

MONTE CARLO SIMULATION-BASED PLANNING METHOD FOR OVERLOAD SAFE ELECTRICAL POWER SYSTEMS

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Abstract: Operational reliability of power systems is one of the most important concerns that engineers have when planning a secure and economical electrical power system. This paper presents a probabilistic power flow analysis, based on the Monte Carlo simulation method, to support an overload safe power system designed to tolerate demand uncertainty and fluctuations of transmission parameters. Generation and transmission capacities in the power system can be estimated on the basis of operational risks and system installation costs. The proposed approach will be very useful for the rational planning of secure power distribution systems.

Key words: Probabilistic security analysis, Overload probability analysis, Risk-Cost efficient power system planning

1. Introduction

Operational reliability of power systems is of great importance in avoiding system failures, which may result in severe outages. The traditional deterministic security analysis methods have limitations in their use of system reliability assessments, because they do not consider stochastic factors affecting normal operation modes [1-3]. As a result, the unpredictable conditions of a complex power system – such as fluctuation of loads, the random failure of generator units, the random failure of transmission and distribution components, ageing of power components, indefinite external factors, and so on – can easily make the results of deterministic system security analysis inconsistent. That is why probabilistic power flow analyses that consider stochastic models and uncertainty in system parameters are more relevant for assessing the operational reliability of a system [1].

Power systems contain uncertainties in parameters of system components, as well as operating conditions. This causes power systems analysis tools to be, at least to some extent, imprecise when they rely on deterministic data. Power flow studies have a substantial role in the analysis and design of power systems [4]. To take into account uncertainties inherent to power systems, probabilistic techniques have been used since the early 1970s [5], where uncertainty in demand was first considered in a standard power flow problem.

Reliability of power distribution systems is an increasingly pressing concern due to the strong

dependence that modern societies and their economies have on power reliability. Many methods have been proposed to assess of power system reliability, including the fault tree analysis approach [6], the graph-theoretical (topological) network analysis method [7], and sequence operation theory-based approaches [8]. These methods aim to identify most critical system elements with respect to failures and attacks. In addition, a risk analysis of undesired states of power systems has been presented; a level-1 probabilistic risk assessment method was proposed to estimate blackout risks in power systems [9]. Optimization algorithms are also used, to improve the reliability of power systems. A genetic algorithm was used to search for the optimal transmission line assignment to the power transmission network so that network reliability is maximized [10]. For planning applications, an analytical probabilistic model for reliability evaluations of competitive electricity markets was proposed [11].

Probabilistic or stochastic load flow methods are commonly used to adjust and model the random nature of the operational load and generation data [4]. In [12], Pereira et al. proposed a methodology to process uncertainty in electrical power systems by using the interval non-linear system. Because of electric system data are uncertain; the proposed method can be considered an effective means in power flow analysis under uncertainty. In [13], author's purpose is to extend the probabilistic power flow to the three-phase field to take into account all the uncertainties in any unbalanced power system.

The Monte Carlo simulation method is a well-known technique, which yields consistent solutions for stochastic load flow problems. This method mainly utilizes repeated trials of a deterministic load flow technique to determine probability distributions of nodal powers, line flows, and losses. In the literature, the Monte Carlo simulation method was used in many engineering applications, owing to its straightforward applicability to complex phenomena, and its controllable accuracy in results. [14-16]. With recent developments in computing technologies and intelligent methods, Monte Carlo simulation can be performed with satisfying accuracy in a reasonable computation time [17-20]. In [21], Stefanopoulos et al. proposed a method, which is based on single phase power flow and non-

conforming load model, for probabilistic power flow. The method was certified via Monte Carlo simulations for each random sample in that study.

Several other methods have been proposed for deterministic load flow analysis: Gauss-Sidel, Newton-Raphson, Fast Decoupled Load Flow, Particle Swarm Optimization, Modified Particle Swarm Optimization [22]. We applied Gauss-Sidel method in the load flow analyses of Monte-Carlo simulation because of its simplicity and minimal computer storage requirements. However, it is slow in computation time [22]. Newton-Raphson method should be preferred for large systems [22].

Probabilistic steady-state security assessment of an electric power system is very useful, not only in the planning, but also in operation and control of electric power systems. In the literature, performance indices were defined to assess the impact of contingencies on power system security [23]. These indices, obtained via Monte Carlo simulations, were used to evaluate the influence of overloads, voltage limit violations, and voltage stability problems in a power distribution network.

