# A High Order Sliding Mode Control for Autonomous Underwater Vehicle (AUV)

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**Abstract**: In this paper, a High Order Sliding Modes Control (HOSMC) approach is used to control an autonomous underwater vehicle. Autonomous underwater vehicle motion in ocean conditions requires investigation of new control solutions that guarantee robustness against external parameter uncertainty. Sliding Mode Control (SMC) is adequate for controlling AUVs, since it offers robustness in the presence of uncertainties and environmental disturbances; however the main drawback is the chattering effects that stimulate high frequency vibration that can damage the actuators. HOSMC control preserves the properties of standard SMC and removes the chattering effects. Therefore it is able to improve a capability to track the desired state of the proposed autonomous underwater model. Different simulations have been carried out to show the performance and effectiveness of the proposed method. **Keywords**: Autonomous underwater vehicles control, High order sliding modes, sliding modes control

### 1 Introduction

Autonomous Underwater Vehicles (AUVs) have gained increasing interest in recent years because of a wide area of possible applications such as maintenance, diver support, pipeline inspection, geological surveying and military. The control of the AUV is a challenging task; this is primarily due to the high and coupled non linearities, environmental disturbances (like ocean currents and wave's effects) and parameter uncertainties. Furthermore, the controller must satisfy two basis requirements: first it has to be robust to develop its

work; secondly, it shouldn't be very complicated, otherwise it could have singularities and their real time performance could be slow. Several control schemes have been proposed for AUVs [9]-[13]. PD controller [9], Optimal control [10], Neural Network [11], Adaptive Sliding Mode Control [4], etc . In this paper, SMC and HOSMC control laws are proposed to solve the problem of accurate trajectory tracking for the desired depth. The paper is organized as follows. Section 2 presents the kinematic and vertical dynamic models of the AUV. A second order sliding mode control structure used for the AUV is described in Section 3. Effectiveness of the proposed schemes is demonstrated by simulation in Section 5. Finally the conclusion is given in Section 6.

## 2 Mathematical Modeling

The equations of motion for underwater vehicles can be presented with respect to a Body-fixed frame relative to an Earth-fixed frame, Fig. 1

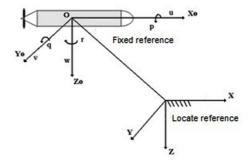


Figure 1 – Reference marks representation

The position of the Body-fixed measured in the Earth-fixed is,

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}; \eta_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \eta_2 = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

$$v = \dot{\eta}$$
(1)

### 2.1 Kinematics model

The vehicles-fixed linear and angular velocity and the time derivative of the earth-fixed vehicle position coordinates are related by the following relationships (Figure 02):

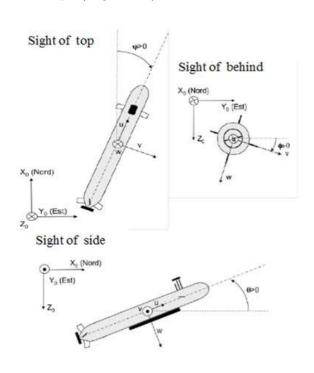


FIGURE 2 – definition of the swing angles

$$\begin{bmatrix} \dot{\eta} = J_c(\eta_2) \upsilon \\ \left[ \begin{matrix} \dot{\eta_1} \\ \dot{\eta_2} \end{matrix} \right] = \begin{bmatrix} J_{c1}(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J_{c2}(\eta_2) \end{bmatrix} \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix}$$
(2)

Where  $J_{c1}(\eta_2)$  is the transformation matrix of the linear velocity and  $J_{c2}(\eta_2)$  is the transformation matrix of the angular velocity. This representation has a singularity for  $\theta = \pm 90^{\circ}$ .

