IMPROVED MODAL TRUNCATION APPROXIMANT: A HYBRID APPROACH

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Abstract: *A hybrid method using modal truncation and singular perturbation to derive the reduced order model for a stable higher order system is presented. The approach is based on retaining the dominant modes of the system and truncating comparatively the less significant once. As the reduced order model has been derived from retaining the dominant modes of the original high order stable system, the reduced order model preserves the stability. The strong demerit of the modal truncation method is that, the steady state value of the reduced model does not match with the original system. This demerit has been overcome by applying singular perturbation approach of the balanced truncation method. Results obtained show the effectiveness of the proposed technique. Numerical examples are carried out to illustrate the procedure. Further it has been also applied to a system of order 1006 to check its applicability to large scale systems as well.*

Keywords: *Modal truncation, Singular perturbation, Steady state value, Eigen value, Large scale systems*.

1. INTRODUCTION

More and more accurate modelling of physical systems such as multi-machine power systems leads to a system with very high dimension. It is often called higher order system. Control design, simulation and analytical understanding of these high order systems become difficult and cumbersome. So the need arises to have the lower dimension representation of the system to better cope with the aforesaid problems.

Researchers have been trying for better approximation of the high order system nearly more than four decades which has now evolved as a promising pasture of research in system and control theory, known as "Model order reduction (MOR)". The target of the MOR is to have a model of considerably lower order for a system with very high order while keeping the key features of the original systems intact; such as stability and passivity etc. Moreover the algorithm or the method adopted should be reliable and computationally efficient. A number of methods for MOR are available in literature both in frequency domain as well as in time-domain [1-7].

One of the popular methods is modal truncation [1-4]. Different methods belonging to this category were also proposed [5,6]. Infect it is a mapping based on a sub-matrix of the full-order system's modal matrix. The elements at the columns of this matrix are its eigenvectors, which can be ordered according to their dominancy. The eigenvector with eigen-values closest to the jw-axis were considered dominant. Another novel approach to select dominant poles is suggested in [7] where the dominant pole is decided by considering the ratio of residue to the real value of pole. E. J. Davison suggested that a system with greater dimension can be represented by a model with lower dimension by considering the effects of the r most dominant eigen-values. The eigen values which were close to jw axis were retained and the eigen values farthest from jw axis were neglected.

It has been observed that the aforesaid methods were able to produce the transient response of the high order system quite well. But, as pointed out in [3] and rectified in [4], there is an error in the steady state value of the system response. An improvement in the method was proposed in [8]. In view of the basic model of Davison gave the deviation in steady state with original system, Chidambara, in his correspondence with Davison had suggested an alternative approach for model order reduction [3]. S. A. Marshall had also proposed an alternate way to compute the reduced order model [5]. The methods discussed above give a reduced order model which is an approximation of the original system. However, it was not clear that to what extent a given system can be reduced while representing a close approximation to the original high order system. It was observed that the proximity of the approximate model can be determined in terms of the principal eigen-value neglected, the size of the original plant, and the reduced plant. This criterion is judged by comparing the time responses of various low order systems. An approach for selecting the order of the model for Davison technique was proposed by Mahapatra [8].

In this present work the reduced order model has been obtained through modal truncation technique. The steady state mismatch has been avoided by utilizing singular perturbation technique. The author has proposed to utilize the singular perturbation method [9] of balance truncation to hybridize with modal truncation method. In singular perturbation the states are grouped into two modes slow and fast modes. The fast mode is made equal to zero thus getting the slow modes of the system along with the contribution of the fast modes also. This is also known as residualization. With this the steady state of the reduced system matches with the higher order system. The advantage of the proposed method is its applicability to large scale systems which has been validated through an example of a system of order 1006 [10].

Statement of the problem

Consider the linear time-invariant stable n^{th} order state space model described by following equations

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}\right\} \quad (1)$$

with the transfer function matrix

$$G(s) = C(sI - A)^{-1}B + D \quad (2)$$

where $x \in R^n$, $y \in R^m$, $u \in R^p$

the aim of the model order reduction is to obtain the low order representation described as

$$\left.\begin{aligned} \dot{x}_r &= A_r x_r + B_r u \\ y_r &= C_r x_r + D_r u \end{aligned}\right\} \quad (3)$$

with transfer function matrix

$$G_r(s) = C_r(sI - A_r)^{-1}B_r + D_r \quad (4)$$

where $x_r \in R^r$, $y_r \in R^m$ such that $y_r(t)$ is a close approximation to $y(t)$ for all input $u(t)$. Moreover the key features of the original system should be preserved in the reduced model such as stability.