This paper presents a probabilistic load flow analysis method for use in capacity planning of power systems, for a cost-effective, overload safe power system design. Authors employ an overload probability analysis method based on the Monte Carlo simulation. In this method, a deterministic power flow analysis scheme, based on a Gauss-Seidel numerical solution method, is used in the Monte Carlo simulation framework. After estimating capacity-exceeding probabilities of power system components, these probabilities are then used for an overall overload safety assessment of the power system. Afterward, on the bases of these safety assessments, a risk-cost analysis can be conducted for cost-efficient capacity planning of the power system components.

2. Method

2.1 Deterministic modeling and power flow analysis of power systems

Power flow analyses are important for power system planning and operation. Deterministic power flow analysis methods were effectively employed in the steady-state analysis of a complex power network modeled by the bus admittance matrix (Y_{bus}) [24]. In our study, the power flow equations written according to Y_{bus} were numerically solved by applying a Gauss-Seidel method [24]. The Gauss-Seidel solution of node voltages in an iterative scheme was given as,

$$V_i^{(k+1)} = \frac{1}{Y_{ii}} \left(\frac{P_i^{sch} - jQ_i^{sch}}{V_i^{*(k)}} - \sum_{j \neq i} Y_{ij} V_j^{(k)} \right) \quad (1)$$

Where $Y_{ij} = -y_{ij}$ and $Y_{ii} = \sum y_{ij}$. The parameter y_{ij} represents elements of Y_{bus} matrix. The parameters P_i^{sch} and Q_i^{sch} denote, respectively, scheduled active and reactive powers at the node i . The parameter k represents iteration steps. The iterative calculation process of voltages continues until the real and imaginary part of the node voltages in subsequent iteration are less than a specified error limit.

The active and reactive power of nodes are calculated by the following formulas,

$$P_i^{(k+1)} = \text{Re} \left\{ V_i^{(k)} [V_i^{*(k)} Y_{ii} + \sum_{j=1, j \neq i} Y_{ij} V_j^{(k)}] \right\} \quad j \neq i, \quad (2)$$

$$Q_i^{(k+1)} = \text{Im} \left\{ V_i^{(k)} [V_i^{*(k)} Y_{ii} + \sum_{j=1, j \neq i} Y_{ij} V_j^{(k)}] \right\} \quad j \neq i \quad (3)$$

After obtaining all node voltages (V_{bus}) of the power system, bus currents are calculated by $I_{bus} = Y_{bus} V_{bus}$.

2.2. Probabilistic overload safety analysis based on Monte Carlo simulation

The Monte Carlo simulation method provides a comprehensive tool for analysis of very complex stochastic factors. The Monte Carlo simulation method was adopted for probabilistic power flow analysis due to its advantage of straightforward principles, uncomplicated realization of complex problems in computer-aided analyses, insensitivity to the dimension of problems, and strong adaptability to any kind of scientific problem [1].

In the Monte Carlo simulation of power systems, the states of stochastic components are sampled randomly according to their stochastic models. A state of the power system is represented by a vector $X = (x_1, x_2, x_3, \dots, x_m)$ where x_i is the state of the i^{th} component. The set of all possible states, containing all combinations of component states, are denoted by S_x . Let $F_i(X)$ denote the overload test function of i^{th} component, which indicates whether or not the state of component i exceeds its upper bound of capacity. It can be analytically expressed as,

$$F_i(X) = \begin{cases} 1 & y_i > c_i, \\ 0 & y_i \leq c_i, \end{cases} \quad (4)$$

Where c_i denotes a safe operation capacity and y_i represents the output of a deterministic system model $f(X)$. The outputs y_i are obtained from the

Gauss-Seidel power flow solution of the power system modeled by a Y_{bus} matrix in this study. For a given N number of random samplings of X , the probability of component i being in an unsafe operation is expressed depending on an expected value of F_i as follows:

$$\tilde{p}_i = \frac{1}{N} \sum_{k=1}^N F_i(X). \quad (5)$$

According to the Kolmogorov Strong Law of Large Numbers, if F_i is a sequence of independent and identically distributed random variables and its numeral expectation exists [1]:

$$P\left(\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N F_i(X) = p_i\right) = 1. \quad (6)$$

Equation (6) tells us that the calculated probability value (\tilde{p}_i) converges to a real probability value (p_i) when $N \rightarrow \infty$ by the Monte Carlo simulation method. Here, convergence error (e) is written as [1],

$$e_i = \frac{\sigma}{\tilde{p}_i}, \quad (7)$$

Where σ is the standard deviation of F_i . The simulation is completed when the convergence error falls below a predefined error threshold (ε). The basic steps of the Monte Carlo simulation of power system are demonstrated in Figure 1 (a).

