Fuzzy NEURAL NETWORK OBSERVER-BASED ADAPTIVE SLIDING MODE CONTROL FOR THE TWO-LINK ROBOT

R.NAOUAL*, E.M. MELLOULI, I. BOUMHIDI

Department of Physics, LESSI Laboratory
Faculty of Sciences Dhar El Mehraz, University Sidi Mohamed Ben Abdellah, FES, Morocco
*Corresponding Author: rachidnaoual72@gmail.com

Abstract: *In this paper, an observer based on Fuzzy Neural Network (FNN) is combined with sliding mode to design an adaptive robust controller without chattering problems for the two-links robot with unknown dynamics and external disturbances. The adaptive fuzzy logic and neural network approaches are used to approximate the unknown models of the MIMO robot system. The observer design is based on the resulting improved model. Moreover the auxiliary sliding mode control term is included in the control law to attenuate respectively the FNN approximation errors and the external disturbances. The stability of the system is provided by the Lyapunov approach. The simulation results demonstrate the efficiency and robustness of the proposed control scheme*

Key words: *Nonlinear observer, Sliding mode control, Fuzzy logic, Neural network, Robot manipulators.*

1. Introduction

Thanks to their use in diversified scientific and technological domains, robots are considered among the vital topics to many researchers [1, 5, 8, 11, 15]. The design of robust control for robot manipulators remains the most challenging task in the area of control systems. Therefore, a robot manipulator is a complex nonlinear system, whose dynamic parameters are difficult to obtain precisely. In fact, it is almost impossible to obtain exact dynamic models; the system is described by a nominal model with large unknown dynamics, which would add more challenges to the control of robots. To overcome this problem, several methods have been proposed, including adaptive control [11, 13, 15], sliding mode control(SMC) [1, 2, 3, 6, 12], intelligent control [4, 8, 9, 13], etc. Sliding mode control has been proved to be successful in the control of nonlinear system. The main advantage of using sliding mode control is its strong robustness with respect to system uncertainties and external disturbances. However, in the presence of large uncertainties, the SMC suffers from the known chattering problem which is caused by the high gain and high speed switching control [1, 2, 10]. In order to alleviate the chattering phenomena, the fuzzy logic [4, 7, 9, 11, 16] and the neural network [8,14, 15] are used to approximate the unknown dynamics of the system.

In this paper, a combined adaptive fuzzy neural network and sliding mode controller are proposed for a two links robot with unknown dynamics and disturbances. The fuzzy-logic is used to estimate the unknown dynamics of the two-links robot model [11], so that the system uncertainties can be reduced which leads to a lower switching gain to be used. The neural network will also be used to estimate the model part [8, 15] in which it is difficult to apply the fuzzy logic, especially during the elaboration of the fuzzy membership functions. In fact, the Neural Network (NN) has an inherent learning ability and can approximate a nonlinear continuous function to arbitrary accuracy. To obtain the approximation of the model part, the network weights are adjusted during the online implementation by using the gradient descent method (GD). The proposed control consists of the so-called equivalent control added to the robust control term. The fuzzy neural network estimation terms are incorporated in the equivalent control component, enabling the robust component to be used with a small gain which is responsible of compensating only the fuzzy neural network errors estimation.

The Adaptive Fuzzy Neural Network Sliding Mode Controller (AFNNSMC) mentioned above has been developed using the full state. However, in this study, the velocity and acceleration are unavailable for measurement, so, a non linear observer is designed based on fuzzy neural network. The control parameters are updated online by the adaptive laws with stability and convergence analysis using the Lyapunov approach for achieving specified tracking performance.

The rest of the paper is organized as follows: In section 2, the model of robot manipulator is presented. The proposed observer-based Fuzzy neural network sliding mode controller is shown in section 3. The simulation results are demonstrated in section 4. A conclusion is conveyed in section 5.

2. The model of the two-link robot

The nominal model of the two-link robot is [5]:

$$\begin{cases} M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau \\ \dot{\tau}=Ji-B\tau-E\dot{q} \end{cases} \tag{1}$$

Where $q, \dot{q}, \ddot{q}$ are vectors that represent respectively the output position, velocity and acceleration. τ is the torque vector actuating the links, i is the current vector input applied to the servo motors. J, B and E are diagonal matrices representing the thermodynamic parameters and dependent on the temperature and the initial conditions.

$M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$: is an inertia matrix that is

symmetric and positive definite
where: $M_{11} = I_1 + I_2 + 4m_2 l_1^2 + 4m_2 l_1 l_2 \cos(q_2)$,

$M_{12} = I_2 + 2m_2 l_1 l_2 \cos(q_2)$,

$M_{22} = I_2$ and $M_{21} = I_2 + 2m_2 l_1 l_2 \cos(q_2)$

With: $q = \begin{bmatrix} q_1 & q_2 \end{bmatrix}^T$ the output positions, l_1, l_2 : the lengths, m_1, m_2 : the masses and I_1, I_2 the inertias respectively of the first and second segment of the pate.

