AN EFFICIENT DECOUPLED POWER FLOW FOR RADIAL DISTRIBUTION SYSTEMS

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Abstract: *This paper presents an efficient decoupled power flow based on line power flows with a view to obtain a reliable convergence and higher computational speed for radial distribution systems. The real and reactive line powers are combined using simple multiplying factors such that the modified set is decoupled into two set of equations without making any assumption on* r/x *ratios. The proposed method is simple and uses a constant sparse sub-jacobian matrix that needs to be factorised only once for both the decoupled sub-problems; and is solved iteratively similar to FDPF technique. This method is applied on three test systems to illustrate its performance.*

Key words: distribution systems, radial power flow, decoupling

Nomenclature

BNPF	Branch-to-Node matrix based Power Flow
FDPF	Fast Decoupled Power Flow
FDGPF	Fast Decoupled G-matrix method for Power Flow
FDDPF	Fast Decoupled Distribution Power Flow
g and f	vector of modified real and reactive set of functions respectively
$G_{km} + jB_{km}$	real and imaginary terms of bus admittance matrix corresponding to *k*-th row and *m*-th column
GS	Gauss-Seidel
H	constant sub-jacobian matrix
m	branch connected between nodes *k* and *m*
NR	Newton and Raphson
nc	not converged in 50 iterations
nn	number of nodes in the system
PM	Proposed Method
$P_m + jQ_m$	real and reactive power at the receiving end of branch-*m*
$P_{L-m} + jQ_{L-m}$	real and reactive power load at node-*m*
$r_m + jx_m$	resistance and reactance of the distribution line-*m*
V_m and δ_m	voltage magnitude and voltage angle at node-*m* respectively
δ_{km}	$\delta_k - \delta_m$
α and β	vector of multiplying factors
Ψ	set of lines leaving node-*m*
ΔP_m and ΔQ_m	real and reactive power mismatches at the receiving end of branch-*m* respectively
ΔV_m and $\Delta\delta_m$	correction of voltage magnitude and voltage angle at node-*m* respectively
Δf and Δg	vector of modified real and reactive set of mismatches respectively

1. Introduction

Distribution Automation Systems (DAS) have evolved both in concept and implementation over a period of time. The distribution power flow has influenced other applications such as network optimisation, VAR planning and switching. The distribution systems, characterized by their prevailing radial nature and high r/x ratio, render them to be ill-conditioned and make the traditional Newton-Raphson (NR) [1] and fast decoupled power flow (FDPF) [2] solution techniques unsuitable. Consequently many power flow algorithms specially suited for distribution systems have emerged and are well documented [3-27]. These methods are roughly viewed as node based and branch based methods. The first category has used node voltages or current injections as state variables and requires information on the derivatives of network equations. The Z-bus method [3], NR based algorithms [4-8] and FDPF based algorithms [9-12] have revolved around this group. The second category has adopted branch currents or branch powers as state variables and involves only basic circuit laws. The backward/forward sweep based methods [13-22] and loop impedance [23] based methods have belonged to this group. However, the formulation and the algorithm are different from the NR technique, rendering this category to be unsuitable for other applications such as optimal power flow, state estimation, etc., for which the former seems to be more appropriate. Although there are many research papers discussing distribution power flow methods, a generalised distribution power flow method is yet to be developed. There is therefore a significant need for developing a specific fast power flow algorithm exclusively for distribution systems.

The objective of this paper is to formulate a robust decoupled power flow algorithm based on line power in order to obtain a faster and reliable convergence. The real and reactive sets of line powers are combined using simple multiplying factors such that the modified set is decoupled into two set of equations without making any assumption on r/x ratios; and is solved similar to FDPF iterative technique. The proposed method is applied on three test systems to illustrate its superior performance and the results are presented.

2. Proposed decoupled power flow

The algorithm is based on real and reactive powers at the receiving end of each branch instead of injected real and reactive powers at each node like in traditional methods. These real and reactive powers and their functions are combined using simple multiplying factors such that the modified set is automatically decoupled into two set of equations without any assumption on line r/x ratios. The resulting jacobian matrices of decoupled set of equations are sparse, constant and identical.

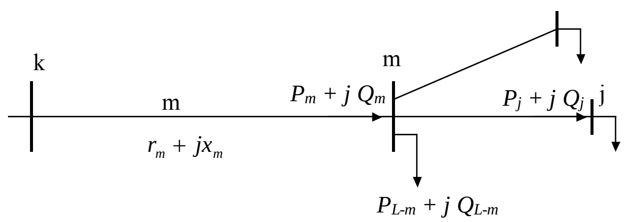

Fig.1 Sample Distribution Line

The equivalent real and reactive powers, P_m and Q_m, at the receiving end of branch-*m*, shown in Fig.1, can be computed from the specified load powers by the following recursive set of equations,

$$P_m = P_{L-m} + \sum_{j\in\Psi}\left\{\left(\frac{P_j^{\,2}+Q_j^{\,2}}{V_j^{\,2}}\right) r_j + P_j\right\} \quad (1)$$

$$Q_m = Q_{L-m} + \sum_{j\in\Psi}\left\{\left(\frac{P_j^{\,2}+Q_j^{\,2}}{V_j^{\,2}}\right) x_j + Q_j\right\} \quad (2)$$

where Ψ is a set of lines leaving node-*m*.