A severity function ($S_v(i)$) is used to specify the degree of abnormality in system functionality. In other words, $S_v(i)$ assigns a severity degree between zero and one for the overloading of system output y_i . The condition of $S_v(i) = 0$ implies that there is no negative effect of capacity exceeding y_i in the normal operation of the power system. The condition of $S_v(i) = 1$ indicates a maximum severity degree that implies a completely undesired system status. Overall capacity-exceeding risk factors of the system can be written as the sum of overload risks of each component as follows,

$$R = \sum_{\forall i} \tilde{p}_i S_v(i). \quad (8)$$

In practice, financial factors are very important during the planning phase of power systems. The capacity allocations of system components should be determined by considering both capacity risk and capacity cost, since a risk reduction causes an additional system installation cost. One can express capacity allocation of the system component i as follows,

$$c_i = E(y_i) + \beta \sigma(y_i), \quad (9)$$

Where operator $E(\cdot)$ and $\sigma(\cdot)$ represent the expected value and the standard deviation of y_i in the Monte Carlo simulation. The parameter β is a capacity enlargement factor and it is used for adjusting the capacity-exceeding risks of a component by increasing the capacity allocation, c_i .

Let us denote a cost function of capacity c_i for the component i by $\phi_i(c_i)$. The ϕ_i function may be a linear or nonlinear function of c_i depending on economical and geographical factors. In this manner, total installation cost of a system can be expressed as,

$$\phi_T = \sum_{\forall i} \phi_i(c_i). \quad (10)$$

For a rational secure system plan, impacts of capacity increments on the system cost (ϕ_T) and the overall capacity-exceeding risk factor R should be evaluated together and the cheapest risk should be found for a cost efficient power system risk plan. In this manner, a minimum of the risk-cost product ($R\phi_T$) in a given range of $(\beta_{\min}, \beta_{\max})$ should be found:

$$\min_{\beta \in (\beta_{\min}, \beta_{\max})} \{R\phi_T\}. \quad (11)$$

An algorithm for the overload-safe power system design, using the Monte Carlo simulation, is presented in Figure 1(b). In the following section, an example analysis is illustrated on a test system.

