions on Power Systems, vol. 29, no. 2, pp: 742–755 (2014).
18. Esmaili R., Gharani K., Shayanfar H A., *Redundant observability PMU placement in the presence of flow measurements considering contingencies*, IEEE Transactions on Power Systems, vol. 28, no. 4, pp: 3795–3773 (2013).
19. Manousakis N M., Korres G N., *A weighted least squares algorithm for optimal PMU placement*, IEEE Transactions on Power Systems, vol. 28, no. 3, pp: 3499–3500 (2013).
20. Mazhari S E., Monsef H., Lesani H., *Comments on Minimizing the number of PMUs and their optimal placement in power systems in 83 (2012) 66–72*, Electrical Power Systems Research, vol. 114, pp: 146–148 (2014).
21. Yang X S., *A new metaheuristic bat-inspired algorithm*, In: Gonzalez JR et al (eds) nature inspired cooperative strategies for optimization (NICSO 2010), vol. 284, Springer, Berlin, pp: 65–74 (2010).
22. Kennedy J, Eberhart., *A discrete binary version of the particle swarm algorithm*, IEEE international conference on computational cybernetics and simulation, Orlando, USA, pp: 4104–4108 (1997).

23 Mirjalili S., Mohd Hashim., *BMOA: binary magnetic optimization algorithm*, 3rd international conference on machine learning and computing, Singapore, pp: 201–206 (2011).

24. Rashedi E., Nezamabadi pour H., Saryazdi S., *BGSA: binary gravitational search algorithm*, Natural Computing, vol. 9, no. 3, pp: 727–745 (2010).