$$J_{c1}(\eta_2) = \begin{bmatrix} c\theta.c\psi & s\theta.s\phi.c\psi - s\psi.c\phi & s\theta.c\phi.c\psi + s\psi.s\phi \\ c\theta.s\psi & s\theta.s\phi.s\psi + c\phi.c\psi & s\theta.c\phi.s\psi - c\psi.s\phi \\ -s\theta & c\theta.s\phi & c\theta.c\phi \end{bmatrix}$$

$$J_{c2}(\eta_2) = \begin{bmatrix} 1 & s\theta.t\theta & c\phi.t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix}, \begin{cases} c = cos \\ s = sin \\ t = tan \end{cases}$$

$$(4)$$

### 2.2 Dynamic model

The vehicle dynamics model derived by [7] is based on a set of dynamic equations that govern the vehicle's translational and rotational motion in 3-D space. Using the Newton laws, the translational and rotational motions are:

$$\begin{cases}
 m * \gamma &= \sum F \\
 I * \ddot{\alpha} &= \sum M
\end{cases}$$
(5)

where F is the net external force applied to the vehicle, m is the mass of the vehicle,  $\gamma$  is the acceleration of its mass center with respect to an inertial frame, M is the net external moment acting on the vehicle at the center of mass, I is the inertia tensor about the vehicle's mass center, and  $\ddot{\alpha}$  is the vehicle's angular acceleration. The external forces and moments on the vehicle are due to gravitational, buoyancy, propulsive, control and hydrodynamic effects. As shown by [5], the motion equations given by (5) can also be written in the form :

$$M_{RB}\dot{v} + C_{RB}(v)v = \Gamma \tag{6}$$

where  $M_{RB}$  is the inertia matrix including added mass, and  $C_{RB}(v)$  is a matrix that includes centrifugal and Coriolis forces

 $\upsilon$  is the generalized velocity vector in the body-fixed frame, and  $\dot{\upsilon}$  is the acceleration vector in the body-fixed frame. Generalized force vector  $\Gamma$  has the following components:

$$\Gamma = \Gamma_h + \Gamma_g + \Gamma_u + \Gamma_p \tag{7}$$

Where  $\Gamma_h$  present the hydrodynamics force vector of the vehicle body,  $\Gamma_g$  is the static force vector ((gravity and buoyancy)),  $\Gamma_u$  is the controlled force vector (include the thruster force and the fin force) and  $\Gamma_p$  is outside disturbances force and moment acting on the AUV.

### 2.2.1 Hydrodynamic forces

In this section, we will discuss the hydrodyna-(3) mic forces for AUV. The balance of efforts due to

the inertia and mass of water added can be put in the form:

$$-(M_a\dot{v} + C_a(v)v) = \Gamma_a \tag{8}$$

where  $M_a$  is the added mass matrix and  $C_a(v)$  is a Coriolis-like matrix

The viscous damping force is given by the damping matrix D(v) is given by

$$-D_a(v)v = \Gamma_d \tag{9}$$

With  $\Gamma_h = \Gamma_a + \Gamma_d$  is the vector of hydrodynamic efforts.

#### 2.2.2 Hydrostatic forces

A solid body submerged in a fluid will have upward buoyant force acting on it equivalent to the weight of displaced fluid, enabling it to float or at least to appear to become lighter. If the buoyancy exceeds the weight, then the object floats; if the weight exceeds the buoyancy, the object sinks. If the buoyancy equals the weight, the body has neutral buoyancy and may remain at its level. Discovery of the principle of buoyancy, which is a result of the hydrostatic pressure in the fluid, is attributed to Archimedes (figure 03). Hydrostatic force includes the gravity part WW

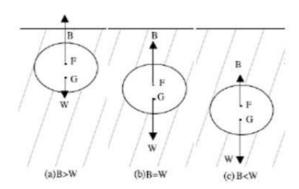


FIGURE 3 – hydrostatic equilibrium

and the buoyancy part BB. The balance of hydrostatic efforts can be put in the form:

$$-g(\eta) = \Gamma_g \tag{10}$$

#### Actuators of the robot 2.2.3

incoming fluid flow, respectively.

