Phase-Only and Elements' Distance-Phase Synthesis of Array Patterns Via Particle Swarm Optimization

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***Abstract*— A computationally efficient global optimization method, the Particle Swarm Optimization algorithm (**PSO**), is proposed for the synthesis of uniform amplitude arrays of two classes, i.e., unequally spaced arrays with equal phases and unequal phases. Phase-only synthesis and the synthesis of uniformly exited unequally spaced arrays (position only synthesis) are compared and it is seen that, by using the unequal spacing, the number of array elements can be significantly reduced for attaining reduced sidelobe levels. From the** PSO **-based synthesis of unequally spaced arrays with uniform amplitudes and unequal phases, it is found that a tradeoff exists between the size of the unequally spaced arrays and the range of phases for the same radiation characteristics. The proposed synthesis technique using uniform amplitudes, unequal spacing, and unequal phases (position-phase synthesis) not only decreases the size of the array for the same sidelobe level compared to both the phase-only synthesis and position-only synthesis but also retains their advantages.**

Keywords: Particle Swarm Optimization, Beam Steering, unequally spaced antenna array, pattern synthesis

I. INTRODUCTION

Phase-only reconfigurable antenna array capable of radiating multiple radiation patterns with fixed amplitude are used in many applications such as cellular, and satellite communication systems. The generation of multiple radiation patterns by an antenna array with prefixed amplitude distribution simplifies the hardware implementation. However the phase-only synthesis with equal element spacing requires a large number of elements compared to the amplitude only arrays.

Controlling the inter element space and elements phases feed we can have the potential to circumvent this design challenge. Theoretically, the unequal spacing of antenna elements corresponds to nonuniform sampling of signals in the time domain.[5] several methods of generating phase-only multiple pattern antenna arrays have been described [1–8]. Phase-Amplitude and phase only beam shaping [9,10] using particle swarm optimization (PSO) was reported.

In this paper, we propose using the PSO algorithm for the design of unequally spaced arrays. Using the PSO, we carry out the synthesis of the uniform amplitude arrays with unequal spacing and equal phases (position-only synthesis) and unequal spacing and unequal phases (position-phase synthesis).We observe that the position-phase synthesis is superior to both the position-only synthesis and phase-only synthesis.

II PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) [11] is one of the artificial life or multiple agents' type techniques. The method finds the optimal solution by simulating such social behavior of groups as bird flocking. A group can achieve the objective effectively by using the common information of every agent, and the information owned by the agent itself. PSO is basically developed through simulation of bird flocking in two-dimension space. The position of each agent (particle) is represented by coordinates on the XY plane, the velocity is expressed by v_x and v_y (the velocity of the agent along X- and Y-axis respectively). The agent position is modified by the position and velocity information. The concept of PSO can be described as in Fig. 1. The velocity of each agent is calculated as follow:

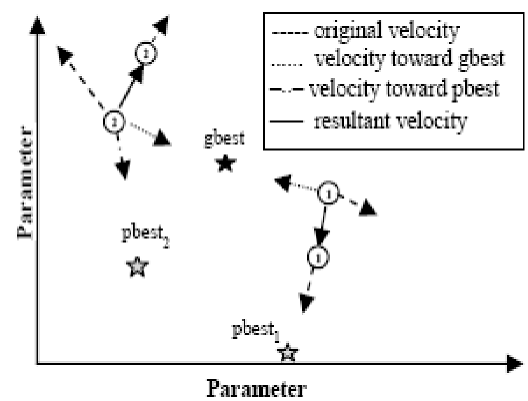

Fig. 1: Individual particle accelerated toward the location of the best solution *gbest*, and the location of its own personal best *pbest*, in a 2-D parameter space.

$$v_i(t+1) = a_i v_i(t) + \eta_1 r\big(pbest_i - s_i(t)\big) + \eta_2 r\big(gbest - s_i(t)\big) \qquad (1)$$

Where:

$v_i(t)$: velocity of agent i at iteration t;
$s_i(t)$: position of agent i at iteration t,
$pbest_i$: best position found by agent i,
$gbest$: best position found by agent group,
η_1, η_2 : coefficients represent the influence of social and cognitive components,
r: is a random number whose upper limit is a constant parameter of the system, used to introduce a stochastic element in the search process,

a_i: velocity inertia factor of agent *i* that control the exploration and exploitation.

