

FOPDT Model fitting to an n th-Order All Pole NMP Processes by an Optimal Method

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Abstract - The aim of this paper is to present a method to approximate the n^{th} -order all pole systems with a zero by the widely used first-order plus dead-time (FOPDT) model in s -plane. To establish this method, impulse response approach is used. This method is applicable for s -shaped functions. The main novelty of the present method is the closeness of the approximated transfer function to the process model. That is the square of the 2-norm of the error signal between the impulse responses of the system and the presented model that is chosen as an index function. By globally minimizing this index function, three nonlinear equations are obtained in terms of the parameters of FOPDT model. Then these equations are analyzed and proved that always have unique solution to the FOPDT model parameters. Finally, by solving the nonlinear equations the optimal values of FOPDT model parameters are achieved. Numerical example is included to present the advantage of the presented method. The proposed research uses optimal method to approximate the n th-order all pole systems by the far-reaching used general form of the first-order plus dead time (FOPDT) model. This proposed investigation has described the structure of optimal FOPDT model, which is analytically modeled for a several industrial processes. The method is based on fitting to special form of transfer function in optimal manner. To compare the performance of the method of this paper and two another, impulse response approach is presented. Further, the obtained results show that the optimal FOPDT model fitting to an n th-order all pole process is capable of optimizing the similarity the model and real systems, less undershoot, and faster settling time than other mentioned method.

Key words- All pole process, FOPDT model Fitting, Zero in s -plane, Optimal manner, optimal parameters, Time delay, Impulse response approach, Nonlinearity, Index function.

Nomenclature

n Process order
 τ Process time-constant
 k_{∞} Steady state gain of process
 ψ Real number

1. Introduction

We can find many complicated systems in the many process control field that their response to a step input is a bounded and monotonic s -shaped function of time. Usually for modeling such systems, first-order plus dead-time (FOPDT) transfer function is used [1,2,12,13]. Since this transfer function is a low-order and simple with three different parameters, named system gain, dead time (time delay), and time constant. Hence it is suitable for identification purpose [14,2].

Furthermore, the basic and main design methods and tuning techniques in the practical controllers such as PID controller are generalized and developed for the FOPDT transfer function [1,2,14]. Primary identification methods for FOPDT model were area-based techniques is analyzed in [1,3]. A direct method has been presented in [3,4]. A modified relay feedback identification method is studied in [5]. A set of general expressions, which is also doable for the FOPDT model systems, is obtained from a single symmetrical relay feedback test for models identification in [6,20].

In [2,7,9] an reality manner for on-line FOPDT model identification and PI controller tuning and adjust is given. A real-coded genetic algorithm is showed for identification FOPDT transfer function from step response [8,15,16]. Erected on using the closed-loop system step response, two identification algorithms are presented for obtaining the FOPDT and second-order plus dead-time (SOPDT) models

[9,10]. An on-line iterative program for identification of a type of nonlinear

FOPDT model is showed in [9,11,21].

Recently, by allowing the parameters of the conventional FOPDT transfer function to be time dependent a time-varying extension of this model is developed in[12,21]. One of the systems that step response is an s-shaped function and therefore it is suitable to be described by FOPDT model[18,19, 22], is the n^{th} -order all pole system that is, a system with n same poles and no zeros [13,17,18]. The purpose of this paper is to present an optimal manner for approximating such systems by a FOPDT model. In this paper is obtained an inequality for boundary that need for selected parameters by global optimization and to use nonlinear equations. In here, major purpose is to give a method for optimization, and is obtained a closer model within achieve desired parameters.

This paper is organized as described below. In section 2 describes with following n^{th} - order processes and stable FOPDT model in order to introducing industrial process. In here we tend to use FOPDT model for optimal of describing of the process. In section 3 focuses how the modeling of distance index between industrial and modeling process by 2- norm of the error signal. In section 4 proposes a nonlinear equation between two last equation. in section 5, will be expressed an example and comparisons with the other method. The section 6 concludes the proposed scheme numerically is carried out. Moreover comparisons of two area method and direct method with brought scheme show better than two another methods.