The expressions for P_m and Q_m of branch-*m* can be written as

$$P_m(V,\delta) = V_m^{\,2} G_{km} + V_m V_k \left[B_{km}\sin\delta_{km} - G_{km}\cos\delta_{km}\right] \quad (3)$$

$$Q_m(V,\delta) = -V_m^{\,2} B_{km} + V_m V_k \left[G_{km}\sin\delta_{km} + B_{km}\cos\delta_{km}\right] \quad (4)$$

The above two equations can be combined to form modified real, g, and imaginary, f, set of expressions using factors α_m and β_m as

$$g_m = \alpha_m P_m + \beta_m Q_m \quad (5)$$
$$f_m = -\beta_m P_m + \alpha_m Q_m \quad (6)$$

The Eqs. 5 and 6 are linearised around a known operating point of δ^o and V^o,

$$\begin{bmatrix} \frac{\partial g}{\partial \delta} & \frac{\partial g}{\partial V} \\ \frac{\partial f}{\partial \delta} & \frac{\partial f}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} \Delta g \\ \Delta f \end{bmatrix} \quad (7)$$

Where

$$\Delta g_m = (\alpha_m \Delta P_m + \beta_m \Delta Q_m)/V_m$$
$$\Delta f_m = (-\beta_m \Delta P_m + \alpha_m \Delta Q_m)/V_m \quad (8)$$
$$\Delta P_m = P_m - P_m(V^o, \delta^o)$$
$$\Delta Q_m = Q_m - Q_m(V^o, \delta^o)$$

$\frac{\partial g}{\partial \delta}, \frac{\partial g}{\partial V}, \frac{\partial f}{\partial \delta} \,\&\, \frac{\partial f}{\partial V}$ are the derivatives of functions g and f with respect to δ's and V's respectively

If the off-diagonal blocks $\frac{\partial g}{\partial V} \,\&\, \frac{\partial f}{\partial \delta}$ of the jacobian matrix of Eq. (7) are made zero, the problem can be decoupled into two sub-problems. It is observed that all the terms in the off-diagonal sub-matrices can be made zero, when $\alpha_m = B_{km}$ and $\beta_m = G_{km}$.

The derivatives of Eq. (7) can be written, while substituting $\alpha_m = B_{km}$ and $\beta_m = G_{km}$. with the assumptions $V_k \approx V_m \approx 1.0\ p.u$, $\cos\delta_{km} \approx 1.0$ and $\sin\delta_{km} \approx 0$, as

$$\frac{\partial g_m}{\partial \delta_k} = G_{km}{}^2 + B_{km}{}^2 \qquad \frac{\partial g_m}{\partial V_k} = 0$$
$$\frac{\partial g_m}{\partial \delta_m} = -\left(G_{km}{}^2 + B_{km}{}^2\right) \qquad \frac{\partial g_m}{\partial V_m} = \frac{\partial g_m}{\partial V_j} = 0$$
$$\frac{\partial g_m}{\partial \delta_j} = 0 \qquad \frac{\partial f_m}{\partial V_k} = \left(G_{km}{}^2 + B_{km}{}^2\right) \quad (9)$$
$$\frac{\partial f_m}{\partial \delta_k} = \frac{\partial f_m}{\partial \delta_m} = \frac{\partial f_m}{\partial \delta_j} = 0 \qquad \frac{\partial f_m}{\partial V_m} = -\left(G_{km}{}^2 + B_{km}{}^2\right)$$
$$\frac{\partial f_m}{\partial V_j} = 0$$

It is observed from Eq. (9) that the diagonal submatrices, $\frac{\partial g}{\partial \delta} \,\&\, \frac{\partial f}{\partial V}$, are sparse, constant and identical. Eq. (7) can be decoupled into two subproblems as

$$[H][\Delta\delta] = [\Delta g] \quad (10)$$
$$[H][\Delta V] = [\Delta f] \quad (11)$$

where

$$H = \frac{\partial g}{\partial \delta} = \frac{\partial f}{\partial V}$$

Equations 10 and 11 can be solved iteratively similar to FDPF algorithm. It should be noted that the sub-jacobian matrix H needs to be factorised only once during the iterative process.

The algorithm of the proposed method is summarized as follows

1. Read the network and load data.
2. Initialise all node voltages.
3. Compute the sub-jacobian matrix, H and factorise it
4. Compute Δg *and* Δf using Eq. (8).
5. Solve Eq. (10) for $\Delta\delta$.
6. Solve Eq. (11) for ΔV.