$a_i = a_{max} t(\frac{a_{max}-a_{min}}{t_{max}})$

a_{max}, a_{min}: initial and final weights,

t_{max}: maximum iteration number.

The current position of an agent is calculated by:

$$s_i(t+1) = s_i(t) + v_i(t+1) \quad (2)$$

For a problem of *r* dimensions (parameters to optimize) and *m* particles.

$$S = (s_{i1}, s_{i2}, \ldots . s_{ir}) \quad (3)$$

$$V = (v_{i1}, v_{i2}, \ldots . v_{ir}) \quad (4)$$

Then for $j = 1,2, \ldots \ldots . . r$ and $i = 1,2, \ldots . . m$

$$v_{ij}(t+1) = a_i v_{ij}(t) + \eta_1 r\left(pbest_{ij} - s_{ij}(t)\right) + \eta_2 r\left(gbest_j - s_{ij}(t)\right) \quad (5)$$

$$s_{ij}(t+1) = s_{ij}(t) + v_{ij}(t+1) \quad (6)$$

PSO algorithm can be summarized as follow:

1. Initiate positioning vector $\boldsymbol{S}$ and velocity vector $\boldsymbol{V}$ by using random values, define the objective function.
2. Evaluate fitness of each position $f(S_i)$.
3. Initially Set $pbest_{ij}$ value equal to $f(s_{ij})$, and $gbest_j = \max_j(pbest_{ij})$.
4. Update S and, V vectors for iteration $t+1$ using equations (5) and (6).
5. Calculate fitness value $f(S_i(t+1))$ for each agent.
6. Update agents' personal best position as follow
   If f_i is better than the $f(pbest_i)$, then set $pbest_i$ to $s_i(t+1)$
7. Update global best position as follow:
   If $f(pbest_i)$ is better than $f(gbest)$, then set $gbest$ to $pbest_i$
8. Increment t by 1, If the iteration number (t) reaches to the pre-determined one, then stop, otherwise, go to step 4

IV. PSO and Constraint problem

This section is devoted to developing swarm intelligence approach to deal with various types of parameters search space constraints. Generally to handle constraints variable, the solutions is applied based on equations 5 and 6 was first randomly done, and the solution is then checked against the constraints. If the constraints were satisfied, the modification is usable, otherwise, the modification is discarded and a new modification is generated and checked. It should be noted that some of the modifications generated were unusable, and therefore more computational time might be needed [11,12].

However we propose to convert the problem to an unconstrained one through using suitable transformations of the constraint parameter [13] and solve for the unconstrained parameter as an example let us consider the case for array elements spacing's usually it is required that the elements has to lie within a specified range to avoid unacceptable practical array dimensions. Stated mathematically in the following form:

$$a_i \le x_i \le b_i \qquad i = 1,2, \ldots . M \quad (7)$$

The transformation to be used in this case is

$$x_i = a_i + (b_i - a_i)\, sin^2\, \acute{x}_i \quad (8)$$

The PSO optimize solution for unconstrained $\acute{x}_i$

V. ANETNNA ARRAY BASICS

The far-field pattern of a linear array of isotropic elements is given as

$$\mathrm{A}(\theta) = \sum_{k=0}^{k=\mathrm{N}} a_k \mathrm{e}^{-\mathrm{j}2\pi\frac{x_k}{\lambda}\sin(\theta)} \quad (9)$$