$C(q, \dot{q}) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$: Represents the centrifugal forces

where:

$C_{11} = -2m_2 l_1 l_2 \dot{q}_2 \sin(q_2)$

$C_{12} = -2m_2 l_1 l_2 (\dot{q}_1 + \dot{q}_2) \sin(q_2)$

$C_{21} = 2m_2 l_1 l_2 \dot{q}_1 \sin(q_2)$

$C_{22} = 0$

$G(q)$ is The coriolis matrix given as:

$G(q) = \begin{bmatrix} m_2 g l_2 \sin(q_1 + q_2) + m_1 g l_1 \sin(q_1) \\ m_2 g l_2 \sin(q_1 + q_2) \end{bmatrix}$

In state space, the system model can be presented as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = f_1(\underline{x}) + g_{11}(x_1, x_4) u_1 + g_{12}(x_1, x_4) u_2 \\ \dot{x}_4 = x_5 \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = f_2(\underline{x}) + g_{21}(x_1, x_4) u_1 + g_{22}(x_1, x_4) u_2 \end{cases} \quad (2)$$

Where: $i = U = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T$
$q = \begin{bmatrix} x_1 & x_4 \end{bmatrix}^T, \dot{q} = \begin{bmatrix} x_2 & x_5 \end{bmatrix}^T, \ddot{q} = \begin{bmatrix} x_3 & x_6 \end{bmatrix}^T,$
$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T$

$f(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) & f_2(\underline{x}) \end{bmatrix}^T$
$= -M^{-1} \left\{ \begin{bmatrix} \dot{M} + C + BM^T \end{bmatrix} \begin{bmatrix} x_3 & x_6 \end{bmatrix}^T + \begin{bmatrix} \dot{C} + BC + E \end{bmatrix} \begin{bmatrix} x_2 & x_5 \end{bmatrix}^T + \begin{bmatrix} \dot{G} + BG \end{bmatrix} \right\}$

and

$M^{-1}(x_1, x_4) J = g_n(x_1, x_4) = \begin{pmatrix} g_{11n}(x_1, x_4) & g_{12n}(x_1, x_4) \\ g_{21n}(x_1, x_4) & g_{22n}(x_1, x_4) \end{pmatrix}$

3. Fuzzy neural network siding mode control design

The strategy of control is based on the fuzzy system, the neural network, the sliding mode control and the Lyapunov approach to ensure stability, tracking and consistent performance. The proposed observer based on Fuzzy-logic and Neural Network has been used to estimate the full state variable of the system.

3.1 Sliding mode control

The control problem is to determine the control law u to force the state $q(t)$ in order to track asymptotically the desired trajectory $q_d(t)$.
The tracking error of the system is defined as:
$e = q - q_d \quad (3)$
With $q_d = \begin{bmatrix} x_{1d} & x_{4d} \end{bmatrix}^T$
The relative degree of the system described by Equation (2) is r = 3.
Then, the sliding surface in the space of the error state can be defined as:
$S = \ddot{e} + \alpha \dot{e} + \beta e \quad (4)$

α, β are diagonal matrices defined as follows :

$\alpha = \begin{pmatrix} \alpha_{11} & 0 \\ 0 & \alpha_{22} \end{pmatrix} \quad , \quad \beta = \begin{pmatrix} \beta_{11} & 0 \\ 0 & \beta_{22} \end{pmatrix}$

Where α, β must satisfy the following Hurwitz polynomial.
The sliding variable derivative is:

$\dot{S} = f + gu - \begin{pmatrix} \dot{x}_{3d} \\ \dot{x}_{6d} \end{pmatrix} + \alpha \ddot{e} + \beta \dot{e} \quad (5)$

To guarantee the existence of sliding mode on a switching surface, the condition given below has to be satisfied [1]:

$$S^T \dot{S} < 0 \qquad (6)$$

The standard sliding mode control law that satisfies Eq. (6) can be given by:

$$\underline{u} = g^{-1}(\underline{x})\left(-f(\underline{x}) + \begin{pmatrix} \dot{x}_{3d} \\ \dot{x}_{6d} \end{pmatrix} - \alpha\ddot{e} - \beta\dot{e} - k\,sign(S)\right) \qquad (7)$$

Where $sign(.)$ is the sign function, given by:

$$sign(S) = \begin{cases} 1 & if & S>0 \\ 0 & if & S=0 \\ -1 & if & S<0 \end{cases} \qquad (8)$$

To eliminate the chattering effect caused by the discontinuous control law, we replace $sign(s)$ with $sat(s)$ as follows:

$$\underline{u} = g^{-1}(\underline{x})\left(-f(\underline{x}) + \begin{pmatrix} \dot{x}_{3d} \\ \dot{x}_{6d} \end{pmatrix} - \alpha\ddot{e} - \beta\dot{e} - k\,sat(S)\right) \qquad (9)$$

Where $sat(.)$ is the saturation function given by:

$$sat(s) = \begin{cases} s/\delta & \text{si } \|s\| < \delta \\ \\ sign(s) & otherwise \end{cases} \qquad (10)$$

With δ is the boundary layer thickness.

k is a large positive constant due to the large uncertainties which can produce the chattering phenomena. Thus, to reduce the system uncertainties, we will use the T-S fuzzy system to approximate $f(x)$ and the neural network to approximate $g(x)$, which can reduce the chattering phenomena.

3.2 Fuzzy unknown dynamics approximation design

The fuzzy system type T-S is used to approximate the system dynamics $f(x)$. The basic configuration of the T–S system includes a fuzzy rule base, which consists of a collection of fuzzy IF THEN rules in the following form:

$$R^j : if\ x_1\ is A_1^j\ and \ldots and\ x_n\ is\ A_n^j\ , then\ \hat{f}_l\ is \theta_{fl}^j\ \text{where:}$$

$j=1,\ldots,M$; $l=1,2$.