2. PROPOSED MODEL REDUCTION ALGORITHM

The proposed algorithm is the result of hybridization of modal truncation [1, 11-13] and singular perturbation [9] approach. It consists of two sections namely modal truncation and singular perturbation.

2.1 Modal truncation

- State space realization of the high order system (HOS) as in (1) is obtained.
- All the Eigen values $\sigma(A)$ of the HOS are evaluated.
- Hankel singular values (HSV) of the HOS is computed and checked for comparatively significant HSV. The number of dominant HSV will be the order of reduction r.
- Modal matrix P is calculated and partitioned as below

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (5)$$

where P_{11} is a matrix of order $r \times r$, P_{22} is a matrix of order $(n-r)\times(n-r)$, P_{12} is a matrix of order $r\times(n-r)$ and P_{21} is a matrix of order $(n-r)\times r$.

- Modal inverse matrix Q which is inverse of matrix P is obtained and partitioned as below

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \quad (6)$$

where Q_{11} is a matrix of order $r \times r$, Q_{22} is a matrix of order $(n-r)\times(n-r)$, Q_{12} is a matrix of order $r\times(n-r)$ and Q_{21} is a matrix of order $(n-r)\times r$.

- Partition the matrices A, B, C and D as given below

$$\left.\begin{aligned} A &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\ B &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \\ C &= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \end{aligned}\right\} \quad (7)$$

Where A_{11} is a matrix of order $r \times r$, A_{22} is a matrix of order $(n-r)\times(n-r)$, A_{12} is a matrix of order $r\times(n-r)$ and A_{21} is a matrix of order $(n-r)\times r$

- Now after the completion of section (i) the reduced order model can be given as

$$\left.\begin{aligned} \hat{A}_{11} &= A_{11} + A_{12}P_{21}P_{11}^{-1} \\ \hat{B}_1 &= B_1 \\ \hat{C}_1 &= C_1 \\ \hat{D}_1 &= D \end{aligned}\right\} \quad (8)$$

Due to mismatch in DC gain of the reduced order model thus obtained with the original system, further singular perturbation is applied. This overcomes the DC gain mismatch problem. Singular perturbation steps are carried out in continuation to modal truncation steps. It has been explained in next sect

2.2 Singular Perturbation

Partitioned forms as above can be used to construct singular perturbation approximation [9]. The matrices of the final reduced system can be given as

$$\left.\begin{aligned} A_r &= \hat{A}_{11} - A_{12}A_{22}^{-1}A_{21} \\ B_r &= \hat{B}_1 - A_{12}A_{22}^{-1}B_2 \\ C_r &= \hat{C}_1 - C_2A_{22}^{-1}A_{21} \\ D_r &= \hat{D} - C_2A_{22}^{-1}B_2 \end{aligned}\right\} \quad (9)$$

The proposed algorithm is the result of hybridization of conventional modal truncation technique with singular perturbation approximation method so as to utilize the merits of the two methods. The technique has been applied to higher order systems which are discussed in the preceding section.

3. NUMERICAL EXPERIMENTS

3.1 Example-1

This example is taken from [9]. It is a continuous, linear, time-invariant and stable system of order four. The state-space matrices are described as follows

$$\left.\begin{aligned} A &= \begin{bmatrix} -0.43781 & 1.1685 & 0.41426 & 0.05098 \\ -1.1685 & -3.1353 & -2.8352 & -0.32885 \\ 0.41426 & 2.8352 & -12.4753 & -3.2492 \\ -0.05098 & -0.32885 & 3.2492 & -2.9516 \end{bmatrix} \\ B &= \begin{bmatrix} -0.11814 \\ -0.1307 \\ 0.05634 \\ -0.006875 \end{bmatrix} \\ C &= \begin{bmatrix} -0.11814 & 0.1307 & 0.05634 & -0.006875 \end{bmatrix} \\ D &= [0] \end{aligned}\right\} \quad (10)$$

The step response and bode diagram of the HOS is shown in Fig. 1 and Fig. 2 respectively.