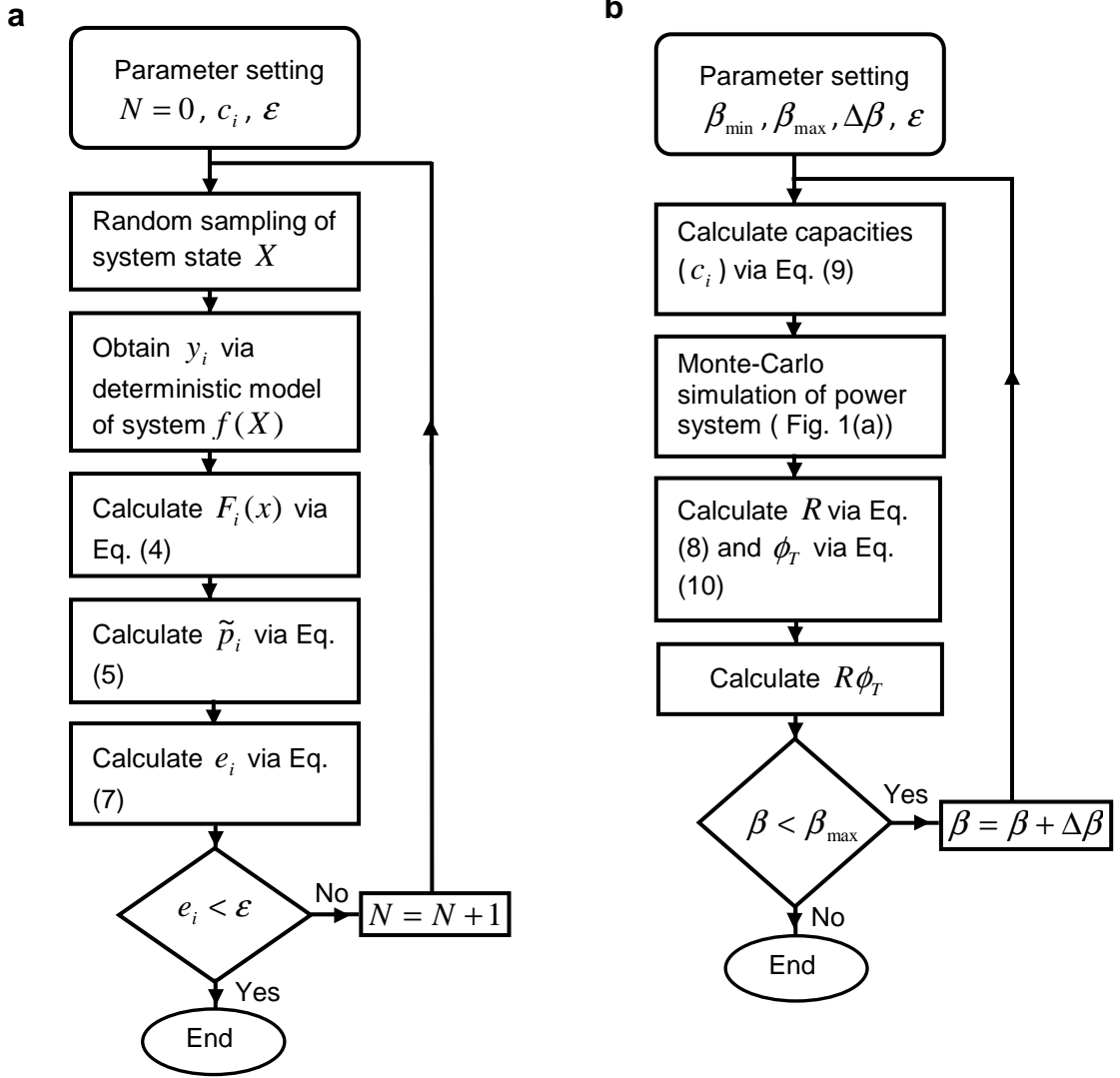


Figure 1. (a) Basic steps of the Monte Carlo simulation [1]. (b) An algorithm for Monte Carlo simulations for the overload safety analysis of power distribution systems

3. A power system planning example

Figure 2 shows the electrical schema of the test system used for the illustration of a safety plan against overloading of transmission and generation components. In this power system, active and reactive powers of Bus 2 ($P_{sch,2}, Q_{sch,2}$) and Bus 3 ($P_{sch,3}, Q_{sch,3}$), and line admittances (y_{12}, y_{13} and y_{23}), are stochastic parameters of the system. Active and reactive power requirement in Bus 1 and Bus 2 were assumed to have a deviation of 60% of its average value for active power and a deviation 40% of its average value for reactive power in order to model fluctuations in consumers demand. Line admittances were assumed to have a 10% deviation from the nominal value of line resistance, and a 15% deviation from the nominal value of line reactance, due to changes in possible external factors (temperature, humidity, dust, mechanical vibrations,

etc.), and structural factors (nonlinearity in power components, ageing). Table 1 lists stochastic components and their parameters. For the random samplings of power system parameters, we used uniformly distributed pseudo-random numbers generated by the Matlab program. Table 2 lists severity degrees (S_v), assigned for defining negative effects of capacity exceeding the power system's normal regime. Our objective in this example is to investigate overload risks for various planned generation capacities (P_{max}, Q_{max}) and transmission current capacities ($I_{12,max}, I_{13,max}, I_{23,max}$).

