

CROSS-ENTROPY APPROACH AND GENETIC ALGORITHMS FOR PSS LOCATION AND TUNING

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Abstract—Mixed Integer Non Linear Programming (MINLP) via the cross-entropy approach for the optimal location and tuning of Power System Stabilizer (PSS) is presented in this paper.

The considered problem is to maximize the damping ratio of the global system under a minimum number of PSS and simultaneously to find out the best candidate machines to be equipped with PSSs. The damping ratio objective is achieved by tuning the controllers to shift the lightly damped and undamped electromechanical modes of all plants to a prescribed zone in the s-plane. This problem of tuning and location of PSS over a wide range of system configurations is formulated as a MINLP problem where the objective is the aggregation of the two objectives on the damping ratio and on the PSS's number. The mixed optimization problem, with a great combinatory aspect, is solved by an extension of the cross-entropy approach from the rare event framework.

The performance of this technique for damping the oscillations in multimachine power systems is confirmed through eigenvalues analysis over many scenarios. The proposed method is assessed by a comparative study with a previous approach based on the genetic algorithms.

Index Terms—Cross-Entropy, Rare event simulation, PSS, Genetic Algorithms, Damping Controller and Transient Stability.

1. Introduction

Modern power systems are more likely to reach stressed conditions than they used to because of the increasing electrical power system demand, and the new deregulated competitive environment which lead to operate them close to their limits of stability. Multiple interarea poorly damped oscillations can then appear more frequently.

In the last decades, PSS have been used by utilities in real power systems as they have proved to be the most cost-effective electromechanical damping control [2][3][4] and increasing the damping of oscillation modes by adequately tuning power system stabilizers (PSSs) has been the topic of many works. Generally the design of the PSS parameters was based on a single machine infinite bus (SIMB) power system model, considering the concepts of synchronizing and damping torque coefficients [1]. However, this procedure

considered that the PSS parameters were chosen to ensure the damping performance of local modes only and the simultaneous coordination of multiple PSSs for multimachine power systems was not attempted.

Several modern control techniques can be used to design different power system stabilizers. However, power systems companies prefer to choose lead-lag structure due to its simplicity and reliability in real power systems implementation. The simultaneous tuning of this type of stabilizers in order to increase their damping performance is very attractive.

Because of its complexity, this simultaneously tuning of PSS has been investigated by heuristic methods and many PSS tuning methods using genetic algorithms (GA) were presented in [9][10][11][12]. These methods investigated the use of GA in order to simultaneously tune the parameters of PSS with values that stabilize multimachine power system over a wide range of scenarios. For example, in [9][10] simultaneous tuning of PSS for the 16-machines and 68-bus power system model was performed in different scenarios. The objective function used for GA optimization was then to maximize the sum of eigenvalues damping and the maximum of the minimum eigenvalues dampings for all scenarios. In [12] a multiobjective optimization technique using GA with a lead lag PSS to set eigenvalues in the left-half side is described. All these papers demonstrated great advantages of using GA for robust PSS tuning. However they consider systems with fixed locations of PSS and different placement of PSSs makes the oscillation behaviors quite different at different scenarios. The PSS design procedure is then a sequence of selection of locations for example by the speed participation factors [17] and tuning in order to achieve optimal stabilization performance leading to a great number of optimal tuning.

A tempting solution is then to solve the problem of localization of PSS and their tuning simultaneously. This problem is a mixed integer (combinatory) non linear programming problem (MINLP) where the discrete variables are the candidate machines to be equipped with PSS, and the continuous variables are their gains. So, it is very hard to

elaborate a 'customized algorithms' which exploits the particular structure of the objective function as cutting planes [5] or branch-and-bound [6] for solving it. Several heuristic methods have been proposed such as genetic algorithms, tabu search, ant colony, and particular swarm to solve it, (see for example [7][8]). So, in [7][13], the authors tried to overcome the above problems by using minimum phase control method, fixing both damping ration and damping factor and minimizing the number of PSS introducing for each machine a control bit (1: PSS installed, 0: PSS not installed) to find out the location of PSS. A micro-GA combined with a hierarchical genetic algorithm (HGA) was used for this optimization and a good improvement in the damping has been achieved with one set of PSS parameters but a great simulation time is required.

The cross entropy method (CE) was proposed by R.Y. Rubinstein [14] for solving rare-event simulation problems and extended to solving combinatorial problems. It has been successfully applied in several engineering problem [15][16] and in this paper an extension of this approach for solving a MINLP is proposed and applied to solve the problem of tuning and location of PSS in power systems.

