

AN ELEGANT GAMMA BASED ALGORITHM FOR FIXED HEAD HYDROTHERMAL SCHEDULING

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Abstract

Hydrothermal scheduling plays an important role in maintaining a high degree of economy and reliability in power system operational planning. The traditional scheduling methods have become inadequate to handle large scheduling problems and tend to be inefficient due to complex computational process. Efficient strategies have thus become an imminent necessity to solve the complex on-line scheduling problems. This paper presents a simple gamma based scheduling algorithm for fixed head hydrothermal problems. It includes the simulation results of three test cases with a view to highlight its superior performance.

Key words: hydrothermal scheduling, $\lambda - \gamma$ iteration method.

Nomenclature

HTS hydrothermal scheduling
CLGM classical lambda gamma iterative method
EGA existing GA based HTS
HP hydel plant
TP thermal plant
NET normalised execution time
PM proposed method
 ng number of generating plants
 nt number of thermal plants
 nh number of hydro plants
 K_{\max} number of intervals
 a_i, b_i, c_i cost coefficients of i^{th} thermal plant
 d_i, e_i, f_i water discharge rate coefficients of i^{th} hydro plant
 t_k duration of interval- k

P_{Tik} generation at i^{th} thermal plant at interval- k
 P_{Hik} generation at i^{th} hydro plant at interval- k
 P_{Ti}^{\min} & P_{Ti}^{\max} minimum and maximum power limits of i^{th} thermal plant respectively
 P_{Hi}^{\min} & P_{Hi}^{\max} minimum and maximum power limits of i^{th} hydro plant respectively
 P_{Dk} total power demand at interval- k
 V_i^{avl} available water for i^{th} hydro plant over the scheduling period
 $F_{ik}(P_{Tik})$ fuel cost function of i^{th} thermal plant at interval- k
 $Y_{ik}(P_{Hik})$ water discharge rate of i^{th} hydro plant at interval- k
 Φ objective function to be minimized
 Φ_T augmented objective function to be minimized
 γ_i fictitious cost of water at i^{th} hydro plant
 λ_k incremental cost of received power at interval- k
 α and β constants computed using cost and water discharge rate coefficients of thermal and hydel plants respectively for a known values of γ

1.0 Introduction

The short range hydrothermal scheduling (HTS) is one of the most important and challenging optimisation problems in economic operation and control of interconnected power systems. The problem, due to insignificant operational cost of hydroelectric plants, is to effectively use the available water to maximise generation from hydro plants and reduce the cost of thermal generation. The HTS problem is therefore to optimally distribute generations among the thermal and hydel plants in such a way to minimise the total thermal production cost for a single day or a week, usually on an hourly basis, satisfying hydraulic and electrical operational constraints such as available water, generation limits, generation-demand balance, etc. The HTS boils down to a typical non-linear optimisation problem involving a non-linear objective function and a combination of linear and non-linear constraints [1]. The power generation in a short range HTS problem, is solely dependent on the water discharge on account of a fixed water head and the net head variation being ignored for relatively large reservoirs [2-6].

The short range HTS problem has been the subject of intensive research work during the past few decades. Several researchers have suggested many methods such as dynamic programming [7], network flow programming [8], mixed integer programming [9] and Lagrangian relaxation [10] to solve this difficult optimisation problem. Dynamic programming among these approaches has been found to tackle the complex constraints directly but suffers from the curse of dimensionality. The other methods have necessitated simplifications in order to easily solve the original model, which may lead to sub-optimal solutions with a great loss of revenue.

In recent years, heuristic optimisation techniques have aroused intense interest due to their flexibility, versatility and robustness in seeking global optimal solution. These evolutionary approaches such as genetic algorithms [11-16], simulated annealing [6], evolutionary strategy [17-19], particle swarm optimisation [20-21] and peak shaving [22] involve a large number of problem variables,

which not only depend on the number of generating plants but also the number of intervals considered in the planning horizon and thus are highly inefficient.

A simple and elegant algorithm for solving fixed-head short range HTS problem is developed in this paper. This method eliminates the λ -iterations through an analytical approach in the λ - γ iteration method to enhance the solution speed. The proposed algorithm is tested on three HTS problems and the results are presented.