The control plane forces in the body frame, acting at the center of pressure of the control planes, are computed as:

When we use fins to control the motion of the AUV, the vehicle must maintain a surge speed. The propeller is used to generate energy to provide the surge force and then keep the vehicle moving ahead. Of course, the number of propellers mounted on the vehicle can be more than one. In the modeling process, we view the propeller as a special module and model it independently. The thrust force generated by a propeller and the rolling moment caused by the propeller can be computed by [1]:

On the vertical control planes

$$\begin{bmatrix} F_{x} \\ F_{z} \\ M_{q} \end{bmatrix} = \begin{bmatrix} -0.5\rho S_{s}V_{0}^{2} (C_{zs}s\delta_{p} + C_{xs}c\delta_{p}) \\ -0.5\rho S_{s}V_{0}^{2} (C_{zs}s\delta_{p} - C_{xs}c\delta_{p}) \\ F_{z} (0.2c_{s}c\delta_{p} - d_{as}) + F_{x} (0.2c_{s}s\delta_{p}) \end{bmatrix}$$
(11)

On the horizontal control planes

$$\begin{bmatrix} F_{x} \\ F_{y} \\ N_{r} \end{bmatrix} = \begin{bmatrix} -0.5\rho S_{s} V_{0}^{2} \left( C_{ys} s \delta_{c} + C_{xs} c \delta_{c} \right) \\ -0.5\rho S_{s} V_{0}^{2} \left( C_{ys} s \delta_{c} - C_{xs} c \delta_{c} \right) \\ F_{y} \left( 0.2 c_{s} c \delta_{c} - d_{as} \right) + F_{x} \left( 0.2 c_{s} s \delta_{c} \right) \end{bmatrix}$$
(12)

Where  $\rho$  is the fluid density,  $S_s$  is the surface of the wing  $S_s = b_s \times c_s$ ,  $V_0$  is the modulus of the fluid flow around the wing,  $0.2c_s$  is the distance from the leading edge of the wing at its point of application of hydrodynamic forces,  $C_{ys}$  and  $C_{xs}$ represent the coefficients of lift and drag.

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$$T_p = \rho D_p^4 K_T (J_0) |n_p| n_p$$
 (13)

$$Q = \rho D_p^4 K_Q(J_0) |n_p| n_p \tag{14}$$

Where  $n_p$  is rotating rate of the propeller,  $\rho$  is the fluid density,  $D_p$  is the propeller diameter, Actuators forces can be decomposed into lift  $K_T$  is the thrust coefficient,  $K_Q$  is torque coeffiand drag forces, perpendicular and parallel to the cient  $J_0 = V_a/(n_p D_p)$  is the advance number.  $V_a$  is the advance speed at the propeller (speed of the water going into the propeller) which has a relationship with surge velocity  $V_0: V_a = (1 - w_a)V_0$  where wa is the wake fraction number (typically: [0.1:0.4]). The final dynamic equation of the robot is as follows [3]:

$$M(\dot{v}) + C(v)v + D(v)v + g(\eta) = \Gamma \qquad (15)$$

where  $M = M_{RB} + M_a$  is the inertia and added mass matrix,  $C(v) = C_{RB}(v) + C_a(v)$  is the matrix of Coriolis and centripetal forces, from inertia and hydrodynamics, D(v) is the hydrodynamic damping and  $g(\eta)$  is the vector of restoring forces and moments.  $\Gamma$  is control-input vector describing forces and moment efforts

$$\Gamma = [\delta_p, \delta_c, n] \tag{16}$$

With  $\delta_p$  is the fin angle of AUV ,  $\delta_c$  is the rudder angle of AUV, n is the revolution of propeller as thrusters of AUV.

### 2.3 General equation of motion

The non linear vehicle equation of motion is written as:

$$\begin{cases} \dot{\eta} = J_c(\eta_2)\upsilon \\ M\dot{\upsilon} + C(\upsilon)\upsilon + g(\eta) = \Gamma \end{cases}$$
 (17)

Hence the global state vector is represented by:

$$(\eta, v) = (x, y, z, \phi, \theta, \psi, u, v, w, p, q, r)$$
(18)

In this article, we only need to position (x,y), the yaw angle  $\psi$ , linear velocity (u,v) and angular velocity r. Then we considered the other variables states as null.