The proposed CE tuning procedure is tested on two power systems models, the Kundur's two area model which is an academic problem. The results are compared with those of the GA approach, and the nonlinear simulation and the eigenvalues analysis demonstrate the excellent improvement of the dynamic oscillations for all the studied scenarios.

2. Problem formulation

The problem of deciding on the location and tuning of the PSS while maximizing the damping ratio and minimizing the number of controllers to be installed is a multiobjective (MO) optimization problem. In the first part we will explain how it is changed in a single objective mixed optimization problem. Then we will propose to adapt the CE method to solve this problem and finally we will explain how the physical problem with the specific PSS is related to the optimization problem.

a. Problem statement

We propose to aggregate the problem of deciding on the location and tuning of the PSS while maximizing the damping ratio and minimizing the number of controllers, as the problem to find the decision variables that maximize:

$$\max S = (F - \alpha \cdot n_{PSS}) \quad (1)$$

Where F characterizes the global damping of the system. As it will be detailed in the next section this damping is linked to the eigenvalues of the linearized system for the various operating conditions.

Of course, the tuning of the aggregation ratio α requires a priori knowledge about the relative importance of the two objectives (minimum number of PSS under a great global damping) and will be of great importance for the final result.

Each controller is tuned with one parameter that is its gain. For simplicity of computations this gain will be considered as $k_{max}(1 - k_i)$ with $0 \leq k_i \leq 1$. The aim of the optimization is then to compute the optimal values for decision variables that maximize the problem specified by (1) i.e. the logical variables that model the location of the PSS and their gain specified by variables k_i .

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This problem is a very combinatory and complex with an implicit objective function, which depends on the evaluation of eigenvalues of a state matrix. It is then very difficult to solve using conventional methods as continuity of objective function cannot be established and Hessian cannot be obtained. We then propose to solve it using CE approach adapted to combinatory and continuous variables.

b. The Cross entropy method

In this section CE method in the rare-event simulation framework is recalled. Then, we explain how we adapt this method for solving Mixed Integer Non Linear Programming (MINLP) problems and finally, we present an algorithm for problems whose search spaces are $n \times m$ -dimensional mixed spaces (i.e. $\mathcal{X} \subset \mathbb{R}^n \times \{0, 1\}^m$ where, n is the number of the continuous variables, and m is the number of binary variables).

The CE method [14] is a method to evaluate the probability that a rare event occurs. Let X be a random variable taking its values in some space \mathcal{X} with a probability density function (pdf) $f(\cdot)$ and $S(\cdot)$ a real value function on \mathcal{X} . The problem is to evaluate $I = \mathbb{P}_{X \sim f}(S(X) \geq \gamma)$ where γ is a given threshold. An estimator of this probability is given by (2) where $I(S(X) \geq \gamma)$ is the indicator function¹ of the event and N is the number of samples.

$$\hat{I} = \frac{1}{N} \sum_{i=1}^N I(S(X_i) \geq \gamma) \quad (2)$$

If the probability to estimate is very low, this estimator requires a great number of samples. For example, in order to estimate $I = 10^{-6}$ with a relative error $\kappa = 0.01$, a number of samples $N \simeq \frac{1}{\kappa^2 I} = 10^{10}$, is required. The basic idea of CE is to introduce an importance sampling pdf $g(\cdot)$ to reduce the required number of samples. The new estimator given by (3) where the samples are drawn from the density $g(\cdot)$ can then be used.

$$\hat{I} = \frac{1}{N} \sum_{i=1}^N I(S(X_i) \geq \gamma) \frac{f(X_i)}{g(X_i)} \quad (3)$$

¹ The function $I(\text{logical expression})$ is defined by $I(\text{logical expression})$ if $\text{logical expression}$ is true and 0 otherwise.

Obviously the best estimator would be based on the ideal importance sampling pdf given by (4) the variance of which is 0 as l is constant but that can't be used as l is unknown and is to estimate.

$$\hat{g}(X) = \frac{I(S(X_i) \geq \gamma) f(X_i)}{l} \quad (4)$$

The main idea of the CE method for rare event simulation is to find inside an, a priori, given set \mathcal{G} of pdfs defined on \mathcal{X} , the element $g(\cdot)$ such that its distance with the 'ideal' sampling distribution is minimal.

