

Effect of Dead-time Approximation on Controller Performance/Robustness Designed for a First Order plus Dead-time Model

Arun R. Pathiran

Dept. of Electronics and Instrumentation Engineering, National Engineering College
Kovilpatti-628503, India. (arun.pathiran@gmail.com)

Abstract: PID controller is considered as the most generally used feedback controller design in process control industries with the longest history and most vital development. It is well-known that the performance of a PID controller is inadequate for processes with long dead-time. Smith Predictor is a kind of predictive control scheme that is effective for processes with long dead-time. PID controllers can be derived from predictive control schemes by approximating the dead-time for FOPDT process models. In this work, the predictive control scheme and dead-time approximated PID controller have been designed with various values of desirable closed loop time constants. A comparative study has been performed for closed loop performance/robustness for the actual and the dead time approximated controllers.

Key words: PID, Smith Predictor, Predictive PI.

1. Introduction

Feedback controller is designed to take the process response from one operating point to another operating point in a desired manner and also the controller is responsible for maintaining the process in the necessary operating point during the process operation. Plethora of feedback control schemes are proposed in the literature. Among the schemes PI/PID controller is considered as the most generally used feedback controller in process control industries with the longest history and most vital development [3]. The fixed structure PI/PID controller parameters are designed or tuned to meet certain performance/robustness measures [4]. Several methods for the selection of PI/PID controller parameters have been reported and huge number of them available in the literature [5-11].

The well known IMC scheme can be viewed as one class of direct synthesis feedback controller, and as pointed out by [12] the DS and IMC design methods can generate equivalent controller transfer function and closed loop performance, under certain circumstances. The IMC and DS method of feedback controller has also been subjugated to tune conventional PI/PID controllers in feedback configurations and several methods are available in the literature [13-17]. It may be noted that the DS and IMC methods do not necessarily result in PI/PID controllers. However, by choosing the appropriate desired closed-loop model and using either pade approximation or a power-series approximation for the dead time in the process model, PI/PID controllers have been derived for certain type of process models (for

example for First order plus Dead-time (FOPDT), and Second order plus dead-time (SOPDT)) that are commonly used in industrial applications. The time delay approximation invites some limitations on the performance and applicability of fixed structure PI/PID controllers [18 - 20].

Many technical difficulties arise in dealing with time-delay systems in continuous time originate from the infinite dimensionality of the delay element. Attractive alternatives in this respect are offered by controllers involving infinite-dimensional dead-time compensators. Smith proposed the first Dead-Time Compensation (DTC) scheme [21] to improve the performance of PI/PID controllers for processes with dead time. This scheme, which became known as the Smith predictor (SP), contained a dynamic model of the dead-time process for the prediction [22]. A SP using a first order plus dead-time (FOPDT) model combined with PI controller requires five parameters to be determined by the operator namely three FOPDT model parameters and two PI controller parameters. Predictive PI (PPI) controller for a FOPDT process model has been proposed in [23] which had all the features of the Smith predictor with only three adjustable parameters. Numerous DTC schemes are proposed in the literature on the basis of different kinds of processes and closed loop objectives.

Since the PID controllers are most widely used, the simplified smith predictor called predictive PI controller is realized into a PID controller by approximating the delay element using pade approximations. Natural questions arising in this respect how a delay element can be approximated by a finite-dimensional one to which standard analysis and design methods can be useful. The rational function approximation of the time-delay element is generally inaccurate. The design of finite-dimensional controllers for (infinite-dimensional) dead-time systems is typically rather conservative.

In this work, the performance and robustness of predictive and dead-time approximated PID controller is analyzed for process control systems. In section 2 predictive PI and PID schemes are discussed. Comparison of the robustness and performance of control schemes is presented in section 3 followed by concluding remarks in section 4.

2. Smith Predictor

The conventional PI/PID controller is inadequate for

processes with long dead-time. Smith proposed a control scheme with PI/PID controller which eliminates the process dead-time from the characteristic equation of the closed loop system. Smith's modification improves performance of PI/PID controller for processes with dead-times. The smith predictor scheme is shown in Figure 1 where $G_{m0}(s)$ is the dead-time free part of model $G_m(s)$. The control signal is passed to two models with and without dead-time. These models used for prediction are often first order approximations of the process.