Where, λ is the radiating wavelength, $\{a_k\}$ is the elements feed and x_k is the distance of the k^{th} element from the reference element let d_i be the distance of i^{th} element from the adjacent element then x_k defined as follow

$$x_k = \sum_{i=0}^{i=k} d_i$$

Let $u = \sin(\theta)$ then

$$\mathrm{A}(\theta) = \sum_{k=0}^{k=\mathrm{N}} a_k \mathrm{e}^{-\mathrm{j}2\pi(\frac{x_k}{\lambda}u)} \quad (10)$$

In the common case where $d_i = \lambda/2$, $\theta \in [-90^\circ, 90^\circ]$ corresponds to $u \in [-0.5, 0.5]$, and the semicircle of physical angles maps exactly to one period of the Fourier transform response [4]. If $d_i < \lambda/2$ the semicircle maps to less than a full period, and thus there exists a range of values of u that do not correspond to any physical angle, and the transform response in that region does not directly affect the array pattern. If $d_i > \lambda/2$ then the semicircle maps to greater than one period of the transform response leading to grating lobes at high angles. Constraint has to be taken care when considering pattern synthesis with unequally spaced elements such that the array dimension is practical and also to avoid grating lobes.

In order to generate a BP fulfilling different applications requirements e.g sidelobe level (SLL) lower than a fixed threshold or reproducing a desired shape $P_{dB}^{ref}(u)$, an array design parameters $\bar{\zeta}$ and corresponding boundaries (search space) are identified then next step it is necessary to define the objective function f that measures the difference between desired and synthesized beam pattern and consequently rank the proposed solution acceptance. A general form for antenna pattern synthesis fitness function can be defined as equation (11).

$$f(\bar{\zeta}) = \frac{1}{c_3 f_{SLL}(\bar{\zeta}) + c_4 f_{BP}(\bar{\zeta}) + c_5 f_N(\bar{\zeta}) + c_6 f_D(\bar{\zeta})} \quad (11)$$

Where

$\bar{\zeta} = [N, d_0, \ldots d_{N-1}; w_0 \ldots . w_{N-1}]$

$f_{SLL}(\bar{\zeta}) = \frac{Q}{\max_{u_{Start} \le u \le 1}\{P_{dB}(u)\}}$;

$f_{Bp}(\bar{\zeta}) = \int_{u \in B}\left(\frac{P_{dB}(u)}{Q} - P_{db}^{ref}(u)\right) du$;

$f_N(\bar{\zeta}) = N$; $f_D(\bar{\zeta}) = D$;

u_{Start} is the value allowing main lobe to be excluded from the calculation of the SLL; D is the array aperture; Q is a normalizing constant; B is the range of values for which $\{P_{dB}(u)/Q\} \ge P_{dB}^{ref}(u)$ $P_{dB}^{ref}(u)$ the desired BP shape;

Finally, c_3, c_4, c_5 and c_6 are normalizing coefficient chosen according to the optimization strategy.

VI. Numerical Results

The PSO is applied to search for the optimum element phases and positions of the uniform amplitude linear arrays to achieve target pattern and minimum side lobe level .We only consider symmetric arrays for the next results however same can be applied for non symmetric array.

Synthesis of an unequally spaced array is carried out separately for the position-only and the position-phase cases for various limits in the distance between the elements. The number of elements considered for the PSO-based synthesis is 32; hence the number of parameters to be optimized is 16 for the position- only synthesis and 32 for the phase-position synthesis.

The prior constraint in the synthesis of the element positions for both the cases is $d_{min} = 0.5\lambda$ where d_{min} is the minimum distance between two adjacent elements. The upper limit in the distance between the elements d_{max} is varied from 0.5 λ to 1λ, each time noting the maximum SLL attained for both the position-only and position-phase synthesis. Fig. 2 shows the achieved maximum SLL for different d_{max} for the PSO-based position-only and position-phase synthesis.