A_i^j Are fuzzy variables characterized by membership functions $\mu_{A_i^j}(x_i)$ and θ_{fl} is the corresponding value of the output fuzzy singleton. The output of the fuzzy system with singleton fuzzification produces inferences

and the defuzzification average centre can be expressed as [9]:

$$\hat{f}_l(\underline{x},\theta_{fl}) = \frac{\sum_{j=1}^{M}\theta_{fl}^j(\prod_{i=1}^{n}\mu_{A_i^j}(x_i))}{\sum_{j=1}^{M}(\prod_{i=1}^{n}\mu_{A_i^j}(x_i))} = \underline{\theta}_{fl}^T \xi^{fl}(\underline{x}) \qquad (11)$$

Where M presents the total number of the fuzzy rules and $\underline{\theta}_{fl} = [\theta_{fl}^1, \theta_{fl}^2, \ldots\ldots\ldots\theta_{fl}^M]^T$ is adjustable. $l=1,2. i=1,\ldots n$

$\xi^{fl}(\underline{x}) = [\xi_1^{fl}(x), \xi_2^{fl}(x), \ldots, \xi_M^{fl}(x)]^T$ is the vector of the fuzzy basis functions.

$$\xi_j^{fl} = (\prod_{i=1}^{n}\mu_{A_i^j}(x_i)) \Big/ \sum_{j=1}^{M}(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)). \qquad l=1,2.$$

The optimal parameters vectors and the fuzzy approximation error are defined as:

$$\underline{\theta}_{fl}^* = \arg\min_{\underline{\theta}_{fl}\in\Omega_f}\left(\sup_{\underline{x}\in R^n}\left|f_l(\underline{x}) - \hat{f}_l(\underline{x},\underline{\theta}_{fl})\right|\right)$$

$\underline{\theta}_{fl}$ Belong to the convex compact set which is

defined as $\Omega_f = \left\{\underline{\theta}_{\varphi fl} / \left|\underline{\theta}_{\varphi fl}\right| \leq M_l\right\}$

3.3 Neural Network design

The neural network is often used to approximate the unknown nonlinear function $g(x_1,x_4)$. In this work, we consider the architecture of NN with two layers of adjustable weights [8] (Fig. 1).

x_1, x_4: are the NN inputs variables, The NN outputs variables are:

$$y_1 = \hat{g}_{11}(x_1,x_4) \qquad y_2 = \hat{g}_{12}(x_1,x_4)\ and\ y_3 = \hat{g}_{22}(x_1,x_4) \qquad (12)$$

With $\hat{g}_{12}(x_1,x_4) = \hat{g}_{21}(x_1,x_4)$

Where:

$$y_k(x_1,x_4) = W_k^T\sigma(W_j^T[x_1\ \ x_4]^T)\ \ k=1,..4\ and\ j=1,..4 \qquad (13)$$

$\sigma(.)$ denotes the hidden-layer activation function, which is sigmoid function given by:

$$\sigma(s) = \frac{1}{1+e^{-s}} \qquad (14)$$

$W_k = [w_{k1}\ \ w_{k2}\ \ w_{k3}\ \ w_{k4}]^T$ represents the interconnection weights between the hidden and the output layers and $W_j = [w_{j1}\ \ w_{j4}]^T$ is the interconnection Weights between the input and the hidden layers.

The actual output $y_{dk}(x_1,x_4)$ (desired output, which is the difference between the actual and nominal functions) is:

$$y_{dk}(x_1,x_4) = y_k(x_1,x_4) + \varepsilon_k(x_1,x_4) \qquad (15)$$

Where: $\varepsilon_k(x_1,x_4)$ is the NN approximation error.

The gradient descent method (GD) is used to update

the network weights during online implementation. This method consists in adjusting iteratively the weights in the direction opposite to the gradient of E, so as to reduce the discrepancy based on:

$$\frac{\partial w_{kj}}{\partial t} = -\eta_k \frac{\partial E}{\partial w_{kj}} \qquad (16)$$

Where $\eta_k > 0$ is the usual learning rate. The gradient terms $\dfrac{\partial E}{\partial w_{kj}}$ can be derived using the back-propagation algorithm [14]. The cost function E is defined as the error index and the least square error criterion is often chosen as follows:

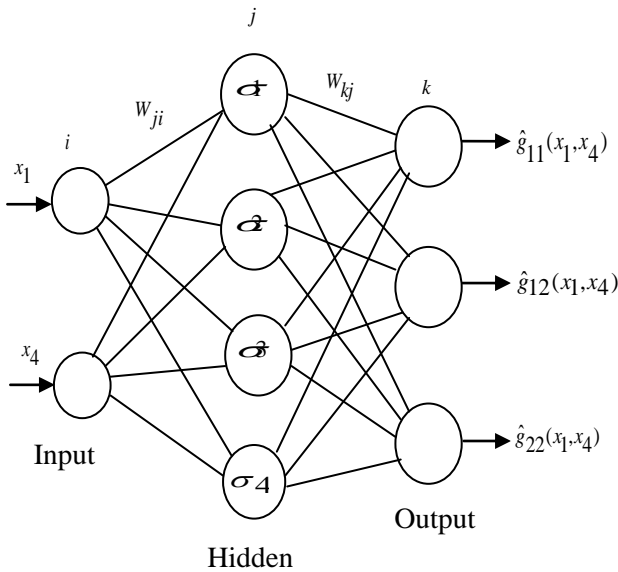$$E = \frac{1}{2}\sum_{k=1}^{4}\varepsilon_k^2 \qquad (17)$$

Fig.1. The architecture of a multilayer neural network for The prediction of the unknown nonlinear function $g(x_1, x_4)$

3.4 Fuzzy neural network based-non linear observer

The control mentioned above in Eq. (7) has been developed using the full state. However, in this study, the velocity and acceleration are unavailable for measurement. So, the fuzzy neural network observer is used to estimate the signals $x_i\,(i=2,3,5,6)$.