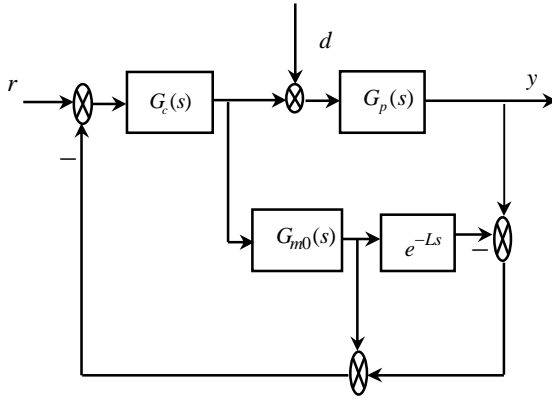


Fig. 1: Smith Predictor

The closed loop transfer function between the setpoint r and the output y is as follows:

$$G_{cl}(s) = \frac{G_c(s)G_m(s)}{1 + G_c(s)(G_{m0}(s) - G_m(s) + G_p(s))} \quad (1)$$

In the case of perfect modeling i.e. $G_p(s) = G_m(s)$ the closed loop transfer function becomes

$$G_{cl}(s) = \frac{G_c(s)G_m(s)}{1 + G_c(s)G_{m0}(s)} \quad (2)$$

Now the characteristic equation is free from delay element and then the primary controller can be tuned for desired performance.

The disturbance response transfer function is as follows:

$$G_d(s) = \frac{y(s)}{d(s)} = G_m(s) \left(1 - \frac{G_c(s)G_m(s)}{1 + G_c(s)G_{m0}(s)} \right) \quad (3)$$

The conventional PI controller has only two tuning parameter where as the Smith scheme has three model parameters additionally. Predictive PI (PPI) controller for a FOPDT process model has been proposed in [23] which had all the features of the Smith predictor with only three adjustable parameters. The PPI scheme combines the prediction of the Smith predictor and the easy tuning of a conventional PI controller. The PPI structure is as similar as the Smith predictor with a PI controller. The uniqueness is the way the parameters

are chosen. The controller gain, ' k_c ' is set to $1/k_m$ and the integral time, ' τ_i ', to τ_m . This leaves only three FOPDT model parameters, ' k_m, τ_m ' and ' L ' to be determined. If one has an estimate of the dead time ' L ', the other two parameters can be tuned as in conventional PI controller.

The PPI scheme was developed for process with long dead-time ' L ' and its design procedure is presented as follows:

The transfer function of the closed loop with error feedback is

$$G_{cl}(s) = \frac{G_c(s)G_m(s)}{1 + G_c(s)G_m(s)} \quad (4)$$

Solving the above equation for controller

$$G_c(s) = \frac{G_m(s)^{-1}G_{cl}(s)}{(1 - G_{cl}(s))} \quad (5)$$

Many industrial processes can be represented by a First Order Plus Dead-time (FOPDT) process models. The main reason that one uses a first order model is that it is pretty easy to find the parameters that approximate such a system. The first order plus dead-time transfer function is given by

$$G_m(s) = \frac{k_m}{\tau_m s + 1} e^{-Ls} \quad (6)$$

where ' k_m, τ_m ' and ' L ' are process gain, time constant and dead-time respectively.

Assume the desired closed loop transfer function is specified as

$$G_{cl}(s) = \frac{1}{\lambda s + 1} e^{-Ls} \quad (7)$$

where the closed loop time constant ' λ ' is the tuning parameter.

Substituting G_{cl} and G_m , the controller transfer function becomes

$$G_c(s) = k_m^{-1} \frac{\tau_m s + 1}{\lambda s + 1 - e^{-Ls}} \quad (8)$$

When dead-time is zero the above equation becomes a PI controller equation with gain $1/k_m$ and integral time constant as λ . The controller given by equation (8) can be interrupted as a Predictive PI controller where the prediction is formed by correcting the effects of the control actions that has been taken but not appeared in the output because of the delay in the process. The prediction given in equation (8) is much better than the prediction obtained in derivative action of a PID controller. The PPI controller gives better performance than the other predictive structure and it is analyzed through comparison.