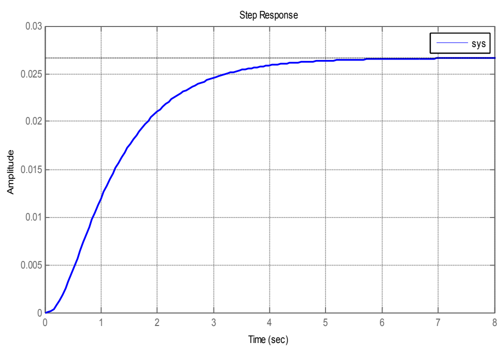

Fig. 1. Step response of the HOS

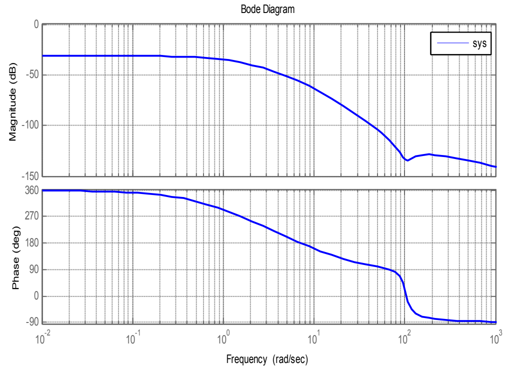

Fig. 2. Bode magnitude and phase plot of the HOS

The various steps to derive the reduced order model has been presented as follows.

The eigen-values of the HOS are given as $\sigma(A)$.

$\sigma(A) = \{-1, -3, -5, -10\}$

As all the poles lie in the left half of s-plane so the HOS is stable.

Henkel singular value of the HOS has been calculated and also plotted in Fig. 3, which gives the indication of the order of reduction. Comparatively the number of nonzero dominant singular values is taken as order of the reduction. Here, from Fig. 3, the first two singular values are significant and the last two singular values have become insignificant.

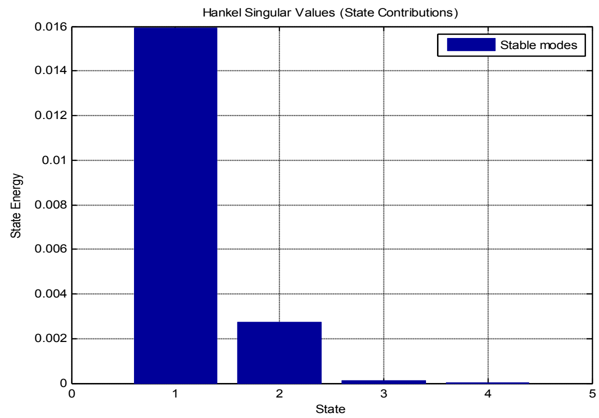

Fig. 3. Hankel singular values plot of the HOS

From the above plot order of the system to which it should be reduced is decided as 2.

The algorithm for modal truncation has been applied to the high order system (10).

The modal matrix P obtained is as:

$$P = \begin{bmatrix} -1.0314 & 0 & 0 & 0 \\ 0 & -2.8885 & 0 & 0 \\ 0 & 0 & -5.1070 & 0 \\ 0 & 0 & 0 & -9.9731 \end{bmatrix} \quad (11)$$

Partitioned matrices P_{11}, P_{12}, P_{21} and P_{22} are given as

$$P_{11}=\begin{bmatrix} -1.0314 & 0 \\ 0 & -2.8885 \end{bmatrix};\; P_{12}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$P_{21}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};\; P_{22}=\begin{bmatrix} -5.1070 & 0 \\ 0 & -9.9731 \end{bmatrix} \quad (12)$$

The inverse modal matrix obtained is as follows

$$Q=\begin{bmatrix} -0.9696 & 0 & 0 & 0 \\ 0 & -0.3462 & 0 & 0 \\ 0 & 0 & -0.1958 & 0 \\ 0 & 0 & 0 & -0.1003 \end{bmatrix} \quad (13)$$