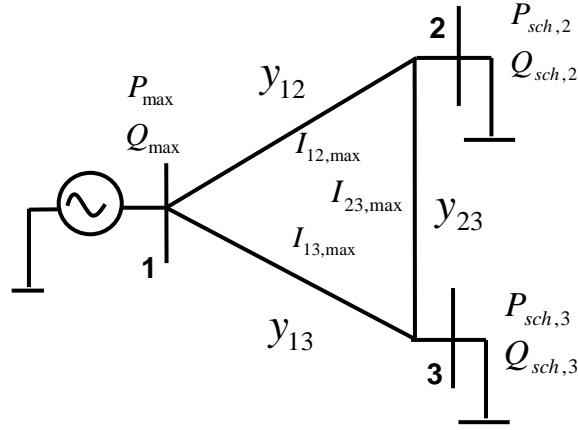


Figure 2. Example of power system in probabilistic overload safety analysis.

Table 1. List of stochastic parameters of the power system shown in Figure 1

	<i>Nominal Values (pu)</i>	<i>Deviation</i>
$P_{sch,2}, Q_{sch,2}$	-1,-1	60%, 40%
$P_{sch,3}, Q_{sch,3}$	-2,-2	60%, 40%
y_{12}, y_{13}, y_{23}	10+30j, 10+20j, 50+50j	10%+(15%)j

Table 2. List of severity degrees ($S_v(i)$) assigned for negative effects of exceeding capacity

<i>Overloading</i>	<i>Impacts on Power Distribution</i>	S_v
$P_1 > P_{max}$	Effects on whole power system due to halt of generator	1.0
$Q_1 > Q_{max}$	Effects on whole power system due to halt of generator	1.0
$I_{12} > I_{12,max}$	Directly affects power at Bus 2, however line 23 supports Bus 2.	0.5
$I_{13} > I_{13,max}$	Directly affects power at Bus 3, however line 23 supports Bus 3.	0.5
$I_{23} > I_{23,max}$	Does not seriously affect any bus, however it results in the loss of the backup line.	0.2

Figure 3(a) illustrates the convergence of active power capacity exceeding the probability of Bus 1 for $\beta = 1.0$. It converges to the probability of 0.19 with the convergence error (e) of 0.025 after 40,000 random samplings. Decreasing of convergence error with number of random samplings is illustrated in Figure 3(b). While convergence error sharply decreases around 3,000 random samplings, it asymptotically approximates to levels of 0.025 throughout 40,000 samplings. The 40,000 random samplings provided an adequate accuracy in the probabilistic estimations. Figure 3(c) illustrates a histogram of active power at Bus 1. In this figure, the scanned area on the left hand side of the P_{\max} line shows the number of tests exceeding maximum capacity allocated. This histogram reveals that roughly 8,000 out of 40,000 random tests resulted in the active power capacity overloading during the simulation for $P_{\max} = 3.94$.

For an illustration of safe capacity planning, the risk-cost (R, ϕ_r) analysis was conducted in (0,2) range of the capacity enlargement factor β with an increment of 0.1 via the Monte Carlo simulation method. Figure 4(a) reveals alterations in the overload probabilities (\tilde{p}) during the increment of β . Effects of increases in β on the parameters of R and ϕ_r are demonstrated in Figure 4(b) and (c), respectively. In addition, Table 3 lists some important system parameters calculated during the simulations. These parameters provide data that is useful in power system capacity planning. An affordable overload risk of power components is determined according to the risk-cost assessment of the system. According to this table, the best risk-cost product is obtained as $R\phi_r = 5.09$, when $\beta = 1.8$.

Figure 4(d) shows numbers of random samplings (N) in the Monte Carlo simulation to have a convergence error (e) lower than 0.05 ($\varepsilon < 0.05$). As the capacity-exceeding probability decreases, it requires more random samplings to retain this convergence error level. It is because of this that there is a need for more tests (random sampling) to approximate lower probability values in the Monte Carlo simulations.

In this power system design example, authors used a fractionated version of the WSCC 9-bus test system for illustrative proposes to avoid unnecessarily complicating analysis result for the benefit of readers. The method can be applied to more complex, large-scale systems, at the expense of larger computational complexity and therefore more computational time requirements.