A particularly convenient measure of distance between two pdfs $a(\cdot)$ and $b(\cdot)$ on \mathcal{X} is the Kullback-Leibler distance, which is also termed the cross-entropy between $a(\cdot)$ and $b(\cdot)$. The Kullback-Leibler distance, which is not a "distance" in the formal sense since it is, for example, not symmetric, is defined as follows:

$$\mathcal{D}(a, b) = E_{X \sim a(\cdot)} [\ln \frac{a(X)}{b(X)}]. \quad (5)$$

The CE method reduces the problem of finding an appropriate importance sampling pdf to the following optimization problem:

$$\arg \min_{g \in \mathcal{G}} \mathcal{D}(\hat{g}, g) \quad (6)$$

One can show through simple mathematical derivations that solving (6) is equivalent to solve (7) which does not explicitly depend on l anymore.

$$\arg \max_{g \in \mathcal{G}} E_{X \sim f(\cdot)} [I(S(X) \geq \gamma) \ln g(X)] \quad (7)$$

Using a given importance sampling with a pdf $h(\cdot)$ ($h \in \mathcal{G}$) it is possible to get an estimation of the solution of (7) by solving its stochastic counterpart (8) where the set of samples X_1, X_2, \dots, X_M is drawn according to $h(\cdot)$ and $W(x) = f(x)/h(x)$ is the *likelihood ratio*, between $f(\cdot)$ and $h(\cdot)$.

$$\arg \max_{g \in \mathcal{G}} \sum_{i=1}^M I(S(X_i) \geq \gamma) \ln g(X_i) W(X_i) \quad (8)$$

Under specific assumptions on \mathcal{X} , $f(\cdot)$ and \mathcal{G} it is possible to analytically solve (8). For example, when $\mathcal{X} \subset \mathbb{R}^n \times \{0, 1\}^m$ is the set of $n \times m$ -dimensional exponential-Bernoulli distributions with independent components specified by (9):

$$Dist_{n,m}(X, u) = \prod_{i=1}^n \frac{1}{u[i]} \exp(-\sum_{i=1}^m \frac{X[i]}{u[i]}) \cdot \prod_{i=n+1}^{n+m} u[i]^{X[i]} (1-u[i])^{1-X[i]} \quad (9)$$

where u is the vector of parameters and $X[i]$ the i^{th} component of X , and f also belongs to this set, the parameters of the solution of (8) are given by (10).

$$u[i] = \frac{\sum_{j=1}^M I(S(X_j) \geq \gamma) X_j[i] W(X_j)}{\sum_{j=1}^M I(S(X_j) \geq \gamma) W(X_j)} \quad (10)$$

Of course the solution of (8) is a better estimation of the solution of (7) when the number of samples such as $(S(X_i) \geq \gamma)$ is high. It is then necessary to adopt an iterative approach to compute the pdf h to favor the occurrence of the desired event. If l is very low it is even necessary to introduce an iteration that increases the value of γ .

This approach evaluates the probability that the function $S(X)$ is greater than or equal to a given value by drawing samples with a pdf that evolves in such a way that the number of event increases *i.e.* such as the value of $S(X)$ is great. It is then attractive to use the approach to evaluate the maximal value of the function.

Let us consider the single objective optimization problem (MINLP) specified by (11) where $\mathcal{X} \subset \mathbb{R}^n \times \{0, 1\}^m$ and $S(x)$ is a real value objective function. The basic idea of CE for SO optimization is to use a stochastic approach based on importance sampling in order to get a good evaluation of \hat{x} .

$$\hat{x} = \arg \max_{x \in \mathcal{X}} (S(x)) \quad (11)$$

Starting from an a priori pdf to draw samples the method iteratively computes series of pdf that increase the probability to draw samples near the global optimal solution. With respect to the previous problem the main difference is that the event that is used to iteratively compute the pdf is not given by the problem but has to be chosen. Generally this is done by choosing a given number k (k is the size of the elite) and considering that the relevant event is that the samples belong to the k better samples according to the objective function $S(x)$. At each step the new pdf is computed according to (12), where *Elite* is the set of the k best samples. When $\mathcal{X} \subset \mathbb{R}^n \times \{0, 1\}^m$, and $n \times m$ -dimensional distributions with independents components specified by (9) are chosen the parameters of the solution of (12) are given by a formula, adapted from (10) by fitting the indicator functions.

$$\arg \max_{g \in \mathcal{G}} \sum_{i=1}^M I(S(X_i) \in Elite) \ln g(X_i) W(X_i) \quad (12)$$

The time constant T_w in the washout stage is considered as a constant parameter classically equal to 10 s. The parameters

T_{1i} , T_{2i} , T_{3i} and T_{4i} are chosen in order that the phase lead provided by lead-lag parameters of the PSS should compensate as well as possible the phase lag between the exciter voltage reference and the generator electric power output. If the PSS exactly compensates this phase, it produces a torque on the generator shaft in antiphase to the generator speed, so, it acts as an ideal damper [3] [17]. In our experiments these parameters were tuned according to [17].