Partitioned matrices Q_{11} ,Q_{12}, Q_{21} and Q_{22} are given as

$$Q_{11}=\begin{bmatrix} -0.9696 & 0 \\ 0 & -0.3462 \end{bmatrix};\; Q_{12}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$Q_{21}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};\; Q_{22}=\begin{bmatrix} -0.1958 & 0 \\ 0 & -0.1003 \end{bmatrix} \quad (14)$$

Now the partitioning the matrices A, B, C and D results into A_{11}, A_{12}, A_{21}, A_{22}, B_1, B_2, C_1 and C_2 shown as below.

$$A_{11}=\begin{bmatrix} -0.4378 & 1.1685 \\ -1.1685 & -3.1353 \end{bmatrix};\; A_{12}=\begin{bmatrix} 0.4143 & 0.0510 \\ -2.8352 & -0.3288 \end{bmatrix}$$
$$A_{21}=\begin{bmatrix} 0.4143 & 2.8352 \\ -0.5098 & -0.3288 \end{bmatrix};\; A_{22}=\begin{bmatrix} -12.4753 & -3.2492 \\ 3.2492 & -2.9516 \end{bmatrix} \quad (15)$$
$$B_1=\begin{bmatrix} -0.1181 \\ -0.1307 \end{bmatrix};\; B_2=\begin{bmatrix} 0.0563 \\ -0.0069 \end{bmatrix}$$
$$C_1=\begin{bmatrix} -0.1181 & 0.1307 \end{bmatrix};\; C_2=\begin{bmatrix} 0.0563 & -0.0069 \end{bmatrix}$$

Now applying the modal truncation only as given by (8) the reduced system matrices obtained are given as $sysr=\{A_r,B_r,C_r,D_r\}$

$$\left.\begin{aligned} A_r &= \begin{bmatrix} -0.4378 & 1.1685 \\ -1.1685 & -3.1353 \end{bmatrix} \\ B_r &= \begin{bmatrix} -0.1181 \\ -0.1307 \end{bmatrix} \\ C_r &= \begin{bmatrix} -0.1181 & 0.1307 \end{bmatrix} \\ D_r &= [0] \end{aligned}\right\} \quad (16)$$

The step response of the reduced system with high order system is compared in the Fig. 4, which is seen to be close approximant of the HOS. From the Fig. 4, it is observed that the steady state value of the reduced system differs from the original high order system.

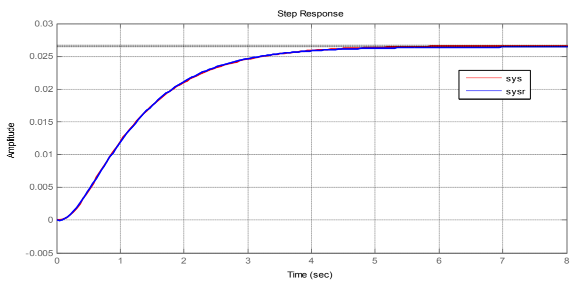

Fig. 4. Step response of reduced system with HOS using modal truncation only

Further the proposed algorithm ie singular perturbation (9) hybridized with modal truncation (8) has been applied to the high order system (10). The reduced system matrices obtained are given as $sysr1=\{A_{r1},B_{r1},C_{r1},D_{r1}\}$

$$\left.\begin{aligned} A_{r1} &= \begin{bmatrix} -0.4249 & 1.2565 \\ -1.2565 & -3.7355 \end{bmatrix} \\ B_{r1} &= \begin{bmatrix} -0.1164 \\ -0.1427 \end{bmatrix} \\ C_{r1} &= \begin{bmatrix} -0.1166 & 0.1412 \end{bmatrix} \\ D_{r1} &= \begin{bmatrix} 2.1019\times10^{-4} \end{bmatrix} \end{aligned}\right\} \quad (17)$$

The comparison of reduced order system with the original HOS is shown in Fig. 5. From the Fig. 5, it is clear that the response of reduced order system is a better approximation by proposed technique than the modal truncation method applied alone.