Identified and optimization and faster settling time, less undershoot or overshoot, continue to be of major concern in system operation. This arise the facts that in steady state. Easier calculation, discover and formulating desired transfer function parameter with minimum settling time a, on the other hand, optimizing with best parameter and most similar model by minimizing 2-norm.

2. Problem formulation

Consider the following n^{th} -order all pole process with a zero:

$$G_p(s) = \frac{k_{\infty}(1 + \psi s)}{(1 + \tau s)^n} \quad (1)$$

Where $\tau > 0$ is the process time-constant and n is the process order that is assume to be at least two, k_{∞} indicates the steady state gain of the process,

$\psi \in R, \psi \neq 0$ are available, without loss of generality it is assumed to be positive.

The stable FOPDT model is defined as follows:

$$G_m(s) = \frac{Ae^{-Ls}}{1 + Ts} \quad (2)$$

Where $T > 0$ is the model time-constant, $L > 0$ is the model time-delay, and A is the dc gain of the model.

Significant and important design procedures and tuning methods in industrial controller and particularly in PID controller are presented based on the model (2) [1].

Therefore, for utilizing and better usage of these procedures there are many interests and efforts to use the model (2) in describing the stable industrial process. In here we are enthusiasm to use model (2) for optimal description of the process (1). In more precise wording, while the parameters of the process (1) k_{∞}, τ , and n are given, it is needed to fit the model (2) in it in an optimal manner. It means that in model (2) the parameters A, L and T should be determined so that a defined distance index (norm) between the process (1) and model (2) to become minimum. Since the index to be used in here, will be in time domain it is suitable to rewrite the $G_p(s)$ and

$G_m(s)$ as follows:

$$G_p(s) = \frac{k(1 + \psi s)}{(s + a)^n}, G_m(s) = \frac{\mu e^{-Ls}}{s + \lambda} \quad (3)$$

Where:

$$a = \frac{1}{\tau}$$

$$k = \frac{k_{\infty}}{\tau^n}$$

$$k\psi = \frac{k_{\infty}}{\tau^n} \psi$$

$$\lambda = \frac{1}{T}$$

$$\mu = \frac{A}{T}$$

Therefore by having a, k and n one should determine μ, λ and L in an optimal manner.

3. Impulse response approach

In here the distance index between the process $G_p(s)$ and model $G_m(s)$ are defined as follows:

$$I = \int_0^{+\infty} \{g_m(t) - g_p(t)\}^2 dt \quad (4)$$

Where, $g_p(t)$ and $g_m(t)$ are the impulse response of the process and the model reactively, that is:

$$g_p(t) = L^{-1}(G_p(S)), \quad g_m(t) = L^{-1}(G_m(S)) \quad (5)$$

Note that the defined index in (4) is the square of the 2-norm of the error signal $g_m(t) - g_p(t)$.

One can easily obtain:

$$g_p(t) = L^{-1}(G_p(S)) = \frac{kt^{n-1}}{(n-1)!} e^{-at} u(t) - \frac{k\psi(n-1)t^{n-2}}{(n-1)!} e^{-at} u(t) \quad (6)$$

$$g_m(t) = L^{-1}(G_m(S)) = \mu e^{-\lambda(t-L)} u(t-L) \quad (7)$$

Where, $u(t)$ is unit step signal. In order to minimize I , it is needed to obtain its derivatives of I with respect to μ, λ , and L ; then setting them all equal to zero. Substituting $g_m(t)$ from (7) into (4), it results:

$$I = \int_0^{+\infty} \{ \mu e^{-\lambda(t-L)} u(t-L) - g_p(t) \}^2 dt \quad (8)$$

Differentiating (8) with respect to μ , gives:

$$\frac{\partial I}{\partial \mu} = 2 \int_0^{+\infty} e^{-\lambda(t-L)} (\mu e^{-\lambda(t-L)} - g_p(t)) u(t-L) dt \quad (9)$$

Solving the equation $\frac{\partial I}{\partial \mu} = 0$ results

$$\mu = 2\lambda e^{\lambda L} \int_L^{+\infty} g_p(t) e^{-\lambda t} dt \quad (10)$$