Then the parameters to be determined by the optimization method are the gains k_i of the controllers and their location.

3. Cases studies

In order to asses our tuning location method we have used two cases. In each of these studies the generator was represented by a sixth-order model.

a. Two area power system

The two-area power system is an academic test problem, it is elaborates to investigate the behaviors of electromechanical oscillations in power systems [2]. The basic topology is depicted in Fig. 4. The system contains eleven buses and two areas, connected by a weak tie between bus 7 and 9. Two loads are applied to the system at bus 7 and 9. Two shunt capacitors are also connected to bus 7 and 9 as shown in the figure. The fundamental frequency of the system is 60 Hz. All generators are equipped with simple exciters and have the same parameters. Two operating conditions with 400 MW of power flowing from area 1 to area 2 were analyzed:

- 1st operation condition: Two lines between bus 7 and 8
2nd operation condition: One line between bus 7 and 8

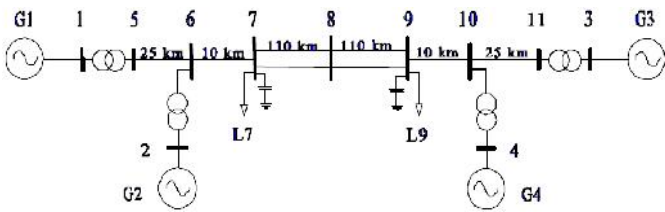


Fig. 4. Single line diagram of the two area power system

The linear analysis of this system around of the above operation points (Fig. 5) show that the system is highly stressed [10], there are two eigenvalues with positive real part.

The maximal value of PSS's gains (k_{max}) is equal to 10. In order to solve the problem described by equations (1), the Power System Toolbox (Cherry software) is used to build the linearized power system models and the Control System Toolbox for the construction of the closed loop MIMO system. In (1) the aggregation coefficient α is set equal to 5.

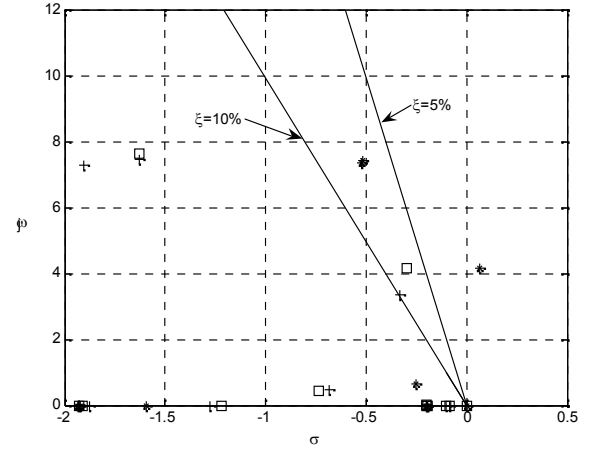


Fig. 5. Linear analysis (with CE and GA)
* w/o control + scenario #1 □ scenario #2

TABLE I

CONVERGENCE OF THE DISTRIBUTION PARAMETERS VECTOR

t	v(1) v(5)	v(2) v(5)	v(3) v(7)	v(4) v(8)	S_{max}	ξ_{min}
0	0.0100000 1/2	0.0100000 1/2	0.0100000 1/2	0.0100000 1/2	#	#
1	0.0002569 0.48	0.0048733 0	0.0027618 0.52	0.0020516 1	2.91605	7.08394
2	0.0009277 1	0.0042278 0	0.0225485 0	0.0009253 1	2.91252	7.08747
3	0.0000787 1	0.0016263 0	0.0149081 0	0.0002498 1	2.90665	7.09335
4	0.0000007 1	0.0099340 0	0.0072095 0	0.0000854 1	2.90560	7.09439
5	0.0000226 1	0.0108531 0	0.0040472 0	0.0000015 1	2.90525	7.09474
6	0.0000005 1	0.0001111 0	0.0020224 0	0.0000001 1	2.90518	7.09481
7	0.0000002 1	0.0129845 0	0.0121039 0	0.0000003 1	2.90518	7.09481
8	0.0000000 1	0.0000348 0	0.0103413 0	0.0000000 1	2.90518	7.09481
9	0.0000000 1	0.0042214 0	0.0046053 0	0.0000000 1	2.90518	7.09481
10	0.0000000 1	0.0092598 0	0.0056280 0	0.0000000 1	2.90518	7.09481

The optimization was performed with the size sample equal to 200, and the p -quantile ratio equal to 0.01. The distributions parameters are given in Table I (these outcomes are verified by ten runs), which confirms that the solution is stable after the sixth iteration. The best solutions sorted accordingly to these parameters are given in Table II:

The PSSs should be installed on generators 2 and 3, which is the same result if we use the speed components of the participation factor associated with the unstable interarea

mode (the mode $0.0670 \pm 4.1528i$ with the frequency 0.6609 Hz) ([10]).