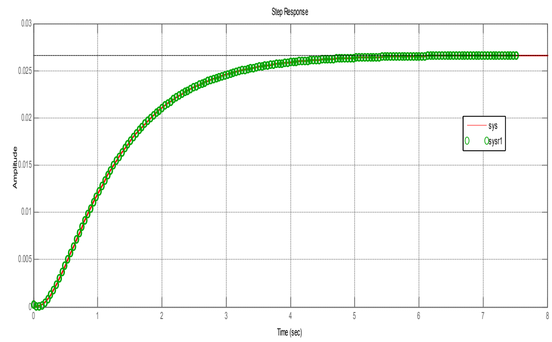

Fig.5. Step response of reduced system with HOS

The comparative responses have been plotted in Fig.6. It clearly shows that with the application of modal truncation alone there is steady state mismatch while with the singular perturbation applied as well the offset has been overcome. Thus it reveals that by the proposed technique the reduced system obtained to be a far superior approximation of HOS than the modal truncation method applied alone.

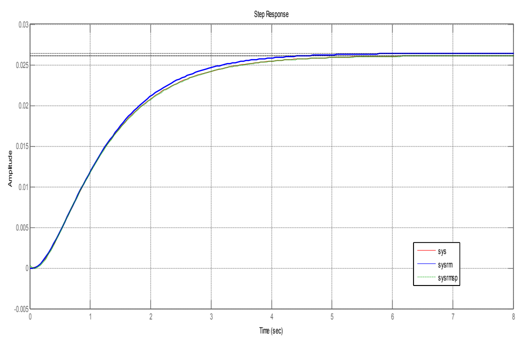

Fig.6. Comparision of response of reduced system with HOS by proposed algorithm and modal method

Integral Square Error (ISE) has been calculated to show the effectiveness of the approach. It is defined as

$$ISE = \int e^2(t)\,dt \quad \text{where } e(t) = y(t) - y_r(t) \qquad (18)$$

$y(t)$ is the step response of high order system and $y_r(t)$ is the step response of reduced order system.
The ISE has been obtained for the above two cases. The table shows the values obtained.

Table.1. Integral Square Error

S.No.	Method used	ISE
1	Modal Truncation	1.27×10^{-6}
2	Proposed Approach	6.21×10^{-8}

ISE value by the proposed approach is much less as compared to modal truncation method. So it confirms the effectiveness of the proposed approach.

3.1 Example-2

This example is taken from [10]. It is a dynamical system of order 1006. The state-space matrices are given by

$$\left.\begin{array}{l} sys = \{A, B, C, D\} \\ A = \begin{bmatrix} A_1 & 0 & 0 & 0 \\ 0 & A_2 & 0 & 0 \\ 0 & 0 & A_3 & 0 \\ 0 & 0 & 0 & A_4 \end{bmatrix} \\ A_1 = \begin{bmatrix} -1 & 100 \\ -100 & -1 \end{bmatrix}; A_2 = \begin{bmatrix} -1 & 200 \\ -200 & -1 \end{bmatrix}; A_3 = \begin{bmatrix} -1 & 400 \\ -400 & -1 \end{bmatrix}; \\ A_4 = -diag(1,2,\ldots,1000) \\ B^T = C = \left[\underbrace{10,10,\ldots,10}_{6}\underbrace{1,1,\ldots,1}_{1000}\right]; D = [0] \end{array}\right\} \quad (19)$$

The step response and bode diagram of the original high order system is shown in Fig.7. and Fig.8. respectively.

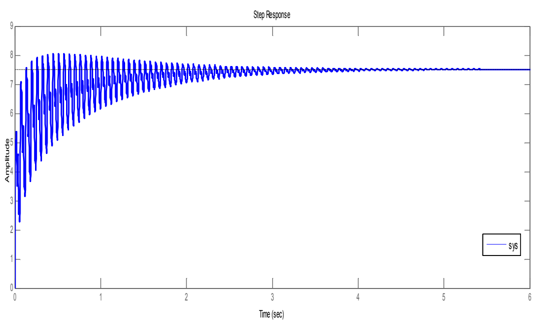

Fig.7. Step response of the high order system (19)

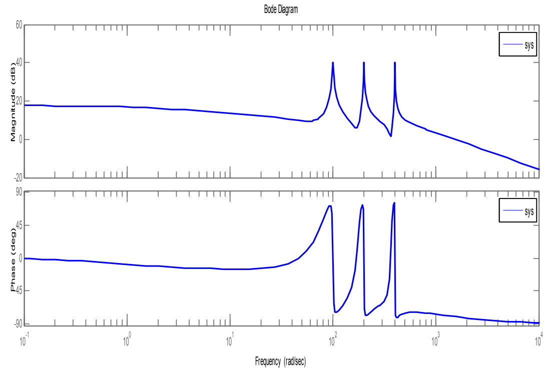

Fig.8. Bode magnitude and phase plot of the HOS (19)