Differentiating (8) with respect to λ , gives:

$$\frac{\partial I}{\partial \lambda} = 2 \int_0^{+\infty} \mu(L-t) e^{-\lambda(t-L)} (\mu e^{-\lambda(t-L)} - g_p(t)) u(t-L) dt \quad (11)$$

Solving the equation $\frac{\partial I}{\partial \lambda} = 0$ and simplifying it

by some mathematical manipulation, results:

$$\mu = 4\lambda^2 e^{\lambda L} \int_L^{+\infty} (t-L) g_p(t) e^{-\lambda t} dt \quad (12)$$

Due to the presence of the step function $u(t-L)$ in (8), determining $\frac{\partial I}{\partial L}$ dose not seems to be simple. In

order to overcome this difficulty, let expand (8) which gives:

$$I = \frac{\mu^2}{2\lambda} - 2\mu e^{\lambda L} \int_L^{+\infty} g_p(t) e^{-\lambda t} dt + \int_0^{+\infty} g_p^2(t) dt \quad (13)$$

Now, the calculation of $\frac{\partial I}{\partial L}$ is doable and from (13)

we get:

$$\frac{\partial I}{\partial L} = 2\mu g_p(t) - 2\mu e^{\lambda L} \int_L^{+\infty} g_p(t) e^{-\lambda t} dt \quad (14)$$

Finally, the result of the equation $\frac{\partial I}{\partial L} = 0$ is:

$$g_p(L) = \lambda e^{\lambda L} \int_L^{+\infty} g_p(t) e^{-\lambda t} dt \quad (15)$$

By solving equation (10), (12), and (15) simultaneously one can obtain the optimal values of μ, λ , and L . Note that, in obtaining these equations $g_p(t)$, given in (6), has not been used. Therefore, these three equations are satisfied and used for any given process. That is, in fitting the FOPDT model based on the index function (4) for any process one must solve the equation (10), (12), and (15) where $g_p(t)$ is the impulse response of the under considering process.

4. Solving the nonlinear equations

To solve the nonlinear equations (10), (12), and (15) simultaneously seems to be a difficult task.

However Eq. (15) is free of the parameter μ and also it can be omitted between the equation (10) and (12). Left sides of the equation (10) and (12) are the same thus, by setting right sides of these equations equal to each other one gets:

$$\int_L^{+\infty} t g_p(t) e^{-\lambda t} dt = \left(\frac{1+2\lambda L}{2\lambda} \right) \int_L^{+\infty} g_p(t) e^{-\lambda t} dt \quad (16)$$

On the other hand, using (6) gives: (the first usage of the given $g_p(t)$)

$$\int_L^{+\infty} t g_p(t) e^{-(\lambda+a)t} dt = \int_L^{+\infty} \frac{kt^n}{(n-1)!} e^{-(\lambda+a)t} dt + \int_L^{+\infty} \frac{kt^{n-1}\psi}{(n-2)!} e^{-(\lambda+a)t} dt \quad (17)$$

Using part by part integration technique, the first term of the right hand of (17) is obtained as follow:

$$\int_L^{+\infty} \frac{kt^n}{(n-1)!} e^{-(\lambda+a)t} dt = \frac{kL^n e^{-(\lambda+a)L}}{(n-1)!(\lambda+a)} - \frac{k\psi L^{n-1} e^{-(\lambda+a)L}}{(n-2)!(\lambda+a)} + \frac{n}{\lambda+a} \int_L^{+\infty} g_p(t) e^{-\lambda t} dt \quad (18)$$

From (17) and (18) results:

$$\int_L^{+\infty} t g_p(t) e^{-\lambda t} dt = \frac{kL^n e^{-(\lambda+a)L}}{(n-1)!(\lambda+a)} - \frac{k\psi L^{n-1} e^{-(\lambda+a)L}}{(n-2)!(\lambda+a)} + \frac{n}{\lambda+a} \int_L^{+\infty} g_p(t) e^{-\lambda t} dt \quad (19)$$