The plot of the eigenvalues obtained from the two linearized model with the controller included is given in Fig. 5. It can be observed that the system is stable and sufficiently damped with all modes placed inside the 5 % damping wedge-shape sector.

TABLE II
PSS PARAMETERS WITH GA AND CE METHODS

No.	K_{PSS}		T_w	T_1	T_2	T_3	T_4
	With GA	With CE					
G1	10	10	10	0.05	0.015	0.08	0.01
G2	no PSS	no PSS	10	0.05	0.015	0.08	0.01
G3	no PSS	no PSS	10	0.05	0.015	0.08	0.01
G4	10	10	10	0.05	0.015	0.08	0.01
δ_{max}	-2.90518	-2.90518					
n_{PSS}	2	2					
ξ_{min}	7.09481	7.09481					
Time to reach 99% of the final value	178.85 s	85.97 s					
δ_{max}							
Iteration to reach 99% of the final value δ_{max}	31	6					

The optimized parameter can be also tested trough the non linear simulation. A three phase fault is applied on line 7 – 8 at 0.1 s, while power (400 MW) is flowing from area 1 to area 2. Then the near end of the line is opened at $t = 0.19$ s. Finally, the line is completely removed at 0.2 s. Fig. 6 shows the responses of the relative angles of this system; it can be observed that the system is well damped and stabilized in less than 12 s.

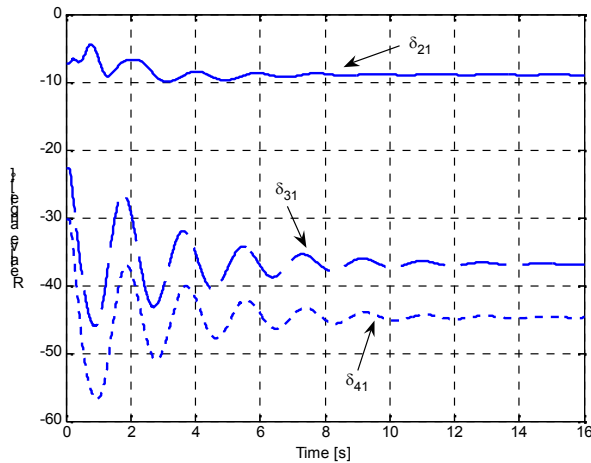


Fig. 6. Relative angles (identical for CE and GA)

The hierarchical genetic algorithm developed in [7], with all GA parameters taken from [8], has been used to asses these results. The generation evolution can be viewed in Fig. 7. The convergence of the GA is reached in less than 31 generations

which corresponds to 178.85 s. However, with the cross entropy approach the solution is reached in less than 6 iterations that corresponds to 85.97s (the simulation is performed with MATLAB (c) under Pentium 3 GHz 1 Go Ram). The execution time of the CE approach is half the GA method one

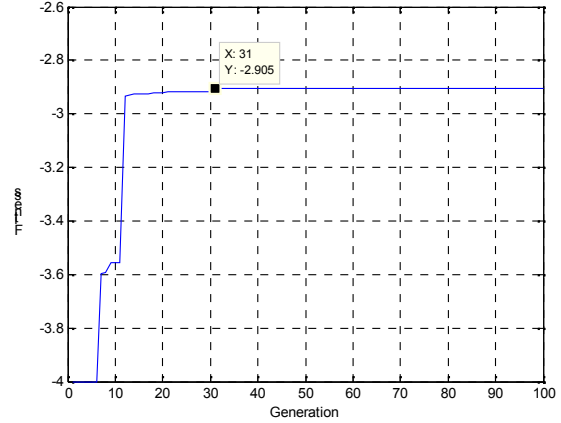


Fig. 7. Fitness – generation (two area power system)

4. Conclusion

In this paper, a new approach for solving the problem of tuning and location of the minimum numbers of PSSs using the cross entropy approach is presented. The main objective was to ensure a significant improvement of dynamical oscillations achieved by a minimum number of PSSs installed and tuned optimally.

The procedure was tested for the two area power systems over several scenarios. The obtained results show that the use of this simulation based method can find the optimal locations and the minimum controller's parameters faster than the hierarchical genetic algorithms.

Also, an original extension of the cross-entropy approach to resolve the mixed integer non linear programming was proposed. This new algorithm is suggested to be tested in other power systems applications where the 'customized algorithms' for the mixed integer nonlinear programming are hard to be developed.

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