B. PID Controller

A standard feedback control scheme is shown in Figure 2. PID controller is used in the standard feedback control scheme. A series form of practical PID controller with filter transfer function is as follows:

$$G_c(s) = k_c \left(1 + \frac{1}{\tau_i s} \right) \left(\frac{\tau_d s + 1}{\tau_f s + 1} \right) \quad (9)$$

where the controller parameter ' k_c ' is the controller gain ' τ_i ', ' τ_d ' and ' τ_f ' is the integral, derivative and filter time constants respectively.

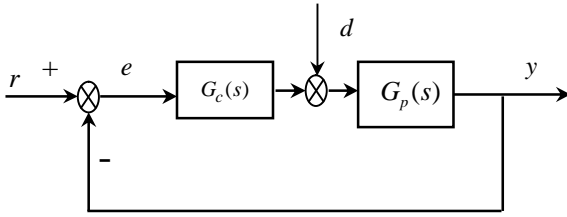


Fig. 2: Feedback control scheme

The PID controller can be obtained by approximating the exponential term in the above PPI controller equation (8). Expand the exponential term by using first order padé approximation [21]

$$e^{-Ls} = \frac{1 - \frac{L}{2}s}{1 + \frac{L}{2}s} \quad (10)$$

And simplifying, the following PID controller is obtained

$$G_c(s) = \frac{\tau_m}{k_m(\lambda + L)} \left(1 + \frac{1}{\tau_m s} \right) \frac{(L/2)s + 1}{\tau' s + 1}$$

Where

$$\tau' = \frac{\lambda L}{2(\lambda + L)} \quad (11)$$

The PID controller parameter tuning relations are as follows:

$$\begin{aligned} k_c &= \frac{\tau_m}{k_m(\lambda + L)} \\ \tau_i &= \tau_m \\ \tau_d &= L/2 \\ \tau_f &= \frac{\lambda L}{2(\lambda + L)} \end{aligned} \quad (12)$$

It should be noted that the both PPI controller (equation 8) and the PID controller (equation 11) which obtained by dead-time approximation has single tuning parameter. The PPI controller tuning parameter is directly related to the desired closed loop model time constant whereas the PID controller parameter is tuned to satisfy a desired closed loop response. Comparing the

PPI and PID controller equations it can be concluded that the gain of the PPI controller is inverse of process gain and it takes a constant value but the PID controller gain is function closed loop time constant. It can be stated that the PPI controller is a constant gain controller but the PID controller is a variable gain controller. The approximated PID controller derivative term is function of dead-time. Hence the prediction in PID controller is also depends on dead-time value. The effect of actual and dead-time approximation is analyzed via simulation of process control systems.

3. Performance/Robustness Comparison of Feedback Controller Design Methods

The prediction in the PPI control scheme and the prediction by a derivative term in the PID controller are compared through evaluating the performance and robustness of the schemes. The Integral Absolute Error (IAE) and Total Variation (TV) of manipulated variable are considered as the performance measure.

The integral absolute error is defined as

$$IAE = \int_0^{\infty} |r(t) - y(t)| dt \quad (13)$$

the total variation (TV) of the manipulated input u is calculated as

$$TV = \sum_{k=1}^{\infty} |u(k+1) - u(k)| \quad (14)$$

The robustness measures of maximum sensitivity (M_s) is the well-known measure of robustness.

The sensitivity function $S(s)$ is defined as follows:

$$S(s) = \frac{1}{1 + G_c(s)G_m(s)} \quad (15)$$

The maximum sensitivity is given by,

$$M_s = \max_{\omega} |S(j\omega)| \quad (16)$$

It is well known that certain practical limits to M_s values is required to guarantee a minimum margin of robustness:

$$1.2 \leq M_s \leq 2.0 \quad (17)$$

The controller parameters are computed for various values of closed loop time constant lambda using the controller relations given in equations 8 and 12. By using the relation (13-16) the performance and robustness of the actual and approximated control schemes are evaluated.

Example 1

The diagram of a typical paper machine is shown in Fig. 3. The paper machine is divided into five sections: the head section, table and press section, dryer section, calender stack, and reel. In this system, there are

numerous control objectives such as basis weight, moisture content, stream pressure, consistency, etc., of which the most important is basis weight, i.e., the weight of one square meter of paper.