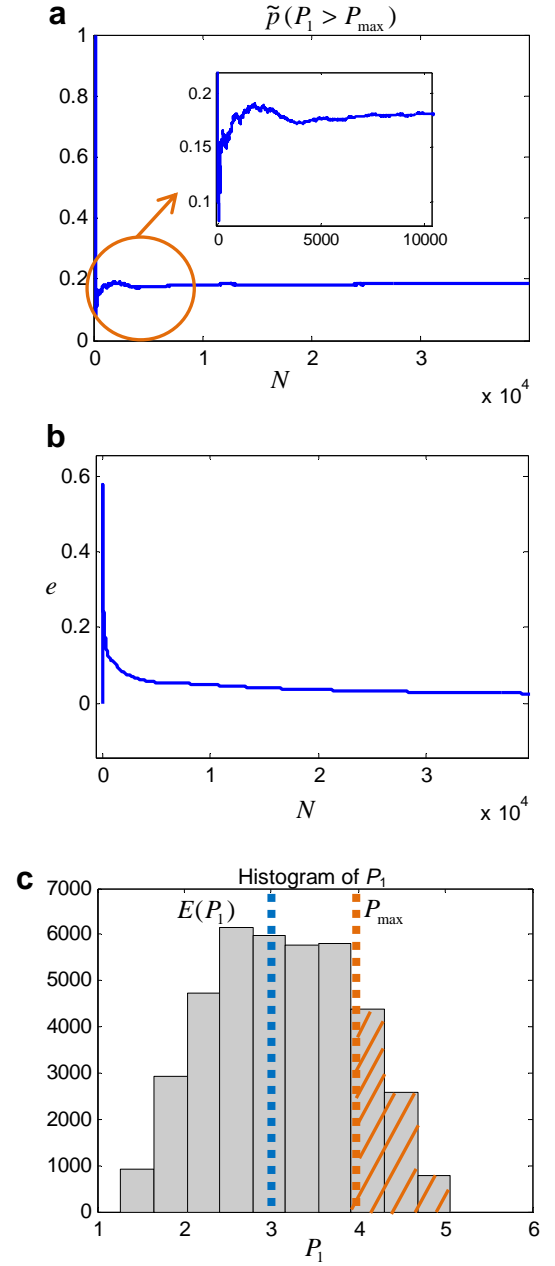


Figure 3. (a) Convergence of active power overload probability ($\tilde{p}(P_1 > P_{\max})$) of Bus 1 in Monte Carlo simulation ($\beta = 1.0, P_{\max} = 3.94$). (b) Convergence error (e) characteristics. (c) Histogram of Bus 1 active power ($E(P_1) = 3.13, \sigma(P_1) = 0.81$).

Table 3. List of allocated capacities ($P_{\max}, Q_{\max}, I_{12,\max}, I_{13,\max}, I_{23,\max}$) in per unit (pu), their calculated overload probabilities (\tilde{p}), overall overload risk factor (R), and installation cost (ϕ_T) for the various values of β

β	$P_{\max} \tilde{p}(P_1 > P_{\max})$	$Q_{\max} \tilde{p}(Q_1 > Q_{\max})$	$I_{12} \tilde{p}(I_{12} > I_{12,\max})$	$I_{13} \tilde{p}(I_{13} > I_{13,\max})$	$I_{23} \tilde{p}(I_{23} > I_{23,\max})$	R	ϕ_T	$R\phi_T$
0.3	3.38 / 0.44	2.83 / 0.44	2.41 / 0.42	1.98 / 0.44	1.04 / 0.44	1.41	29.56	41.85
0.6	3.62 / 0.33	2.96 / 0.32	2.50 / 0.30	2.07 / 0.31	1.14 / 0.32	1.04	31.20	32.53
0.9	3.86 / 0.24	3.10 / 0.23	2.61 / 0.22	2.16 / 0.22	1.23 / 0.23	0.75	32.93	24.84
1.2	4.12 / 0.16	3.25 / 0.15	2.72 / 0.15	2.26 / 0.15	1.33 / 0.15	0.50	34.77	17.52
1.5	4.36 / 0.09	3.39 / 0.08	2.82 / 0.08	2.35 / 0.08	1.42 / 0.08	0.28	36.5	10.27
1.8	4.60 / 0.04	3.53 / 0.03	2.93 / 0.04	2.44 / 0.04	1.52 / 0.04	0.13	38.21	5.09