The proposed observer is based on [3, 6].

$$
\begin{cases}
\dot{\hat{x}}_1 = \hat{x}_2 + k_1 sign\,(x_1 - \hat{x}_1) \\
\dot{\hat{x}}_2 = \hat{x}_3 + k_2 sign\,(\tilde{x}_2 - \hat{x}_2) \\
\dot{\hat{x}}_3 = f_1(x_1, \tilde{x}_2, \tilde{x}_3, x_4, \tilde{x}_5, \tilde{x}_6) + g_{11}(x_1, x_4)u_1 \\
\qquad + g_{12}(x_1, x_4)u_2 + K_3 sign\,(\tilde{x}_3 - \hat{x}_3) \\
\dot{\hat{x}}_4 = \hat{x}_5 + k_4 sign\,(x_4 - \hat{x}_4) \\
\dot{\hat{x}}_5 = \hat{x}_6 + k_5 sign\,(\tilde{x}_5 - \hat{x}_5) \\
\dot{\hat{x}}_6 = f_2(x_1, \tilde{x}_2, \tilde{x}_3, x_4, \tilde{x}_5, \tilde{x}_6) + g_{21}(x_1, x_4)u_1 \\
\qquad + g_{22}(x_1, x_4)u_2 + K_6 sign\,(\tilde{x}_6 - \hat{x}_6)
\end{cases} \qquad (18)
$$

With $\tilde{x}_2 = \hat{x}_2 + K_1 sign\,(x_1 - \hat{x}_1)$, $\tilde{x}_3 = \hat{x}_3 + K_2 sign\,(x_2 - \hat{x}_2)$, $\tilde{x}_5 = \hat{x}_5 + K_4 sign\,(x_4 - \hat{x}_4)$ and $\tilde{x}_6 = \hat{x}_6 + K_5 sign\,(x_5 - \hat{x}_5)$

Where $\hat{x} = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5, \hat{x}_6]^T$ is the estimate of state vector $x = [x_1, x_2, x_3, x_4, x_5, x_6]^T$ and $k_i\,(i=1,...6)$ represents the observer gain that would be chosen such that the observer state $\hat{x}$ converges in finite time to x.

The functions f_1 and f_2 are approximated by the fuzzy logic design proposed in section 3.2 and the functions g_{11}, $g_{12}=g_{21}$ and g_{22} are approximated by the neural network design proposed in section 3.3.

Theorem 1. Considering the system described by Eq. (2) and the observer defined by Eq. (18) with unknown dynamics approximated by the proposed fuzzy neural network estimators, for any initial state $x(0)$, $\hat{x}(0)$ and any bounded input u there exists a choice of k_i such that the observer state $\hat{x}$ converges in finite time to x.

Proof:

Let's define the observation error dynamics as $e_{ob} = x - \hat{x}$. The system is assumed to be bounded input and bounded state in finite time. Consequently, the observation error state is also bounded. Now considering the initial state condition such that $x_1(0) \neq \hat{x}_1(0)$ and $x_4(0) \neq \hat{x}_4(0)$ (if not, we begin with the second step).

Step1:

Using a model system described by Eq. (2) and observer defined by Eq. (18), we obtain the dynamic error as:

$$
\begin{cases}
\dot{e}_{1ob} = e_{2ob} - k_1 sign\,(x_1 - \hat{x}_1) \\
\dot{e}_{2ob} = e_{3ob} - k_2 sign\,(\tilde{x}_2 - \hat{x}_2) \\
\dot{e}_{3ob} = f_1(x_1, x_2, x_3, x_4, x_5, x_6) - f_1(x_1, \tilde{x}_2, \tilde{x}_3, x_4, \tilde{x}_5, \tilde{x}_6) \\
\qquad - K_3 sign\,(\tilde{x}_3 - \hat{x}_3) \\
\dot{e}_{4ob} = e_{5ob} - k_4 sign\,(x_4 - \hat{x}_4) \\
\dot{e}_{5ob} = e_{6ob} - k_5 sign\,(\tilde{x}_5 - \hat{x}_5) \\
\dot{e}_{6ob} = f_2(x_1, x_2, x_3, x_4, x_5, x_6) - f_2(x_1, \tilde{x}_2, \tilde{x}_3, x_4, \tilde{x}_5, \tilde{x}_6) \\
\qquad - K_6 sign\,(\tilde{x}_6 - \hat{x}_6)
\end{cases} \qquad (19)
$$

Consider: $V_1 = \dfrac{e^2_{1ob}}{2}$ and $V_4 = \dfrac{e^2_{4ob}}{2}$

using Eq. (19) we have:

$\dot{V}_1 = e_{1ob}(e_{2ob} - k_1 sign(x_1 - \hat{x}_1))$

and $\dot{V}_4 = e_{4ob}(e_{5ob} - k_4 sign(x_4 - \hat{x}_4))$

If $k_1 > |e_{2ob}|_{max}$ then the observation errors e_{1ob} converge to zero in finite time τ_1. If this condition is maintained after τ_1 we have $\dot{e}_{1ob} = e_{2ob} - k_1 sign(x_1 - \hat{x}_1) = 0$ then $x_2 = \tilde{x}_2$.

If $k_4 > |e_{5ob}|_{max}$ then the observation errors e_{5ob} converge to zero in finite time τ_4. If this condition is maintained after τ_4 we have $\dot{e}_{4ob} = e_{5ob} - k_4 sign(x_4 - \hat{x}_4) = 0$ then $x_5 = \tilde{x}_5$.

We define τ_{14} as: $\tau_{14} = Max(\tau_1, \tau_4)$, after τ_{14} the observer dynamics become:

$$\begin{cases} \dot{e}_{1ob} = e_{2ob} - k_1 sign(x_1 - \hat{x}_1) = 0 \\ \dot{e}_{2ob} = e_{3ob} - k_2 sign(x_2 - \hat{x}_2) \\ \dot{e}_{3ob} = f_1(x_1, x_2, x_3, x_4, x_5, x_6) - f_1(x_1, x_2, \tilde{x}_3, x_4, x_5, \tilde{x}_6) \\ \qquad - K_3 sign(\tilde{x}_3 - \hat{x}_3) \\ \dot{e}_{4ob} = e_{5ob} - k_4 sign(x_4 - \hat{x}_4) = 0 \\ \dot{e}_{5ob} = e_{6ob} - k_5 sign(x_5 - \hat{x}_5) \\ \dot{e}_{6ob} = f_2(x_1, x_2, x_3, x_4, x_5, x_6) - f_2(x_1, x_2, \tilde{x}_3, x_4, x_5, \tilde{x}_6) \\ \qquad - K_6 sign(\tilde{x}_6 - \hat{x}_6) \end{cases} \quad (20)$$