The dynamic model of the paper machine has been developed for basis weight control objective is [24]

$$G_m(s) = \frac{5.15}{1.8s + 1} e^{-2.8s}$$

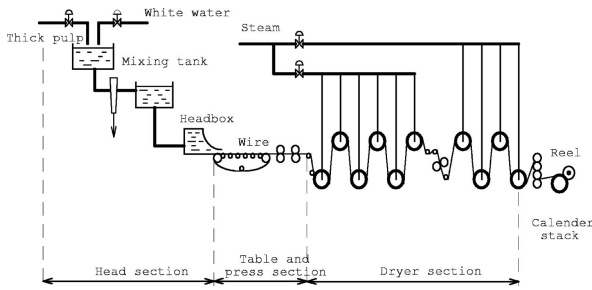


Fig. 3: Fourdrinier paper machine

The process dead-time is relatively high compared to the process time constant. The servo performance measures IAE and TV values of PPI and dead-time approximated PID controller for various values of closed loop time constants are shown in Figure 4. From the Figure it is inferred that the PPI controller performance (IAE) Vs closed loop time constant has linear relation and also it gives improved performance for lower values of closed loop time constant. One important thing is that the PPI control energy (TV value) is less compared with approximated PID controller.

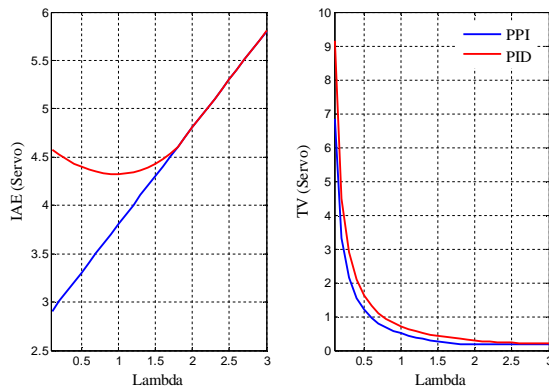


Fig. 4: Servo performance analysis for paper machine process

The regulatory performance measures IAE and TV values of PPI and dead-time approximated PID controller is shown in Figure 5 for various values of closed loop time constants. The PPI and dead-time approximated PID controller shows identical performance. However, the PPI control energy is less and constant all values of closed loop time constant.

The robustness of control schemes is analyzed and the robustness graph is shown in Figure 6. From the

graph it is inferred that the PPI controller M_s Value is within the limit (1.2 to 2.0) where as the approximated PID controller M_s Values are out of the limit for lower values of closed loop time constant. The dead-time approximation leads to degraded servo performance, increased control energy and reduced robustness for the paper machine basis weight control objective.

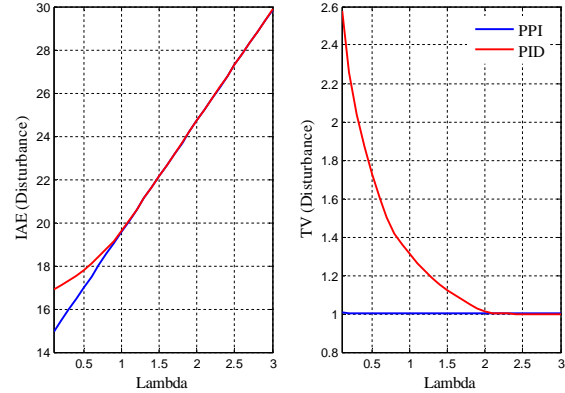


Fig. 5: Regulatory performance analysis for paper machine process

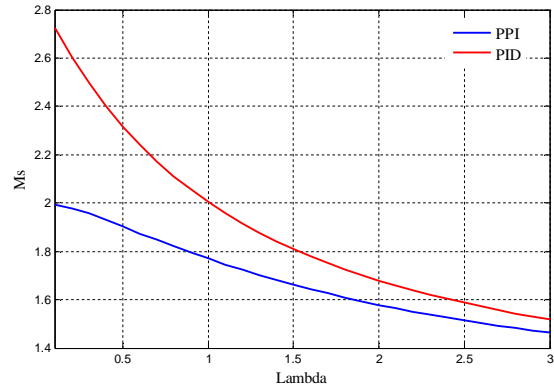


Fig. 6: Robustness analysis for paper machine process

Example 2

In a heat exchanger process which is shown in Fig. 7 the outlet temperature (T) of the cold water is controlled by manipulating the input flow of the hot water using control valve (V). The temperature of the hot water is controlled by an independent controller.