Left sides of the equation (16) and (19) are the same thus, by setting the right sides of them equal to each other we have:

$$\int_L^{+\infty} g_p(t) e^{-\lambda t} dt = \frac{2k\lambda L e^{-(\lambda+a)L}}{(n-1)! \{(\lambda+a)(1+2\lambda L) - 2\lambda n\}} + \frac{2k\lambda L^{n-1} e^{-(\lambda+a)L}}{(n-2)! \{(\lambda+a)(1+2\lambda L) - 2\lambda n\}} \quad (20)$$

Substituting $\int_L^{+\infty} g_p(t) e^{-\lambda t} dt$ from (20) into (15) gives:

$$\lambda = \frac{a}{2n-1-2aL} \quad (21)$$

Now, by replacing λ from (21) into (15) the following equation is obtained:

$$(2n-1-2aL) \left\{ \frac{L^{n-1} e^{-aL}}{(n-1)!} + \frac{\psi L^{n-2} e^{-aL}}{(n-2)!} \right\} = a e^{\frac{aL}{(2n-1-2aL)}} \times \int_L^{+\infty} \frac{t^{n-2}}{(n-2)!} \left\{ \frac{t}{n-1} + \psi \right\} e^{\left(1+\frac{1}{2n-1-2aL}\right)at} dt \quad (22)$$

In Eq. (22) the values of a and n are known and the only unknown parameter of the equation is L . That is by solving equation (22) the optimal value of L is obtained.

By defining the parameter θ and the variable t' in the following form

$$\theta = aL, \quad t' = at - \theta \quad (23)$$

and with a little mathematical manipulation Eq. (22) can be written as follows:

$$(2n-1-2\theta)\theta^{n-2} \{L + \psi(n-1)\} = \int_0^{+\infty} (t' + \theta)^{n-2} e^{-\left(1+\frac{1}{2n-1-2\theta}\right)t'} dt' \quad (24)$$

Expanding $(t' + \theta)^{n-2}$:

$$(t' + \theta)^{n-2} = \sum_{m=0}^{n-2} \binom{n-2}{m} t'^m \theta^{n-2-m} \quad (25)$$

Where:

$$\binom{n-2}{m} = \frac{(n-2)!}{m!(n-2-m)!} \quad (26)$$

According to (25) the existed integration in Eq. (24) is obtained as follow:

$$\int_0^{+\infty} (t' + \theta)^{n-2} e^{-\left(1+\frac{1}{2n-1-2\theta}\right)t'} dt' = \sum_{m=0}^{n-2} \frac{(n-2)! \theta^{n-2-m}}{(n-2-m)! \left(1+\frac{1}{2n-1-2\theta}\right)^m} \quad (27)$$

From (24) and (27) results:

$$(2n-1-2\theta)\theta^{n-2} \{L + \psi(n-1)\} = \int_0^{+\infty} (t' + \theta)^{n-2} e^{-\left(1+\frac{1}{2n-1-2\theta}\right)t'} dt'$$

The above equation can be simplified in the following form:

$$f(\theta) = 0 \quad (28)$$

Where:

$$f(\theta) = \theta^{n-2} - \frac{1}{\{L + \psi(n-1)\}} \sum_{m=0}^{n-2} \frac{(n-2)!(2n-1-2\theta)^{m-1} \theta^{n-2-m}}{2^m (n-2-m)! (n-\theta)^m} \quad (29)$$

By solving Eq. (28) one can obtain θ . According to the definition of θ given in (23), θ must be positive. Having obtained θ , the value of L is determined

from $L = \frac{\theta}{a}$. Then, from (21) the value of λ is obtained. The achieved value of λ is acceptable if it

is positive, thus θ should be less than $\frac{2n-1}{2}$. That

means it is needed to look for the root of $f(\theta) = 0$ in

the interval $0 < \theta < \frac{2n-1}{2}$. Using some mathematical

manipulation the function $f(\theta)$ becomes to:

$$f(\theta) = \frac{(2(n-\theta)-1)}{2(n-\theta)} g(\theta) \quad (30)$$

Where:

$$g(\theta) = \theta^{n-2} - \frac{1}{\{L + \psi(n-1)\}} \sum_{m=0}^{n-2} \frac{(n-2)!(2n-1-2\theta)^{m-1} \theta^{n-2-m}}{2^m (n-2-m)! (n-\theta)^m} \quad (31)$$

Thus, it is needed to look for the root of the $g_n(\theta) = 0$ in the interval $\theta \in \left(0, \frac{2n-1}{2}\right)$. The following theorem is about the roots of the equation $\theta \in (0, n-0.5)$ in interval of $\theta \in (0, n-0.5)$.