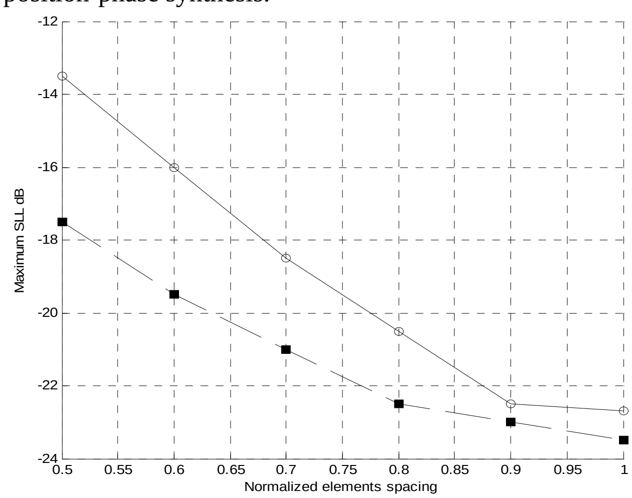

Fig. 2. Maximum SLL of an equal amplitude, unequally spaced 2-element array. dotted position-only. solid position-phase

As can be seen from Fig. 2, when d_{max} is smaller, the maximum SLL for the position-phase synthesis is much lower compared to that of the position-only synthesis. It is to be noted that, when $d_{max} = d_{min} = 0.5\lambda$ i.e. phase-only synthesis, the maximum SLL is lower by about 5 dB compared to the case when the array is uniformly exited. From Fig. 2, we can also see that, when d_{max} is increased, the maximum SLL decreases for both the cases. When approaches λ, there is no significant reduction in the maximum SLL for the position-phase synthesis compared to the position-only synthesis.

The PSO synthesis results of positions and phases for the cases when $d_{max} = 0.6\lambda$ and $d_{max} = \lambda$ array patterns are shown in Figs. 3 and 4, respectively. From Fig. 3, we can see that the maximum SLL for the position-phase synthesis is lower than that for the position-only synthesis. In Fig. 4 When $d_{max} = \lambda$, the maximum SLL of the position- phase synthesis and position-only synthesis is 23.34 and 22.53 dB, respectively

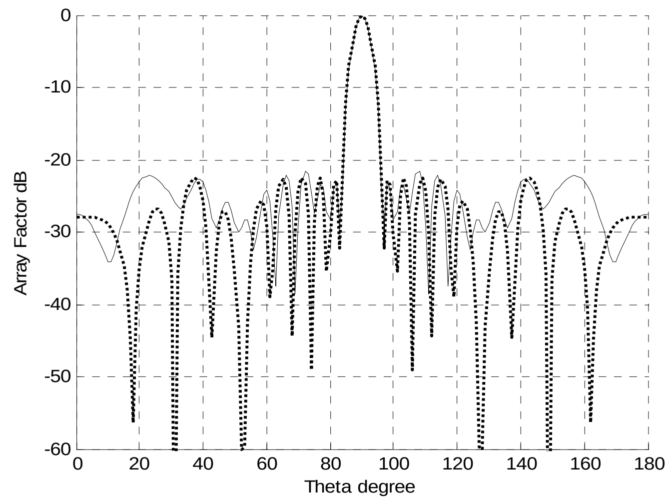

Fig. 3. Array patterns for the PSO-based position-only (dashed line) the position-phase (solid line) for $d_{max} = 0.6\lambda$

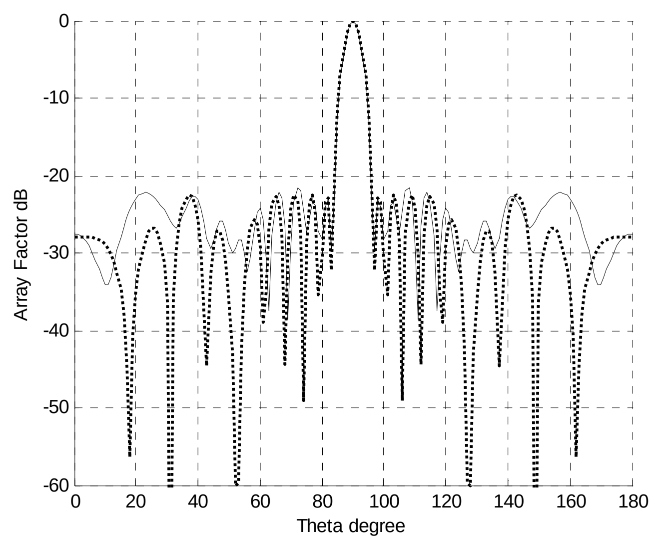

Fig. 4. Array patterns for the PSO-based position-only (dashed line) the position-phase (solid line) for $d_{max} = \lambda$