2.0 Problem Formulation

The main objective of HTS problem is to determine the optimal schedule of both hydro and thermal plants of a power system in order to minimise the total system operating cost, represented by the fuel cost required for the system's thermal generation. It is intended to meet the forecasted load demand over the scheduling period, while satisfying various system and unit constraints. The HTS problem without accounting the losses is formulated as

$$\text{Minimise } \Phi = \sum_{k=1}^{K_{\max}} \sum_{i=1}^{nt} t_k \cdot F_{ik}(P_{Tik}) \quad (1)$$

subject to the power balance constraint

$$\sum_{i=1}^{nt} P_{Tik} + \sum_{j=1}^{nh} P_{Hjk} - P_{Dk} = 0 \quad k = 1, 2, \dots, K_{\max} \quad (2)$$

and to the water availability constraint

$$\sum_{k=1}^{K_{\max}} t_k \cdot Y_{ik}(P_{Hik}) = V_i^{avl} \quad ; \quad i = 1, 2, \dots, nh \quad (3)$$

with

$$\begin{aligned} P_{Ti}^{\min} &\leq P_{Tik} \leq P_{Ti}^{\max} \quad ; \quad i = 1, 2, \dots, nt \\ P_{Hi}^{\min} &\leq P_{Hik} \leq P_{Hi}^{\max} \quad ; \quad i = 1, 2, \dots, nh \end{aligned} \quad (4)$$

where

$$F_{ik}(P_{Tik}) = a_i \cdot P_{Tik}^2 + b_i \cdot P_{Tik} + c_i \quad \$/\text{h} \quad (5)$$

$$Y_{ik}(P_{Hik}) = d_i \cdot P_{Hik}^2 + e_i \cdot P_{Hik} + f_i \quad \text{m}^3/\text{h} \quad (6)$$

2.1 Classical $\lambda - \gamma$ iteration method [1]

The augmented lagrangian function for the HTS problem is written as

$$\Phi_T = \sum_{k=1}^{k \max} \left[\sum_{i=1}^{nt} t_k F_{ik}(P_{Tik}) - \lambda_k \left(\sum_{i=1}^{nt} P_{Tik} + \sum_{j=1}^{nh} P_{Hjk} - P_{Dk} \right) \right] + \sum_{i=1}^{nh} \gamma_i \left[\sum_{k=1}^{K \max} t_k Y_{ik}(P_{Hik}) - V_i^{avl} \right] \quad (7)$$

The co-ordination equation from the above function can be obtained as

$$t_k \frac{\partial F_{ik}(P_{Tik})}{\partial P_{Tik}} = \lambda_k \quad (8)$$

$$\gamma_i t_k \frac{\partial Y_{ik}(P_{Hik})}{\partial P_{Hik}} = \lambda_k \quad (9)$$

The above co-ordination equations along with constraint Eqs. 2, 3 and 4 is iteratively solved to obtain optimal HTS.

3.0 Proposed Methodology

The solution process of the $\lambda - \gamma$ iteration method involves time consuming three iterative loops, in which the λ -iterations itself accounts for two iterative loops in each γ -iteration. The solution speed can be enhanced, if λ -iterations is eliminated, thereby avoiding two iterative loops. An analytical non-iterative approach is developed instead of λ -iterations in the proposed approach. The co-ordination equations for a system with two thermal and a hydro units are written from Eqs. 8 and 9 as:

$$t_k \frac{dF_{1k}(P_{T1k})}{dP_{T1k}} = t_k (2 a_1 P_{T1k} + b_1) = \lambda_k \quad (10)$$

$$t_k \frac{dF_{2k}(P_{T2k})}{dP_{T2k}} = t_k (2 a_2 P_{T2k} + b_2) = \lambda_k \quad (11)$$

$$t_k \gamma_1 \frac{dY_{1k}(P_{H1k})}{dP_{H1k}} = t_k (2 \gamma_1 d_1 P_{H1k} + \gamma_1 e_1) = \lambda_k \quad (12)$$

Equating Eqs. 10, 11 and 12,

$$P_{T2k} = \frac{a_1}{a_2} P_{T1k} + \frac{b_1 - b_2}{2 \cdot a_2} = \alpha_{T2} P_{T1k} + \beta_{T2} \quad (13)$$

$$P_{H1k} = \frac{a_1}{\gamma_1 d_1} P_{T1k} + \frac{b_1 - \gamma_1 e_1}{2 \gamma_1 d_1} = \alpha_{H1} P_{T1k} + \beta_{H1} \quad (14)$$