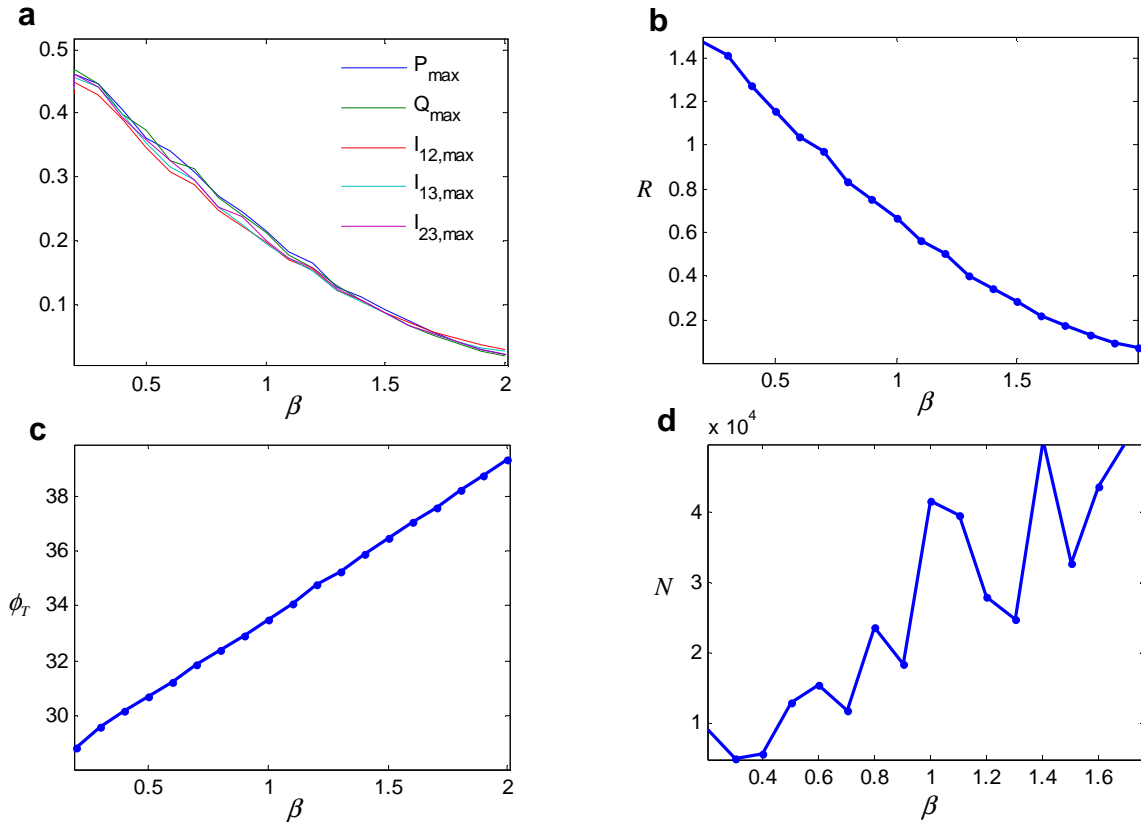


Figure 4. (a) Overload probabilities versus capacity enlargement factor plots for $\beta \in (0,2)$. (b) Overall capacity risk factor (R) versus capacity enlargement factor. (c) System cost (ϕ_T) versus capacity enlargement factor. (d) Number of random sampling (N) which have a convergence error (e) lower than 0.05 ($\varepsilon = 0.05$).

4. Conclusion

This paper presented a Monte Carlo simulation method for the overload security analysis of power systems in the case of uncertain power demand and fluctuation of line parameters. A risk-cost analysis was also conducted for a rational, secure power system plan. In an affordable and reliable power system design, risk-cost analyses on the bases of stochastic modeling of system parameters are more appropriate than deterministic methods of analysis, because deterministic models cannot provide a bird's eye view of a power system on the risk-cost plane. This disadvantage of deterministic security analysis methods may result in wasting security budgets, due to lack of risk-cost optimization of system parameters. As power systems get more complicated, deterministic planning of complex, large scale power systems become more vulnerable to unpredictable contingencies. Hence, probabilistic analyses based on stochastic models of system components are more reliable in risk analyses of large scale complex systems. Although the method was demonstrated on electrical power systems in this paper, the technique presented can also be adopted for other capacity-limited systems, such as mechanical structures with limited stress capacities, or channel systems with limited flow capacities.

As a methodology, it contributes to solution of risk-cost effective design problems of complex systems with uncertain parameters. The method offers a simple and reliable solution for power system planning problems.

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