Therefore, from Figs. 2–4, we can conclude that for smaller , $d_{max}$the element phases have a larger effect in lowering the SLL of an unequally spaced array with no significant difference in the directivity.

For the case $d_{max} = \lambda$, The time taken to reach -20 dB SLL was about 10 min, and the total time taken for 300 iterations was about 23 min for a swarm of 320 agents. The simulations were carried out on a PC based on an Intel Pentium-IV 3-GHz processor.

We have seen that the unequally spaced array derived using the position-phase synthesis has lowered SLL compared to that of the unequally spaced arrays derived using the position-only synthesis. Let us consider the PSO-based position-phase synthesis and phase-only synthesis for designing a pencil beam array. The number of elements has

to increase to meet beam requirement we consider symmetric array of 60 elements. For the position-phase synthesis, the prior limits assumed in the minimum and maximum distance between the elements are $d_{min} = 0.5\lambda$ and $d_{max} = 0.7\lambda$, respectively.
For phase-only synthesis, the uniform distances between the elements are assumed to be 0.5λ. Fig. 5 shows the corresponding array patterns shows the phases and positions derived using the PSO-based phase-only synthesis and position-phase synthesis we can see that for the position-phase synthesis, the SLL is lower compared to that of the phase-only synthesis.

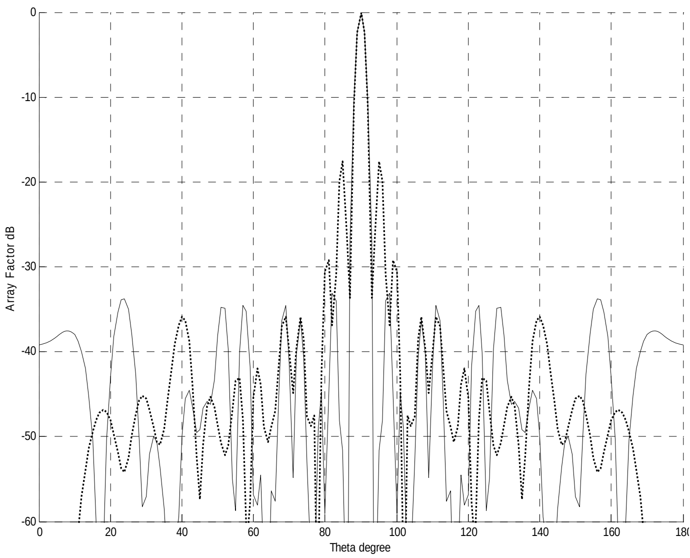

Fig. 5 Array patterns for the PSO -based position-phase synthesis (solid line) and the phase-only synthesis (dashed line) of a pencil beam array of 60 elements.

IV. CONCLUSION

The PSO is applied to develop a computationally efficient synthesis tool for the design of uniform amplitude, unequally spaced arrays. From the study of the unequally spaced arrays derived using the PSO-based position-only and position-phase synthesis, it is seen that, for lower upper limits in the distance between the elements, the position-phase synthesis resulted in reduced sidelobe levels compared to that of the position-only y synthesis. However, as the upper limit in the distance between the elements increases, the difference between the maximum SLL between the position-phase and position-only synthesis decreases. Therefore, we can have a tradeoff between the element phases and spacings for attaining the same radiation characteristics. Improved array efficiency and decreased sidelobe levels have been achieved in the design of the pencil beam array using PSO-based position-phase synthesis compared to the phase-only synthesis. We can conclude that the algorithm is promising to be applied for the synthesis of planar arrays.

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