Step 2:

Consider: $V_2 = \dfrac{e^2_{1ob}}{2} + \dfrac{e^2_{2ob}}{2}$ and $V_5 = \dfrac{e^2_{4ob}}{2} + \dfrac{e^2_{5ob}}{2}$

using Eq. (20), we have:

$\dot{V}_2 = e_{2ob}(e_{3ob} - k_2 sign(x_2 - \hat{x}_2))$ and $\dot{V}_5 = e_{5ob}(e_{6ob} - k_5 sign(x_5 - \hat{x}_5))$

If $k_2 > |e_{3ob}|_{max}$, then the observation errors e_{2ob} converge to zero in finite time $\tau_2 > \tau_{14}$. If the conditions on k_1, k_2 are maintained after τ_2, we have $\dot{e}_{1ob} = 0$ and $\dot{e}_{2ob} = e_{3ob} - k_2 sign(x_2 - \hat{x}_2) = 0$ then $x_3 = \tilde{x}_3$.

If $k_5 > |e_{6ob}|_{max}$, then the observation errors e_{6ob} converge to zero in finite time $\tau_5 > \tau_{14}$. If the conditions on k_4, k_5 are maintained after τ_5 we have $\dot{e}_{4ob} = 0$ and $\dot{e}_{5ob} = e_{6ob} - k_5 sign(x_5 - \hat{x}_5) = 0$ then $x_6 = \tilde{x}_6$.

We define τ_{25} as: $\tau_{25} = Max(\tau_2, \tau_5)$, after τ_{25} the observer dynamics become:

$$\begin{cases} \dot{e}_{1ob} = 0 \\ \dot{e}_{2ob} = e_{3ob} - k_2 sign(x_2 - \hat{x}_2) = 0 \\ \dot{e}_{3ob} = - K_3 sign(x_3 - \hat{x}_3) \\ \dot{e}_{4ob} = 0 \\ \dot{e}_{5ob} = e_{6ob} - k_5 sign(x_5 - \hat{x}_5) = 0 \\ \dot{e}_{6ob} = -K_6 sign(x_6 - \hat{x}_6) \end{cases} \quad (21)$$

Step 3:

Consider: $V_3 = \dfrac{e^2_{1ob}}{2} + \dfrac{e^2_{2ob}}{2} + \dfrac{e^2_{3ob}}{2}$ and $V_6 = \dfrac{e^2_{4ob}}{2} + \dfrac{e^2_{5ob}}{2} + \dfrac{e^2_{6ob}}{2}$

Using Eq. (21), we have: $\dot{V}_3 = e_{3ob}(-k_3 sign(x_3 - \hat{x}_3))$ and $\dot{V}_6 = e_{6ob}(-k_6 sign(x_6 - \hat{x}_6))$

If $k_3 > 0$, then the observation errors e_{2ob} converge to zero in finite time $\tau_3 > \tau_{25}$.

If $k_6 > 0$, then the observation errors e_{6ob} converge to zero in finite time $\tau_6 > \tau_{25}$.

3.5 Observer-based fuzzy neural network sliding mode control

The overall architecture of the observer-based fuzzy neural network sliding mode control is shown in Fig. 2. The proposed adaptive controller is given as:

$$u = \hat{u}^* + u_s \quad (22)$$

With $\hat{\underline{u}}^* = \hat{g}^{-1}(\hat{\underline{x}})(-\hat{f}(\hat{\underline{x}}, \theta) + \begin{pmatrix} \dot{\hat{x}}_{3d} \\ \dot{\hat{x}}_{6d} \end{pmatrix} - \alpha \ddot{\hat{e}} - \beta \dot{\hat{e}})$ \quad (23)

and $u_S = [u_{s1} \quad u_{s2}] = [-k_{s1} \, sat(s_1(\hat{x})) \quad -k_{s2} \, sat(s_2(\hat{x}))]$ \quad (24)

The selection of left the minimum approximation error as follow:

$$\begin{cases} w_{f_l} = f_l(\hat{x}, t) - \hat{f}_l(\hat{x}, \theta^*) \\ w_{g_{l1}} = (g_{l1}(\hat{x}_1, \hat{x}_4) - \hat{g}_{l1}(\hat{x}_1, \hat{x}_4)) u^*_1 \\ w_{g_{l2}} = (g_{l2}(\hat{x}_1, \hat{x}_4) - \hat{g}_{l2}(\hat{x}_1, \hat{x}_4)) u^*_2 \\ w_l = w_{f_l} + w_{g_{l1}} + w_{g_{l2}} \quad l = 1, 2. \end{cases} \quad (25)$$

We assume that:

1-There exist a positive constant such that:

$$\begin{cases} |w_{f_l}| < w_{f_{l max}} \\ |w_{g_{l1}}| < w_{g_{l1 max}} \\ |w_{f_l}| < w_{f_{l max}} \\ |w_{g_{l2}}| < w_{g_{l2 max}} \\ |w_l| < w_{l max} \quad\quad l = 1, 2. \end{cases} \quad (26)$$

2- The positive constant $k_S = [k_{s1} \quad k_{s2}]$ is chosen with respect to the following condition:

$$k_{s1} \geq \left|\frac{1}{g_{11}}\right|(w_{1max} + |g_{12}||u_{s2}|) \quad (27)$$

and

$$k_{s2} \geq \left| \frac{1}{g_{22}} \right| (w_{2\max} + |g_{21}||u_{s1}|) \qquad (28)$$

The adjustable fuzzy parameters of $\hat{f}(\hat{x},\theta)$ are tuned online using the Lyapunov approach. In order to guarantee that the parameters are bounded, we introduce the projection algorithm to restrict them in the closed set Ω_f.