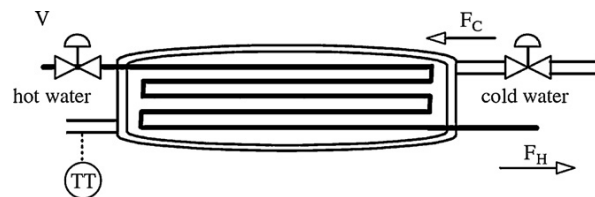


Figure 7: Heat Exchanger

This heat exchanger process can be represented by the following simple first order model at the nominal

operating point.

$$G_m(s) = \frac{0.12}{6s+1} e^{-3s}$$

For the heat exchanger process the dead-time is relatively low compared to the process time constant. The servo and regulatory performance measures IAE and TV values of PPI and dead-time approximated PID controller for various values of closed loop time constants are shown in Figure 8 and 9 respectively. From the performance analysis it is inferred that the PPI Controller shows improved servo response. Both controllers show identical regulatory performance. One important thing is that the PPI control energy (TV value) is less compared with approximated PID controller.

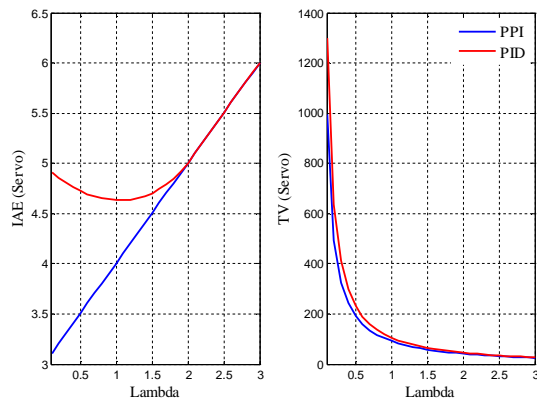


Fig. 8: Servo response analysis for heat exchanger process

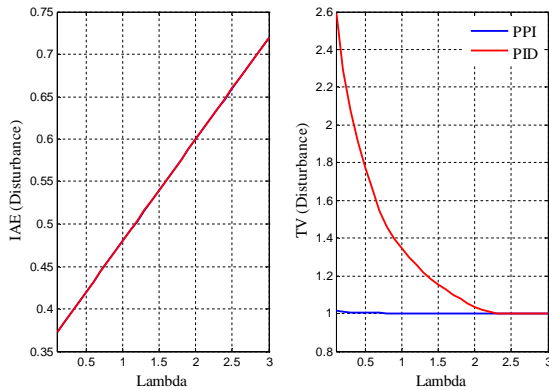


Fig. 9: Regulatory response analysis for heat exchanger process

The robustness graph is shown in Figure 10. It is inferred that the PPI controller M_s Value is within the limit (1.2 to 2.0) where as the approximated PID controller M_s Values are out of the limit for lower values of closed loop time constant. The dead-time approximation affects the robustness for heat exchanger process.

4. Conclusion

The effects of dead-time approximation on controller performance/robustness designed for a first order dead-time model is analyzed for process control systems. Comparing the PPI and PID controller equations it can be concluded that the gain of the PPI controller is inverse of process gain but the PID controller gain is function closed loop time constant. The Predictive PI controller shows improved servo performance and the robustness is within limit for various values of closed loop time constant. The dead-time approximation leads to degraded servo performance, increased control energy and reduced robustness. The PPI and approximated PID controller shows almost identical regulatory performance. The prediction in PPI controller is much better than the prediction obtained in derivative action of a PID controller.