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -0.209 & -0.605 & 0 \\ -0.054 & -0.569 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} 0.145 \\ -0.152 \\ 0 \end{bmatrix} \delta_{c}$$
(19)

## 3 Sliding mode controller

Nonlinear model based control systems offer a level of dynamic capabilities which linear techniques are incapable of providing when dealing with parameter uncertainties and unmodelled dynamics. Sliding mode [13], is categorized as a variable structure control system which has excellent stability, robustness, and disturbance rejection characteristics. This type of control is not new to submarines, in fact it is widely used due to its capability to overcome modeling errors (due in this case to the hydrodynamic terms and modeling as an uncoupled system). Sliding mode has been used in : spacecraft [14], robotics [15], missiles [16], and many other applications where modelling error is a concern.

The idea behind SMC is to define a surface along which the process can slide to its desired final value. The structure of the controller is intentionally altered as its state crosses the surface in accordance with a prescribed control law. The SMC control law, consists of two additive parts. That is:

$$u = u_{eq} + u_{qlis} \tag{20}$$

where  $u_{eq}$ : Nominal control, which is determined by the robot model.  $u_{glis}$ : Sliding part, which is useful to compensate model uncertainties. Sliding surface in the state error space for tracking problem is defined and now the state errors are:

$$\tilde{x} = x - x_d \tag{21}$$

where  $x_d$  desired tracking state. Now, sliding surface can be defined in the error state space form as follows

$$s = \left[\begin{array}{cc} \varpi & \chi & \vartheta \end{array}\right] \left[\begin{array}{c} \tilde{v} \\ \tilde{r} \\ \tilde{\psi} \end{array}\right] = S\tilde{x} \tag{22}$$

where  $\chi, \varpi$  and  $\vartheta$  are the sliding surface constants. The sliding surface s must obey the next condition :

 $\dot{s} \to 0$  and  $s \to 0$  as  $t \to \infty$  so, this imply  $\tilde{x} \to 0$  as  $t \to \infty$  The simplified model considered here given by the expression (23) and takes into account the following three states. Then we rewrite:

$$M_c \dot{Y} = C D_c Y + g_c(\eta) + V_c u_0 + U_c \qquad (23)$$

The sliding surface s is defined as a function of the state space of errors :

$$s = \varpi (v - v_d) + \chi (r - r_d) + \vartheta (\psi - \psi_d) \quad (24)$$

The derivative of s is expressed by the equation :

$$\dot{s} = \varpi \dot{v} + \dot{r} + \vartheta \dot{\psi} = S \dot{Y} \tag{25}$$

The equivalent command is determined from the solution of the following expression

$$\dot{s} = 0 \tag{26}$$

 $\dot{Y}$  is replaced by equation (23), we have :

$$\dot{s} = SM_c^{-1} (CD_c Y + g_c + V_c u_0 + U_c) = 0$$
 (27)

The equivalent command is:

$$U_c = -(CD_cY + g_c + V_c u_0)$$
  

$$u = U_c + K_{glis} sign(s)$$
(28)

Finally the sliding surface and the command are given by the expression :

$$u = 0.5260v + 0.162r + 4.3465 (\psi - \psi_d) + 1.5sign ((0.15v + 1.65r + (\psi - \psi_d)) / 0.05)$$
(29)

In the standard SMC, is discontinuous; this is the main reason why high frequency switching appears in the output signal (chattering effect), which causes problems in practical application. In order to avoid chattering, a high order sliding control is used [17][18]. HOSMC acts on the higher order time derivative of the system deviation, instead of influencing the first deviation derivative as it happens in SMC [18]. Its principal characteristic is that it keeps the main advantages of the SMC, removing the chattering effects. The sliding order is a number of continuous total derivatives of in the vicinity of the sliding mode (Fig. 04) The proposed 2nd order sliding mode control can be used as an alternative natural approach for smoothing the input signal. The sliding order is a measure of the degree of smoothness of the sliding variable in the vicinity of the sliding mode. This proposed technique can be considered an extension of the first order sliding regime. It is composed of two parts [3]:

$$u = \int u_{eq} + K_{glis} \int u_{glis} \tag{30}$$

 $\int u_{glis}$  is composed of the integral of the function **4** sign multiplied by a constant  $K_{glis}$ .