Substituting Eqs. 13 and 14 in power balance Eq. (2),

$$P_{Dk} = (1 + \alpha_{T2} + \alpha_{H1}) P_{T1k} + (\beta_{T2} + \beta_{H1})$$

or

$$P_{T1k} = \frac{P_{Dk} - (\beta_{T2} + \beta_{H1})}{(1 + \alpha_{T2} + \alpha_{H1})} \quad (15)$$

Eqs. 15, 13 and 12 for a general system having nt -thermal and nh -hydro plants can be written as

$$P_{T1k} = \frac{P_{Dk} - (\beta_{T2} + \beta_{T3} + \dots + \beta_{Tns} + \beta_{H1} + \beta_{H2} + \dots + \beta_{Hnh})}{(1 + \alpha_{T2} + \alpha_{T3} + \dots + \alpha_{Tns} + \alpha_{H1} + \alpha_{H2} + \dots + \alpha_{Hnh})} = \frac{P_{Dk} - \beta}{(1 + \alpha)} \quad (16)$$

$$P_{Tjk} = \alpha_{Tj} P_{T1k} + \beta_{Tj} \quad ; \quad j = 2, 3, \dots, nt \quad (17)$$

$$P_{Hjk} = \alpha_{Hj} P_{T1k} + \beta_{Hj} \quad ; \quad j = 1, 2, \dots, nh \quad (18)$$

where

$$\alpha_{Tj} = \frac{a_1}{a_j} \quad \alpha_{Hj} = \frac{a_1}{\gamma_j d_j}$$

$$\beta_{Tj} = \frac{b_1 - b_j}{2a_j} \quad \beta_{Hj} = \frac{b_1 - \gamma_j e_j}{2\gamma_j d_j}$$

$$j = 1, 2, \dots, nh \quad (19)$$

Eq. (16) can be solved for P_{T1k} for a given power demand with a suitable initial value of γ for the hydel plants. Eqs. 17 and 18 are to be solved for the remaining thermal and hydro power generations respectively taking into account the generator power limits. If there is any limit violation of generation, the respective limit value is assigned as the generation for the violated plant; and the limit violated plant is omitted by treating the respective α and β values zero and reducing the net power demand by the corresponding limit value in the process of computation for the remaining units. These computations are non-iterative in nature and are performed in the γ -iterative loop till the available volume of water is fully utilised in the planning horizon. This procedure avoids the λ -iterative loop in finding the optimal schedule and helps in reducing the overall computation time. A flow chart for solving the hydrothermal scheduling problem by this method is presented in Fig. 1.

4.0 Simulation Results

The proposed method (PM) is tested on three HTS examples, which are taken from Ref. [4]. The results of the PM are compared with that of classical λ - γ iteration method and EGA method in order to bring out the supremacy of the developed algorithm.

Problem-1

This problem under study consists of one thermal and one hydel plant. The system characteristics are

$$F_{1k}(P_{T1k}) = 0.001991 P_{T1k}^2 + 9.606 P_{T1k} + 373.7 \quad \$/h$$

$$Y_{1k}(P_{H1k}) = 0.0007749 P_{H1k}^2 - 0.009079 P_{H1k} + 61.53$$

M cubic ft / hour

The allowable volume of water for the dispatch period of one day is

$$V_1^{avl} = 2559.6 \quad M \text{ cubic ft.}$$

The hourly power demand over the scheduling for a day is given in Table 1.