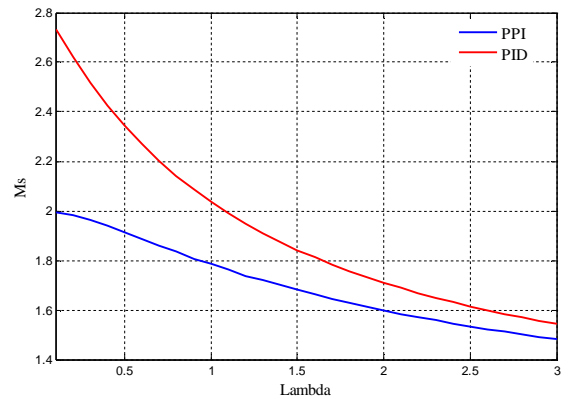


Fig. 10: Robustness analysis for heat exchanger process

References

1. K. J. Astrom and T. Haggund, *Advanced PID Controller*, Research Triangle Park, North Carolina, Instrument Society of America, 2006.
2. Vilanova R., Visioli A. *PID Controller in the Third Millennium*, Springer, 2012.
3. A. Visioli, Research trends for PID controllers, *Acta Polytech.* 52 (5) (2012)144–150.
4. O'Dwyer A. *Handbook of PI and PID controller tuning rules*. Imperial College Press, London: 2006.
5. M. Shamsuzzoha, S. Skogestad, The setpoint overshoot method: a simple and fast closed-loop approach for PID tuning, *Process Cont.* 20 (10) (2010)1220–1234.
6. Q.-G. Wang, Z. Zhang, K.J. Astrom, L.S. Chek, Guaranteed dominant pole placement with PID controllers, *Process Cont.* 19 (2) (2009) 349–352.
7. D.-J. Wang, Synthesis of PID controllers for high-order plants with time-delay, *Process Cont.* 19 (2) (2009) 349–352.
8. Skogestad, S. (2003). Simple Analytic Rules for Model Reduction and PID Controller Tuning, *Journal of Process Control*; 13: 291-309.
9. A. Besanion-Voda, Iterative auto-calibration of digital controllers. Methodology and applications, *Control Engineering Practice*, vol. 6, no. 3, pp. 345-358, 1998.

10. R.E. Precup, S. Preitl, G. Faur, PI predictive fuzzy controllers for electrical drive speed control: Methods and software for stable development, *Computers in Industry*, vol. 52, no. 3, pp. 253-270, 2003.
11. V.J. Ginter, J.K. Pieper, Robust gain scheduled control of a hydrokinetic turbine, *IEEE Transactions on Control Systems Technology*, vol. 19, no. 4, pp. 805-817, 2011.
12. Chen, D. and Seborg, D.E. (2002). PI/PID Controller Design Based on Direct Synthesis and Disturbance rejection. *Ind. Eng. Chem. Res.*, 25, pp.4807-4822.
13. Fruehauf, P. S., Chien, I. L., & Lauritsen, M. D. (1994). Simplified IMC-PID tuning rules. *ISA Transactions*, 33(1), 43-59.
14. Lee, Y., Lee, M., Park, S., & Brosilow, C. (1998). PID controller tuning for desired closed-loop responses for SI/SO system. *A.I.Ch.E. Journal*, 44(1), 106-115.
15. S. Alcántara, W.D. Zhang, C. Pedret, R. Vilanova, S. Skogestad, IMC-like analytical H_∞ design with S/SP mixed sensitivity consideration: utility in PID tuning guidance, *Journal of Process Control* 21 (6) (2011) 976–985.
16. R. Vilanova, IMC based robust PID design: tuning guidelines and automatic tuning, *Journal of Process Control* 18 (2008) 61–70.
17. Jin QB, Lin Q, IMC-PID design based on model matching approach and closed loop shaping. *ISA Transactions*; 53:462-473, 2014.
18. Wang QG, Hang CC, Yang XP. Single loop controller design via IMC Principles. *Automatica*. 2001, 37, 2041-48.
19. Saurabh S, Pandit V.S. A PI/PID controller for time delay systems with desired closed loop time response and guaranteed gain and phase margins. *Journal of Process control*. 2016, 37, 70-77.
20. Arun R, Prakash J. Design and Implementation of a model-based PI-like control scheme in a reset configuration for stable single-loop systems. *Can. J. Chem. Eng.* 2014;92(9):1651-1660.
21. Smith OJ. A controller to overcome dead time. *ISA J* 1959;6(2):28–33.
22. Normey-Rico JE, Camacho EF. Dead-time compensators: a survey. *Cont Eng Pract* 2008;16(4):407–428.
23. Hägglund T. An industrial dead-time compensating PI controller. *Cont Eng Pract* 1996;4(6):749–756.
24. Zhang W, He X, Xu X. Comparison of several well-known controllers used in process control. *ISA Trans.* 2003;42(2):317-325.