 $\int u_{eq}$  is designed by using the equivalent command method for the new sliding surface s.

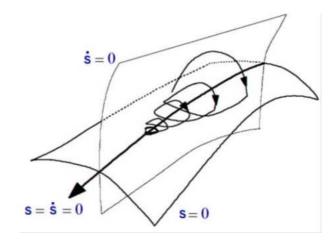


FIGURE 4 – Second order sliding mode trajectory

It is necessary to express the previous system Finally the sliding surface and the command are (23) to appear the derivative of the control output

$$\begin{bmatrix} \dot{v} \\ \dot{\dot{r}} \\ \dot{\psi} \\ \ddot{v} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.209 & -0.605 \\ 0 & 0 & 0 & -0.054 & -0.569 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \\ \dot{v} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.145 \\ -0.152 \end{bmatrix} \delta_{c}$$

$$(31)$$

The new sliding surface is defined as follows:

$$s_{c} = \dot{s} + \beta s$$

$$= \omega \dot{v} + \chi \dot{r} + \omega \beta v + (v + \chi \beta) r + v \beta \tilde{\psi}$$
(32)

The equivalent command for the second order sliding technique is defined as follows:

$$u_{eq} = -0.36215\dot{v} + 1.67170\dot{r} + 1.09150\dot{\psi} \quad (33)$$

Finally, the command output is written:

$$\delta_c = -0.36215\dot{v} + 1.67170\dot{r} + 1.09150\dot{\psi} + 0.75sign((0.15\dot{v} + 1.65\dot{r} + 0.0375v + 1.4125r + 0.25(\psi - \psi_d))/0.05)$$
(34)

## 4 Simulation Results

In this section, simulation results are implemented by using second order SMC controllers

as described in previous section. In the heading control, the desired heading angle is 2°. Figures 05-09 depict the results of plots for heading control without disturbance. As can be seen, the desired trajectory is followed almost without error after 17s and 25s for SMC and HOSM controller respectively

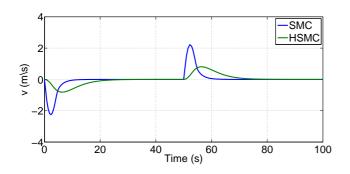


FIGURE 5 – Sway velocity of robot

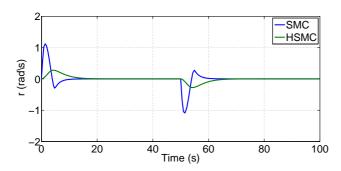


FIGURE 6 – Heading velocity of robot

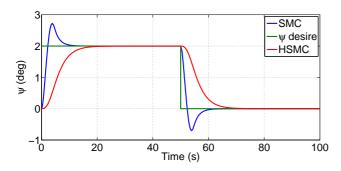


FIGURE 7 – Heading Angle of robot

## 5 Conclusion

This paper describes SMC and HOSMC control algorithms, which are evaluated in autonomous

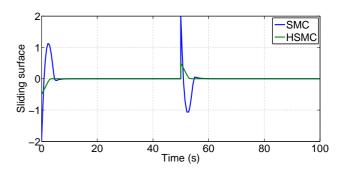


FIGURE 8 – Sliding surface

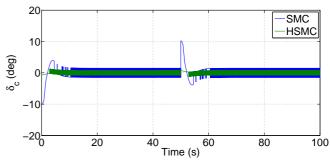


FIGURE 9 – Rudder control of the heading

underwater vehicle (AUV) heading control. The second order sliding mode controller has been introduced to provide system's stability. It guarantees that an AUV is able to converge to a desired heading and depth with constant speed as demonstrated in the simulation. From simulation, a second order sliding mode controller is able to provide accurate control for subsystems, which gives satisfactory results. The first Sliding mode control consumes a lot of control energy and damages AUV. By using a second order sliding mode controller reduce control energy and prevent the chattering effect. Moreover, the proposed control exhibits a satisfactory performance when used with disturbance and dynamics uncertainty.

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