$$\dot{\theta}_{f_1} = \begin{cases} -\gamma_1 S_1 \xi^{f1}(\hat{x}) & \text{if } (|\theta_{f1}| < M_1 \text{ or } (|\theta_{f1}| = M_1 \text{ and } \gamma_1 S_1 \xi^{f1}(\hat{x}) > 0 \\ -\gamma_1 S_1 \xi^{f1}(\hat{x}) + \gamma_1 S_1 \dfrac{\theta_{f1} \theta_{f1}^T \xi^{f1}}{|\theta_{f1}|^2} & \text{otherwise} \end{cases} \qquad (29)$$

$$\dot{\theta}_{f_2} = \begin{cases} -\gamma_2 S_2 \xi^{f2}(\hat{x}) & \text{if } (|\theta_{f2}| < M_2 \text{ or } (|\theta_{f2}| = M_2 \text{ and } \gamma_2 S_2 \xi^{f2}(\hat{x}) > 0 \\ -\gamma_2 S_2 \xi^{f2}(\hat{x}) + \gamma_2 S_2 \dfrac{\theta_{f2} \theta_{f2}^T \xi^{f2}}{|\theta_{f2}|^2} & \text{otherwise} \end{cases} \qquad (30)$$

γ_1 and γ_2 are positive constants.

Theorem 2. Considering that the non linear system described by Eq. (2) satisfies the assumptions 1) and 2). The observer-based fuzzy neural network sliding mode control law is chosen as $u = \hat{u}^* + u_s$. The closed-loop system is stable so that all the signals are bounded and tracking performance is achieved.

Proof:

Consider the following Lyapunov function:

$$V_1 = \frac{1}{2}\hat{S}_1^2 + \frac{1}{2\gamma_1}\Phi_{f_1}^T \Phi_{f_1} \quad \text{with} \quad \Phi_{f_1} = \theta_{f_1} - \theta_{f_1}^* \qquad (31)$$

$$\dot{V}_1 = \hat{S}_1 \dot{\hat{S}}_1 + \frac{1}{\gamma_1}\Phi_{f_1}^T \dot{\Phi}_{f_1} \qquad (32)$$

$$\dot{\hat{S}}_1 = \dot{\hat{x}}_3 - \dot{\hat{x}}_{3d} + \alpha_{11}\ddot{e}_1 + \beta_{11}\dot{e}_1$$

$$\dot{\hat{S}}_1 = f_1(\underline{\hat{x}},t) + g_{11}(\hat{x}_1,\hat{x}_4)u_1 + g_{12}(\hat{x}_1,\hat{x}_4)u_2 - \dot{\hat{x}}_{3d} + \alpha_{11}\ddot{e}_1 + \beta_{11}\dot{e}_1$$

Using Eqs. (22) and (23), the time derivative of $\dot{\hat{S}}_1$ may be expressed as:

$$\dot{\hat{S}}_1 = f_1(\underline{\hat{x}},t) - \hat{f}_1(\underline{\hat{x}},\theta) + (g_{11}(\hat{x}_1,\hat{x}_4) - \hat{g}_{11}(\hat{x}_1,\hat{x}_4))u^*_1 + (g_{12}(\hat{x}_1,\hat{x}_4) \\ - \hat{g}_{12}(\hat{x}_1,\hat{x}_4))u^*_2 + g_{11}(\hat{x}_1,\hat{x}_4)u_{s1} + g_{12}(\hat{x}_1,\hat{x}_4)u_{s2} \qquad (33)$$

Using Eqs. (11), (25) and (31) into Eq. (33):

$$\dot{\hat{S}}_1 = w_{f_1} + \Phi_{f_1}\xi^{f1}(\underline{\hat{x}}) + w_{g11} + w_{g22} + g_{11}(\hat{x}_1,\hat{x}_4)u_{s1} + g_{12}(\hat{x}_1,\hat{x}_4)u_{s2} \qquad (34)$$

By substituting Eq. (34) in Eq. (32) and using the update laws described by Eq. (29), $\dot{V}_1$ becomes:

$$\dot{V}_1 = \hat{S}_1(w_{f_1} + w_{g11} + w_{g22} + g_{12}(\hat{x}_1,\hat{x}_4)u_{s2}) + \hat{S}_1 g_{11}(\hat{x}_1,\hat{x}_4)u_{s1}$$

Using Eqs. (24), (25) and $g_{11}(\hat{x}_1,\hat{x}_4) > 0$, we obtain $\dot{V}_1$ as:

$$\dot{V}_1 \leq |\hat{S}_1|(w_{1\max} + |g_{12}(\hat{x}_1,\hat{x}_4)||u_{s2}|) - \hat{S}_1 g_{11}(\hat{x}_1,\hat{x}_4)k_{s1}sat(\hat{S}_1) \qquad (35)$$

Consider the following Lyapunov function:

$$V_2 = \frac{1}{2}\hat{S}_2^2 + \frac{1}{2\gamma_2}\Phi_{f_2}^T \Phi_{f_2}$$

$$\dot{V}_2 = \hat{S}_2 \dot{\hat{S}}_2 + \frac{1}{\gamma_2}\Phi_{f_2}^T \dot{\Phi}_{f_2}$$

with $\Phi_{f_2} = \theta_{f_2} - \theta_{f_2}^*$ and $\dot{\hat{S}}_2 = \dot{\hat{x}}_6 - \dot{\hat{x}}_{6d} + \alpha_{22}\ddot{e}_2 + \beta_{22}\dot{e}_2$

by applying the same previous steps, we obtain:

$$\dot{V}_2 \leq |\hat{S}_2|(w_{2\max} + |g_{21}(\hat{x}_1,\hat{x}_4)||u_{s1}|) - \hat{S}_2 g_{22}(\hat{x}_1,\hat{x}_4)k_{s2}sat(\hat{S}_2) \qquad (36)$$

For any $\varepsilon_l > 0$, if $\hat{S}_l > \varepsilon_l$ and $sat(\hat{S}_l) = sign(\hat{S}_l)$, applying Eqs. (27), (28) on Eqs. (35) and (36) gives $\dot{V}_1 < 0$ and $\dot{V}_2 < 0$.