The eigen-values of the high order system are given as $\sigma(A)$ and corresponding pole zero plot is given in Fig.9.

$$\sigma(A) = \{-1, -2, \ldots, -1000, -1 \pm 100j, -1 \pm 200j, -1 \pm 400j\}$$

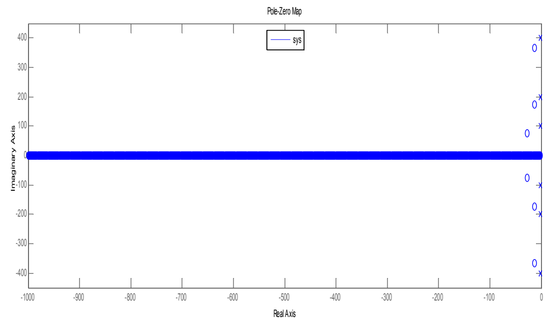

Fig.9. Pole-zero plot of the high order system (19)

Henkel singular value of the high order system has been calculated and also plotted in Fig.10, which is indication of the order of reduction. Comparatively the number of nonzero dominant singular values is taken as order of the system to be reduced. Here the first eight singular values are significant and after that ninth singular values onwards have decayed sharply [14], becoming insignificant. So the order of reduction is decided as eight.

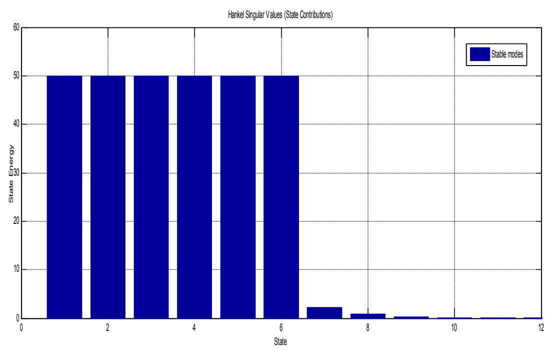

Fig.10. Henkel singular value plot of the HOS (19)

From the plot of Fig.11., order of the system to which it should be reduced is decided as 8.

The Modal truncation method alone has been applied to the high order system (19). The reduced system matrices obtained are given as $sysr = \{A_r, B_r, C_r, D_r\}$

$$\left.\begin{aligned}
A_r &= \begin{bmatrix} -1 & 100 & 0 & 0 & 0 & 0 & 0 & 0 \\ -100 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 200 & 0 & 0 & 0 & 0 \\ 0 & 0 & -200 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 400 & 0 & 0 \\ 0 & 0 & 0 & 0 & -400 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix} \\
B_r &= \begin{bmatrix} 10 & 10 & 10 & 10 & 10 & 10 & 1 & 1 \end{bmatrix}^T \\
C_r &= \begin{bmatrix} 10 & 10 & 10 & 10 & 10 & 10 & 1 & 1 \end{bmatrix} \\
D_r &= \begin{bmatrix} 5.9855 \end{bmatrix}
\end{aligned}\right\} \quad (20)$$

The step response of the reduced system (20) with high order system (19) is compared in the Fig.11, which is seen to be approximant of the system with dc gain mismatch.

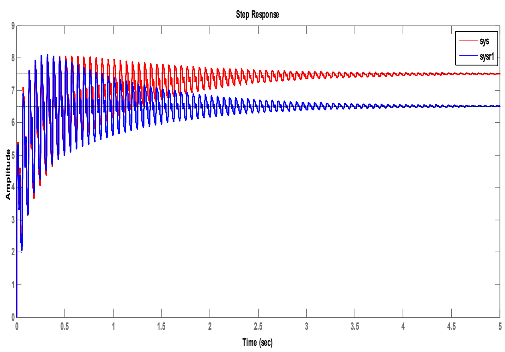

Fig.11. Step response of reduced system with high order system using Modal truncation only

Frequency response of the reduced system with the higher order system is also presented in Fig.12, which reveals the reduced system to be close approximation of the high order system. Fig.12 shows deviation at very high frequency which is equivalent to high initial transient behaviour as shown in Fig.12.