Theorem1: Equation $g_n(\theta) = 0$ has a unique solution in the interval $\theta \in (0, n-0.5)$.

Proof:

From (31) it is achieved:

$$g(0) = -\frac{(n-2)!}{2n} \frac{1}{\{L + \psi(n-1)\}} \left(\frac{2n-1}{2n}\right)^{n-3} < 0 \quad (32)$$

$$g\left(\frac{2n-1}{2}\right) = \left(\frac{2n-1}{2n}\right)^{n-3} \left(\frac{2n-1}{2} - \frac{n-2}{\{L + \psi(n-1)\}}\right) > 0$$

$$\Leftrightarrow L + \psi(n-1) > 1 - 3 / (2n-1) \quad (33)$$

Remark1: we have $g\left(\frac{2n-1}{2}\right) > 0$ if only if

$$L + \psi(n-1) > 1 - 3 / (2n-1)$$

Therefore, the function $g_n(\theta) = 0$ at the beginning and the end points of the interval $(0, n-0.5)$ has two different signs. Beside $g_n(\theta) = 0$ is continuous in the interval $(0, n-0.5)$.

Consequently, in this interval, certainly $g_n(\theta) = 0$ has at least one root. Due to the lengthiness of the proof of root uniqueness, the rest of the proof is not brought here. This is end of the proof. ■

Let the normalized functions $g_{nN}(\theta) = 0$ to be defined in the following form:

$$g_{nN}(\theta) = \frac{g_n(\theta)}{g_{n \max}} \quad (34)$$

Where $g_{n \max}$ indicates the maximum value of the $g_n(\theta) = 0$ in interval of $(0, n-0.5)$.

The graphs of $g_{nN}(\theta) = 0$ for $n = 2$ are brought in Fig.1. This figure also illustrates that $g_{nN}(\theta) = 0$ and therefore $g_n(\theta) = 0$ has a unique root in interval of $(0, n-0.5)$.

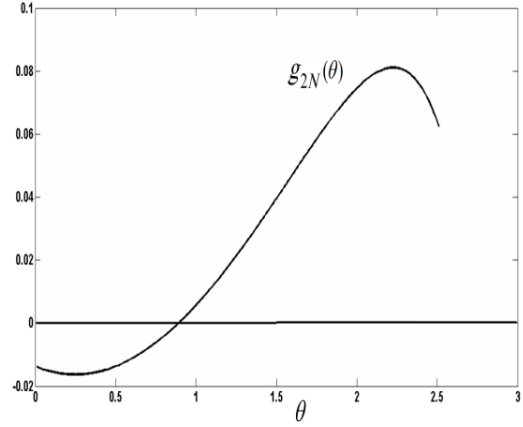


Fig.1. Plots of $g_{nN}(\theta) = 0$ versus θ (for $n = 2$).

By having n and employing f-zero command in MATLAB, one can easily solve equation $g_n(\theta) = 0$ in the interval $(0, n-0.5)$ and obtain the unique solution for $g_n(\theta) = 0$, which is denoted by θ^* . In table1 the obtained values of θ^* are brought for $n = 2$ to $n = 16$.

n	2	3	4	5	6
θ^*	1.34	1.87	3.9	4.12	5.5
n	7	8	9	10	11
θ^*	5.91	7.83	8.33	9.17	10.08
n	12	13	14	15	16
θ^*	11.67	11.84	13.11	14.89	15.38

Table 1. The root of $g_\theta(n) = 0$ in the interval $(n-0.5)$ (for $n = 2$ up to $n = 16$).