However, in a small ε_l-vicinity of the origin (boundary layer), $sat(\hat{S}_l) = \dfrac{\hat{S}_l}{\varepsilon_l}$ is continuous and the system trajectories are confined to boundary layer of the sliding manifold $\hat{S}_l = 0$. $\quad l=1,2$
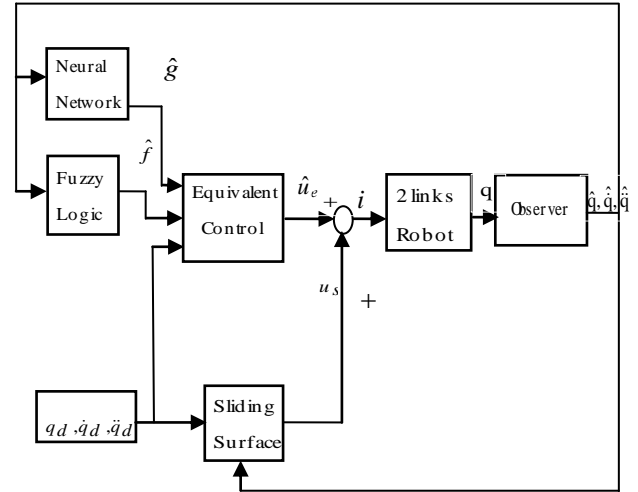

Fig.2 Observer-based fuzzy neural network sliding mode controller scheme.

4. Results and Discussion

This section presents the simulation results of the proposed control strategy compared with the standard sliding mode control for a two-links robot manipulator described by Eq. (2).

Where $B = \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$ and $E = \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

The considered uncertain parameters are the load variations considered as: $m_1 = 0.6 + 0.06*randn(t_0,t_f)$

$m_2 = 0.4 + 0.04*randn(t_0,t_f)$

and $J = \begin{pmatrix} 100 + 10*rand(t_0,t_f) & 0 \\ 0 & 100 + 10*rand(t_0,t_f) \end{pmatrix}$

with t_0 is the initial time and t_f is the final time.

The control objective is to maintain the system in order to track the desired angle trajectory: $x_{1d}=(\pi/3)\cos(t)$

and $x_{4d}=(\pi/3)\sin(t)$

The observer gain k_1, k_2, k_3, k_4, k_5 and k_6 respectively equal to 1, 0.7, 0.1, 1, 0.7 and 0.1. The switching functions coefficients are defined as:
$\alpha_{11}=14, \alpha_{22}=9, \beta_{11}=6$ and $\beta_{22}=16$

To construct the fuzzy logic system $\hat{f}_l(\hat{\underline{x}},\theta_{fl})$ ($l=1,2$) as given in Eq. (11). We select the membership functions for x_i ($i=1,2,3,4,5,6$) from the following fuzzy sets:

$\mu_{f_i^1}(\hat{x}_i)=\exp(-((\hat{x}_i-cn)/(2*sig))^2)$;

$\mu_{f_i^2}(\hat{x}_i)=\exp(-((\hat{x}_i)/(2*sig))^2)$;

$\mu_{f_i^3}(\hat{x}_i)=\exp(-((\hat{x}_i-cp)/(2*sig))^2)$;

with: $cp=1, cn=-1$ and $sig=4$. $(i=1,2,3,4,5,6)$

The sampling time is defined as 0.1 ms and the running time as 60s.

The performance of the observer sliding mode control is shown in Fig. 3.

The corresponding control current signals u_1 and u_2 are given in Fig. 4.

In Fig. 5 it can be seen that the good tracking position performance is obtained with unknown non linear dynamics and in the presence of disturbances, when compared to the standard sliding mode control.
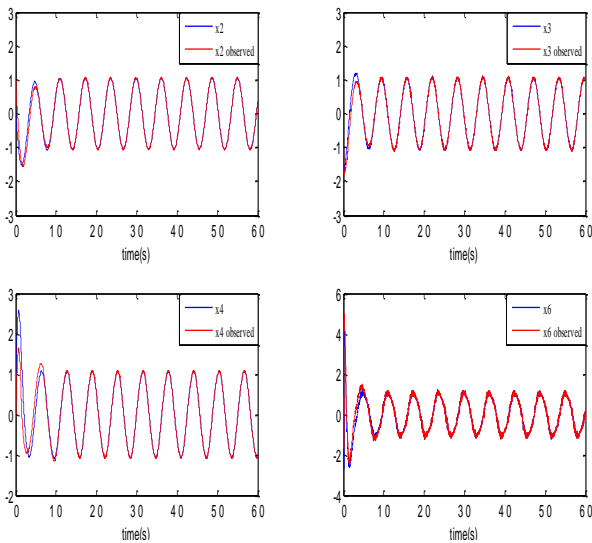

Fig.3.The Outputs x_2, x_3, x_5 and x_6 with their observer states $\hat{x}_2, \hat{x}_3, \hat{x}_5$ and $\hat{x}_6$.
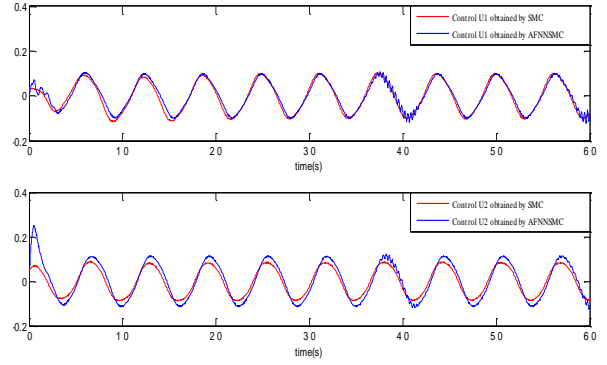