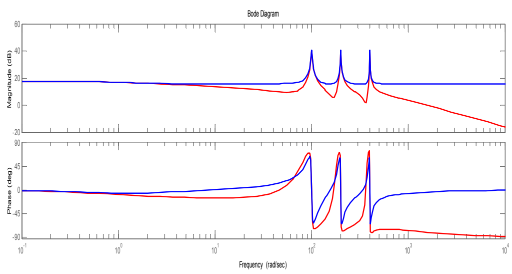

Fig.12. Frequency response of reduced system with high order system using modal truncation only

Further, proposed approach has been applied to the system. The reduced system matrices obtained are given as $sysr1 = \{A_{r1}, B_{r1}, C_{r1}, D_{r1}\}$

$$\left.\begin{aligned}
A_{r1} &= \begin{bmatrix} -6.112 & 1.276 & -80.26 & 0.3737 & -69.88 & -180.3 & 9.427 & 9.229 \\ 1.276 & -0.2662 & 361.8 & -0.07799 & 44.53 & 81.91 & -1.968 & -1.927 \\ 80.26 & -361.8 & -0.0003607 & 12.95 & -0.0004454 & -0.00143 & 0.0791 & 0.07371 \\ 0.3737 & -0.07799 & -12.95 & -0.02285 & 154.6 & 50.48 & -0.5769 & -0.5648 \\ 69.88 & -44.53 & -0.0004454 & -154.6 & -0.00055 & -0.001766 & 0.09769 & 0.09104 \\ 180.3 & -81.91 & -0.00143 & -50.48 & -0.001766 & -0.005669 & 0.3137 & 0.2924 \\ 9.427 & -1.968 & -0.0791 & -0.5769 & -0.09769 & -0.3137 & -90.56 & -120.5 \\ 9.229 & -1.927 & -0.07371 & -0.5648 & -0.09104 & -0.2924 & -120.5 & -189.3 \end{bmatrix} \\
B_{r1} &= \begin{bmatrix} 24.73 \\ -5.159 \\ -0.1899 \\ -1.511 \\ -0.2344 \\ -0.7526 \\ -19.91 \\ -19.03 \end{bmatrix} \\
C_{r1} &= [24.73{-}5.1590.1899{-}1.5110.23440.7526{-}19.91{-}19.03] \\
D_{r1} &= [5.9855]
\end{aligned}\right\}$$

(21)

The step response of the reduced system (21) with high order system (19) is compared in the Fig.13, which is seen to have close approximant with steady state matching from the original high order system.

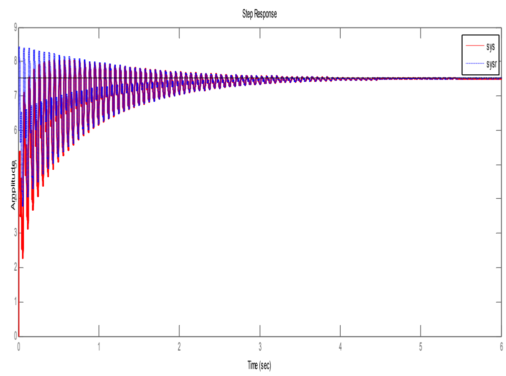

Fig.13. Step response of reduced system with high order system by proposed technique

Frequency response of the reduced system with the higher order system is also shown in Fig.14, which reveals the reduced system to be close approximation of the high order system.

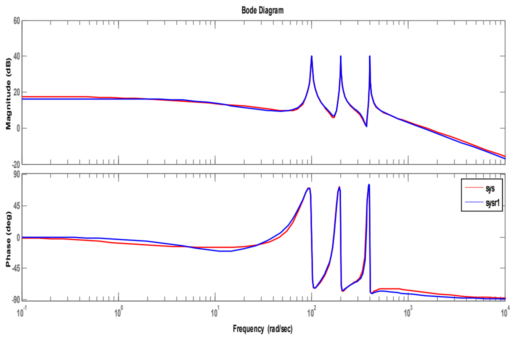

Fig.14. Frequency response of reduced system with high order system by proposed technique

Further another step response of the reduced system with the high order system for one second has also been shown in Fig.15 to clearly view the transient behaviour comparison. From Fig.15, the response show considerable deviation from the high order response but it lasts for 0.03 seconds only. This short high dynamic behaviour, if ignored the rest of the response is quite close.