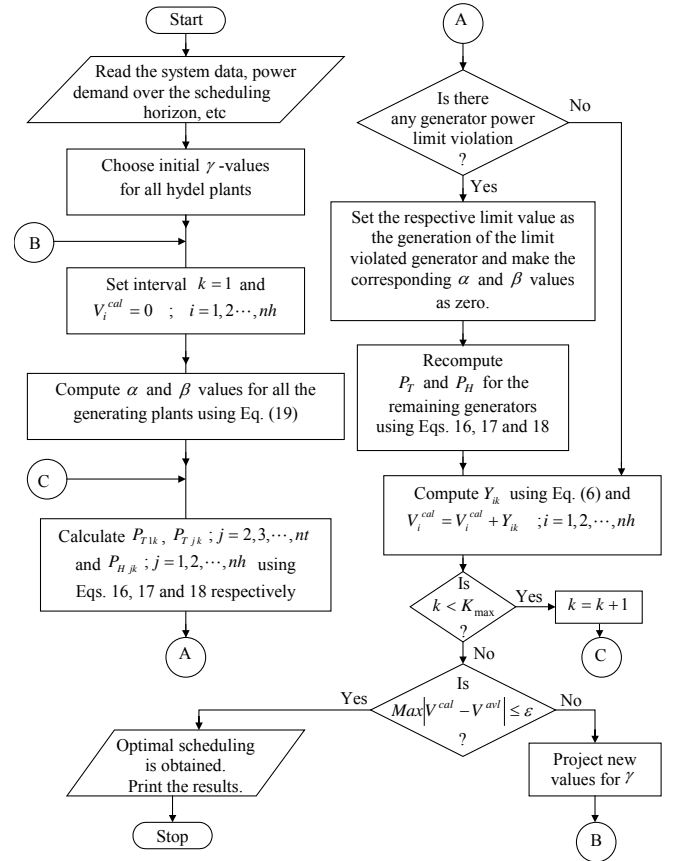


Fig. 1 Flow chart for proposed γ iterative method

Table 1
Power demand at different intervals of Problem-1

k	P _{Dk}	k	P _{Dk}	k	P _{Dk}
1	455	9	665	17	721
2	425	10	675	18	740
3	415	11	695	19	700
4	407	12	705	20	678
5	400	13	580	21	630
6	420	14	605	22	585
7	487	15	616	23	540
8	604	16	653	24	503

* Duration of each interval $t_k = 1$ hour

The optimal thermal and hydel generations of each plant at each interval over the scheduling horizon for this problem, obtained by the PM, is shown in Fig. 2.

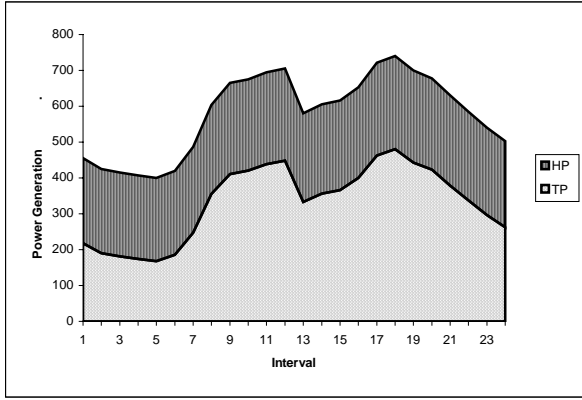


Fig. 2 Optimal thermal and hydel generations over the scheduling period for Problem-1

Problem-2

The second system under study is made up of one thermal and two hydel plants. The cost characteristics of thermal plants, the discharge characteristics of hydel plants and water available for each hydel plant of this problem are given below. The power demand at each time interval over the scheduling period is given in Table 2 for this problem.

$$F_{1k}(P_{T1k}) = 0.01 P_{T1k}^2 + 3.0 P_{T1k} + 15.0 \quad \$ / h$$

$$Y_{1k}(P_{H2k}) = 0.000105 P_{H1k}^2 + 0.03 P_{H1k} + 0.2 \quad M \text{ cubic ft / hour}$$

$$Y_{2k}(P_{H2k}) = 0.0001 P_{H2k}^2 + 0.06 P_{H2k} + 0.4 \quad M \text{ cubic ft / hour}$$

$$V_1^{avl} = 25.0 \quad M \text{ cubic ft}$$

$$V_2^{avl} = 35.0 \quad M \text{ cubic ft}$$

Table 2

Power demand at different intervals of Problem-2

k	P _{Dk}	k	P _{Dk}	k	P _{Dk}
1	30	9	61	17	71
2	33	10	58	18	62
3	35	11	56	19	55
4	38	12	57	20	50
5	40	13	60	21	43
6	45	14	61	22	33
7	50	15	65	23	31
8	59	16	68	24	30

* Duration of each interval $t_k = 1$ hour

The optimal thermal and hydel generations of each plant at each interval over the scheduling horizon for this problem, obtained by the PM, is shown in Fig. 3.