7. Check for convergence. i.e., check whether all the values in ΔV and $\Delta\delta$ are sufficiently small in magnitude. If not converged, update the voltages

$$V = V + \Delta V$$

$$\delta = \delta + \Delta\delta$$

and go to step (4)

8. Stop

3. Simulation

The proposed algorithm is tested to evaluate its solution accuracy and computational efficiency on 15, 29 and 69 node distribution systems [13,14,28] using a Pentium-IV, 2 GHz, Personal Computer. The convergence sensitivity of the proposed method (PM) to r/x ratio of the distribution lines is also tested. Two series of tests are generated, one by varying the base case resistance and the other by changing the base case reactance using a uniform scaling factor, keeping the latter parameter unchanged. The results obtained by the PM are compared with that of FDDPF [12], BNPF [18] and FDGPF [10] methods to highlight its superior performance. The algorithms are tested with a flat start and a convergence tolerance of 0.0001 per unit.

The solution of the PM for the 15-node system with base case resistance multiplied by a scaling factor of 1.5 is compared with the solution obtained by FDDPF, BNPF and FDGPF methods in Table-1. This table indicates that the PM offers the same solution as that obtained by the other methods, which validates its solution accuracy. Table-2 explains the convergence characteristics in terms of number of iterations. The PM reliably converges for all test systems with wide variation in r/x ratio of the distribution lines similar to FDDPF and BNPF methods. But the FDGPF needs a higher r/x ratio for convergence even for smaller systems and diverges for 69 node system irrespective of the r/x ratio. It is very clear that the PM is insensitive to r/x ratio and provides solution for larger systems unlike FDGPF method. The execution time of the PM, shown in Table-3, is very less for all the test cases when compared with FDGPF, FDDPF and BNPF methods.

These results indicate that the PM is accurate, fast and robust and is suitable for larger distribution systems.

Table-1 Power Flow Solution obtained for 15 node system

Node No	PM		FDDPF		BNPF		FDGPF	
	V	δ	V	δ	V	δ	V	δ
1	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
2	0.9635	0.0078	0.9635	0.0078	0.9635	0.0078	0.9635	0.0077
3	0.9450	0.0120	0.9450	0.0120	0.9450	0.0120	0.9449	0.0119
4	0.9377	0.0137	0.9377	0.0137	0.9377	0.0137	0.9376	0.0136
5	0.9364	0.0142	0.9364	0.0142	0.9364	0.0142	0.9363	0.0141
6	0.9344	0.0151	0.9344	0.0151	0.9344	0.0151	0.9344	0.0150
7	0.9347	0.0150	0.9347	0.0150	0.9346	0.0150	0.9346	0.0149
8	0.9361	0.0156	0.9361	0.0156	0.9361	0.0156	0.9361	0.0156
9	0.9307	0.0179	0.9307	0.0179	0.9307	0.0179	0.9307	0.0178
10	0.9290	0.0186	0.9290	0.0186	0.9290	0.0186	0.9289	0.0186
11	0.9592	0.0096	0.9592	0.0096	0.9592	0.0096	0.9592	0.0095
12	0.9578	0.0101	0.9578	0.0101	0.9578	0.0101	0.9578	0.0101
13	0.9464	0.0147	0.9464	0.0147	0.9464	0.0147	0.9464	0.0147
14	0.9435	0.0159	0.9435	0.0159	0.9435	0.0160	0.9434	0.0159
15	0.9447	0.0154	0.9447	0.0154	0.9447	0.0154	0.9447	0.0154

Table-2 Number of Iterations

	15 node				29 node				69 node			
	PM	Ref. [12]	Ref. [18]	Ref. [10]	PM	Ref. [12]	Ref. [18]	Ref. [10]	PM	Ref. [12]	Ref. [18]	Ref. [10]
r + j x	4	4	4	nc	5	5	5	nc	5	5	5	nc
0.5 r + jx	3	3	3	nc	5	5	5	nc	4	4	4	nc
1.5 r + jx	4	4	4	16	7	7	7	15	6	6	6	nc
r + j 0.5x	3	3	3	9	5	5	6	10	4	5	5	nc
r + j 1.5x	4	4	4	nc	6	6	6	nc	5	5	5	nc

Table-3 Normalised Execution Time (milliseconds)

	PM	Ref. [12]	Ref. [18]	Ref. [10]
15 node	14	15	16	31
29 node	15	16	16	62
69 node	28	31	31	nc

4. Conclusions

An efficient and robust decoupled algorithm, based on equivalent line power flows, has been described and investigated to solve power flow problem of the distribution systems. The problem has been decoupled into two sub-problems without making any assumptions on r/x ratios, which has endeared the algorithm to be robust. This method has used a constant and identical jacobian matrix for both the decoupled sub-problems, which has enabled the algorithm to be computationally efficient. The developed algorithm is thus well suited for online applications of distribution systems.

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