Having θ^* and considering equations (23), (25), (12), and (22) the optimal values of λ, L , and μ are obtained as follows:

$$\lambda^* = \frac{a}{2n-1-2\theta^*} \quad (35)$$

$$L^* = \frac{\theta^*}{a} \quad (36)$$

$$\mu^* = \frac{4k \lambda^{*2} L^{*n} e^{-aL^*}}{(n-1)! \{(\lambda^* + a)(1 + 2\lambda^* L^*) - 2\lambda^* n\}} \quad (37)$$

Theorem 2: The index function I , which is defined in (4), has a global minimum at point (μ^*, λ^*, L^*)

Proof:

It is omitted for limitation in the paper pages number. Here, the algorithm in the flowchart is shown.

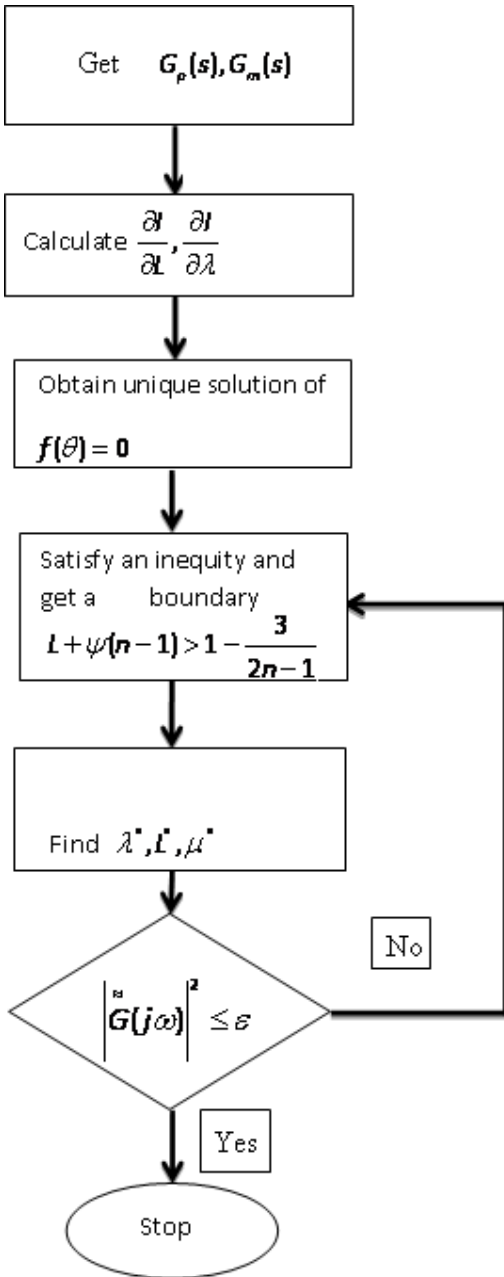


Fig. 2. Flow chart for optimal parameters.

5. Illustrative example

An eight order all pole process is given:

$$G_p(s) = \frac{-2s + 1}{(s + 1)^5}$$

Comparing with (3) we have:

$$a = 1, k = 1, n = 8, \psi = -2$$

From table 1 and equations (37-39) one gets:

$$L^* = 7.83, \lambda^* = 0.0908, \mu^* = 0.016$$

Therefore, the obtained FOPDT model for this process is:

$$G_m(s) = \frac{Ae^{-Ls}}{1 + Ts} = \frac{0.1762e^{-7.83s}}{1 + 11.013s}$$

The impulse responses of the process and the obtained FOPDT model are given in Fig.4. The square magnitude of $G(j\omega)$ is also shown in Fig. 5.

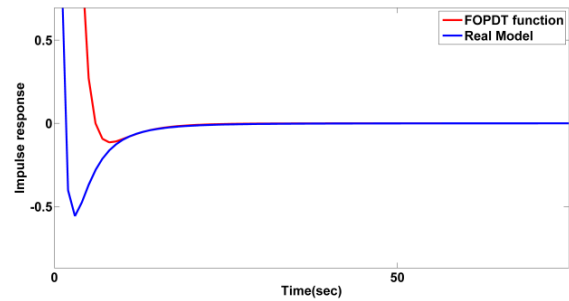


Fig. 4. Impulse responses of the process $(g_p(t))$ and the model $(g_m(t))$.