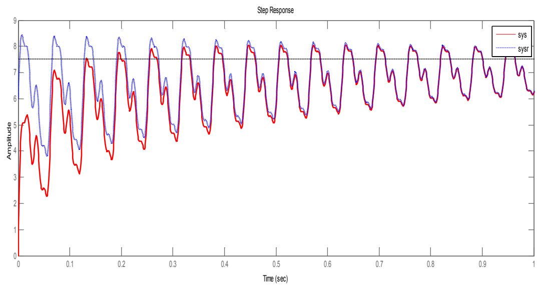

Fig.15. Step response of reduced system with high order system for short duration (one second)

Integral Square Error (ISE) has been calculated to show the effectiveness of the approach. The ISE obtained ignoring the short transient is 0.1966, which ensures the effectiveness of the method.

4. CONCLUSION

A hybrid method utilizing modal truncation and singular perturbation approximation approach has been applied successfully to determine the reduced replica of a high order system. One of the important demerits of the modal truncation method is dc-gain mismatch i.e., steady state value of the reduced order system do not agree with high order system. This demerit has been removed in the proposed approach. Further the method has been tested on high order system of order 1006. The results obtained confirm the validity of the approach. Applicability to large scale systems makes the method more advantageous. The high frequency transients show approximation error though it is of very short duration which is open for further investigation.

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APPENDIX

```
% Program for the Modal Truncation and
Singular Perturbation

% Defining FOM real time model from
benchmark examples
A1=[-1 100;-100 -1]
A2=[-1 200;-200 -1]
A3=[-1 400;-400 -1]
A4=diag(-1:-1:-1000)
A=blkdiag(A1,A2,A3,A4)
C1a=[10 10 10 10 10 10]
C2a=ones(1,1000)
C=[C1a C2a]
B=C'
D=[0]

% Eigen analysis and Hankel singular
values
eig(A)
sys=ss(A,B,C,D)
hankelsv(sys)

% Defining Example-1 4th order model
A=[-0.43781 1.1685 0.41426 0.05098;
    -1.1685 -3.1353 -2.8352 -0.32885;
    0.41426 2.8352 -12.4753 -3.2492;
    -0.5098 -0.32885 3.2492 -2.9516]
B=[-0.11814;-0.1307;0.05634;-0.006875]
C=[-0.11814 0.1307 0.05634 -0.006875]
D=[0]

% Algorithm for Modal truncation in
time domain
A=input('A=')
B=input('B=')
C=input('C=')
D=input('D=')
E=eigs(A)
M=modreal(A)
n=size(M)
r=input('r=')
M1=M(1:r,1:r)
M2=M(1:r,r+1:n)
M3=M(r+1:n,1:r)
M4=M(r+1:n,r+1:n)
N=inv(M)
A11=A(1:r,1:r)
A12=A(1:r,r+1:n)
A21=A(r+1:n,1:r)
A22=A(r+1:n,r+1:n)
B1=B(1:r,:)
B2=B(r+1:n,:)
C1=C(:,1:r)
C2=C(:,r+1:n)

% Applying Only Modal Truncation
Ar=A11+A12*M3*inv(M1)
Br=B1
Cr=C1
Dr=D
Dr=D-C*inv(A)*B+Cr*inv(Ar)*Br

% Applying Singular Purturbation to
match steady state
A11n=A11+A12*M3*inv(M1)
Ar=A11n-A12*inv(A22)*A21;
Br=B1-A12*inv(A22)*B2;
Cr=C1-C2*inv(A22)*A21;
Dr=D-C2*inv(A22)*B2;

% Ploting rsponse and comparision
sys=ss(A,B,C,D)
sysr=ss(Ar,Br,Cr,Dr)
fig1=figure
step(sys,'r',sysr,'bo')
fig2=figure
bode(sys,'r',sysr,'bo')

% ISE CALCULATION..
t=0:0.1:6;
[y,t]=step(sys,t)
[y1,t]=step(sysr,t)
e=y-y1
j=e.^2
ISE=sum(j)
% END of program%
```