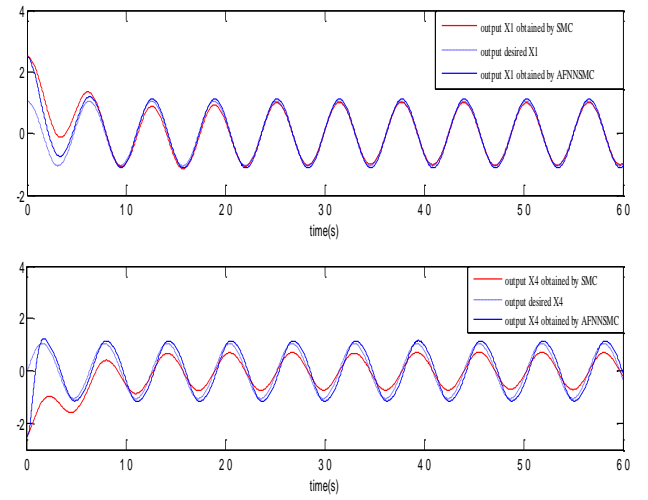Fig.4. the controls u_1 and u_2 using our method compared to SMC.

Fig.5. Responses of the desired output x_{1d} (——), x_{4d} (——) versus the output x_1 (——), x_4 (——) with AFNNSMC and x_1 (- - -), x_4 (- - -) with SMC.

5. Conclusion

In this paper, an observer-based indirect adaptive Fuzzy Neural Network sliding mode control scheme has been developed for the two-links robot manipulator.

The proposed observer based on Fuzzy-logic and Neural Network has been used to estimate the full state variable of the system, where the Fuzzy-logic and the Neural Network have been employed to approximate the nonlinear unknown model functions. This provides a better description of the plant, and hence leads to a lower switching gain to be used despite the presence of large uncertainties. Furthermore, the parameters can be tuned online by adaptive law based on Lyapunov synthesis. The simulation results illustrate a better performance of the proposed method to track the desired trajectory, compared to the standard sliding mode control.

References

1. Alaoui,C., Boumhidi, J., Mrabti, M.: *Sliding surface controller design based on the LMIs with application to a two links robot*. IJ-STA,Special Issue, CSC, 2007, p.300-311.

2. Basin, M., Rodriguez-Ramirez P.: *Sliding mode controller design for linear systems with unmeasured states*. Journal of the Franklin Institute. 349, 2012, p.1337–1349.

3. Barbot, J.P., Boukhobza, T., Djemai, M. : *Sliding mode observer for triangular input form*. Proceedings of the 35th Conference on Decision and Control Kobe, Japan, 1996, 1489-1490.

4. Bennassar, A., Barara, M., Abbou, A., Akherraz, M.: *performance evaluation of sliding mode control for induction motor based on fuzzy luenberger observer and kalman filter. Journal of Electrical and* Engineering , Vol 17, No.2, 2017, p.78-85.

5. Dawson, D.M., Qu, Z., Carroll, J.J. : *Tracking control of rigid-link electrically-driven robot manipulators*. International Journal of Control Vol 56, No.5, 1992, p.991–1006.

6. Drakunov, S., Utkin, V.: *Sliding mode observer*. Proceedings of the 34th Conference on Decision and Control New Orleans,LA, 1995, p.3376-3378.

7. Kung, C.C, Chen, T.H.: *Observer-based indirect adaptive fuzzy sliding mode control with state variable filters for unknown nonlinear dynamical systems*. Fuzzy Sets and Systems, 155, 2005. p.292-308.

8. Lewis, V.F.L., Jagannathan, S., Yesildirek, A.: *Neural Network Control of Robot Manipulators and Nonlinear Systems*, Taylor and Francis, London, 1999.

9. Lin, F.C., Yang, S.M.: *Adaptive Fuzzy Logic Based Velocity observer for Servo Motor Drives*. Mechatronics, 13, 2003, p.229-241.

10. Lin, P., Ma, J., Zheng, Z.: *Robust adaptive sliding mode control for uncertain nonlinear MIMO system with guaranteed steady state tracking error bounds*. Journal of the Franklin Institute. (in press).

11. Naoual, R., Mellouli, E.M., Sefriti, S., Boumhidi, I.: *Fuzzy sliding mode control for the two-link robot*. Int J. Systems, Control and Communications, Vol 6, No.1, 2014, p. 84-97.

12. Nasiri, A., Nguang, S.K. and Swain, A.: *Adaptive sliding mode control for a class of MIMO nonlinear systems with uncertainties*. Journal of the Franklin Institute. No.351, 2014, p. 2048–2061.

13. Sudheer, H., Kodad, SF., Sarvesh, B.: *Direct Torque and Flux control of Induction Machine using Fuzzy Logic controller*. Journal of Electrical and Engineering, Vol 17, No.2, 2017, p.122-128.

14. Rumelhart, D.E., Hinton, G.E. and Williams, R.J.: *Learning internal representations by error propagation,* D.E. Rumelhart, J.L. McClelland (Eds.), Parallel Distributed Processing, MIT Press, Cambridge, MA,1, p.318–362, 1986.

15. Sefriti, S., Boumhidi, J., Naoual, R. and Boumhidi,I.: *Adaptive Neural Network Sliding Mode Control For Electrically-Driven Robot Manipulators*. In:Control engineering and applied informatics, Vol 14, No.4, 2012, p.27-32.

16. Wang, L. X. :*Adaptive fuzzy systems and control: (designing and stability analysis)*. Prentice Hall, 1994.