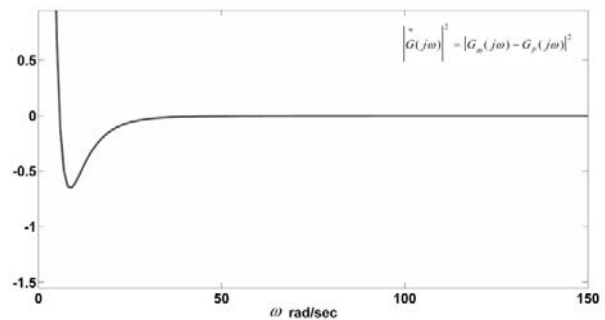


Fig.5. Plot of $\left| \tilde{G}(j\omega) \right|^2 = \left| G_m(j\omega) - G_p(j\omega) \right|^2$ versus ω .

Simulations shows that the settling time impulse response for FOPDT model is approximately 10 seconds, while for real model is about 20 seconds. In undershoot, when is used FOPDT model, more less than when it is implemented real model. For

example function, undershoot in FOPDT model is %89.51 less than in real model.

In the other hand, The area under the function error $\left| \overset{\approx}{G}(j\omega) \right|^2$ is $S = \int_0^{\infty} \left| \overset{\approx}{G}(j\omega) \right|^2 d\omega$ can be a good indicating index of the closeness of the transfer functions of $G_m(s)$ to $G_p(s)$, in this method much less than other method, this means that high similarity of presented method and real systems.

Process	The name of method	Obtained FOPDT model	$S = \int_0^{\infty} \left \overset{\approx}{G}(j\omega) \right ^2 d\omega$
$\frac{1-2s}{(1+s)^8}$	Area Method [1,2]	$\frac{0.86e^{-7.55s}}{1+6.8s}$	0.0473
	Direct Method [4,5]	$\frac{e^{-7.31s}}{1+5.98s}$	0.0432
	The method in [9]	$\frac{1.28e^{-7.31s}}{1+6.91s}$	0.0399
	The method in this paper	$\frac{0.1762e^{-7.83s}}{1+11.013s}$	0.0377

Table 2. Achieved results for $S = \int_0^{\infty} \left| \overset{\approx}{G}(j\omega) \right|^2 d\omega$ from other methods.

For the purpose of trade off of the presented procedure in this paper to the other methods, the obtained result from this paper for the under considered process with the obtained results from three other procedures are illustrated in Fig. 6 and Table 2.

According to the Fig.5, it is seen that the area under the function $\left| \overset{\approx}{G}(j\omega) \right|^2$ in the presented procedure is less than the others. Table 2 can prove this fact further.

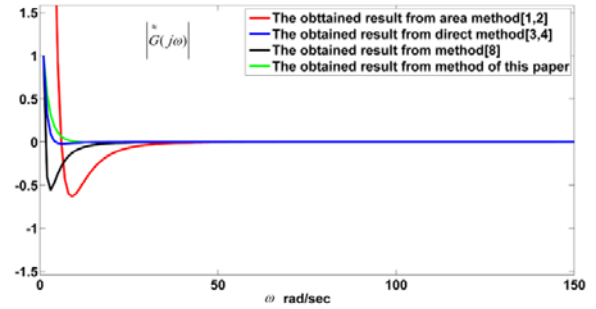


Fig. 5. Obtained results for the graph of $\left| \overset{\approx}{G}(j\omega) \right|^2$ versus ω for different methods.

6. Future work

The quality of the resulting presented method is very dependent on the quality of model estimate. The use of parameters optimizations and obtained area method is planned to be extended to less error between model estimate and real function, which is capable to better approximate a much wider class of systems. The method of state form to solving differential equations with initial value or used a data base cloud be the next work.

7. Conclusion

The problem of optimal FOPDT model fitting to n th-order all pole processes was formulated. It was shown that in general case the problem has a unique solution. By solving a set of nonlinear equations by determining the parameters of FOPDT model in terms of the process parameters. To verify the proposed scheme numerically, the model was compared with two other identification methods which are the area based and the directed identification techniques, and also compared with a rather complicated method which used genetic algorithm for identification. The proposed method shows better performance. This result is predictable because the showed technique is based on global optimization and but not on local optimization.

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