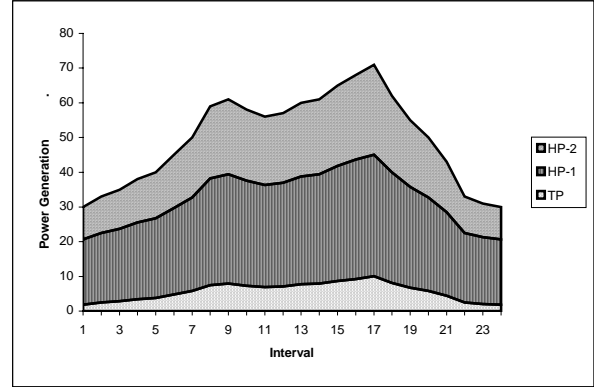


Fig. 3 Optimal thermal and hydel generations over the scheduling period for Problem-2

Problem-3

The third problem has two thermal and two hydel plants. The system characteristics and available water for each hydel plant are as follows:

$$F_{1k}(P_{T1k}) = 0.0025 P_{T1k}^2 + 3.2 P_{T1k} + 25.0 \quad \$ / h$$

$$F_{2k}(P_{T2k}) = 0.0008 P_{T2k}^2 + 3.4 P_{T2k} + 30.0 \quad \$ / h$$

$$Y_{1k}(P_{H1k}) = 0.000216 P_{H1k}^2 + 0.306 P_{H1k} + 1.98 \quad M \text{ cubic ft / hour}$$

$$Y_{2k}(P_{H2k}) = 0.000136 P_{H2k}^2 + 0.612 P_{H2k} + 0.936 \quad M \text{ cubic ft / hour}$$

$$V_1^{avl} = 2500.0 \quad M \text{ cubic ft}$$

$$V_2^{avl} = 2100 \quad M \text{ cubic ft}$$

Table 3

Power demand at different intervals of Problem-3

k	P _{Dk}	k	P _{Dk}	k	P _{Dk}
1	400	9	1230	17	1350
2	300	10	1250	18	1470
3	250	11	1350	19	1330
4	250	12	1400	20	1250
5	250	13	1200	21	1170
6	300	14	1250	22	1050
7	450	15	1250	23	900
8	900	16	1270	24	600

* Duration of each interval $t_k = 1$ hour

Table 4 Comparison of results with existing methods

Problem	γ values			NET (seconds)			Fuel Cost (\$/day)		
	PM	$\lambda - \gamma$	EGA	PM	$\lambda - \gamma$	EGA	PM	$\lambda - \gamma$	EGA
1	29.236	29.236	---	1.2	12.43	80.46	91344.573	91344.573	91348.642
2	95.275 49.102	95.275 49.102	---	0.609	9.27	72.03	780.084	780.084	780.023
3	9.398 5.673	9.398 5.673	---	0.625	10.16	68.87	49118.171	49118.171	49119.241

The hourly power demand over the scheduling period is given in Table-3. The optimal scheduling obtained by the PM is graphically represented in Fig. 4.

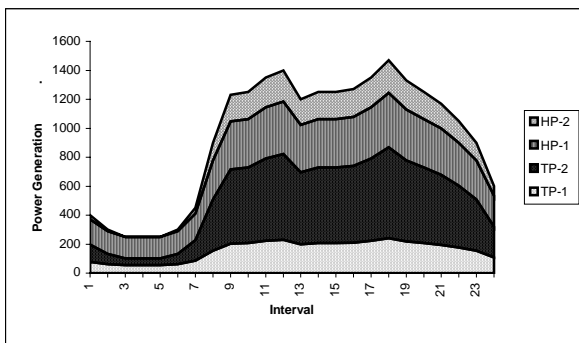


Fig. 4 Optimal thermal and hydel generations over the scheduling period for Problem-3

In addition, in order to validate the results, Table 4 compares γ -values, the NET and fuel cost obtained by the PM with that of the $\lambda - \gamma$ iteration method and EGA for the three problems. The γ -values of the PM and $\lambda - \gamma$ iteration method are the same, which infers that the developed non-iterative steps to replace λ -iterative loop of $\lambda - \gamma$ iteration method does not affect the solution process. Besides, these steps being non-iterative in nature serve to enhance the computational efficiency. The NET values given in the same table indicate that the PM is more than ten times faster than the $\lambda - \gamma$ iteration method and around hundred times faster than EGA, thus rendering the PM suitable for on-line applications. The accuracy of the solution is evident from the fact that the fuel cost obtained by the PM is same as that of the

$\lambda - \gamma$ iteration method and nearer to that obtained by EGA.

5.0 Conclusion

An elegant gamma algorithm for solving HTS problem by reducing the number of iterative loops in the classical $\lambda - \gamma$ iteration has been developed.

The non-iterative procedure to replace λ -iterative loop is very simple and has served to reduce the computational burden without affecting the accuracy of the solution. The ability of the developed algorithm to converge at an optimal solution with smaller execution time will find its role as a powerful on-line tool in energy management systems.

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