# LOW COMPLEXITY VLSI DESIGN USING STEPWISE EXPONENTIAL COMPANDING QPSK DETECTION IN MIMO SYSTEM

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#### **ABSTRACT:**

MIMO is a modern mobile scheme used for the transmission of digitized data at high speed, which has a large number of narrow band subcarriers. It needs multiple antennas at both the transmitter and receiver side. In real time, the implementation of Multiple-Input Multiple-Output (MIMO) is more complicated. To overcome this problem, one new method was implemented, that is SEC(Stepwise Exponential Companding) based Quadrature Phase Shift Key (QPSK) detection which is implemented in Field Programmable Gate Array (FPGA) device. This method has a better feature for both Communications as well as the VLSI domain. This method has a great future for the wireless 5G system because of its spectral competence and robustness of the channel. Our investigation observed results are promising one to minimize MIMO detection complexity which can enhance the MIMO scheme operation.

Keywords:-MIMO,Stepwise Exponential Companding, FPGA, QPSK,

#### **INTRODUCTION:**

Presently, mobile communication is playing a vital role in everyone's daily life. The data in wireless communication is transmitted over a long distance or short distance. The traveled data are sent through over a wireless channel means, there is no cable or fiber in between the sender and receiver communication. There are many applications are presently depending on wireless communication. For example, WIFI, WLAN is commonly used in wireless technology. To improve efficiency and throughput, a new method is introduced. That is QPSK modulation based on the SEC technique. MIMO is the advanced technology which is commonly used in mobile transmission and secured data transmission for military and other application. MIMO detection strategies could also be separated into 2 groups, i.e., hard and soft detection. The fundamental knowledge of hard- detection is to pursue the possible settings for the symbol vector directly adjacent that to the communicated signal vector and the detector outcome as an output.

The major key challenges in exploiting the potential of MIMO systems are to design maximum throughput and minimum complexity detection algorithms whereas achieving near-

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optimal performance. In this thesis, there is a new method is implemented to reduces the complexity and get a high throughput output signal. The VLSI implementation is based on an FPGA kit with a suitable device. FPGA is a field programmable Gate array that has a great future for VLSI related implementation. PROM hips and FPGA are relatively same but, FPGA has more advantages than PROM (programmable read-only memory.

#### **PILOT SEQUENCE:**

In the MIMO technique, the receiver should know the characteristics of the channel. To know the knowledge of the channel, a pilot sequence is used. This is a well-known technique, which is used in wireless communication mostly. The other name of this sequence is the training sequence. This sequence is transmitted and the corresponded matrix estimation is calculated. Based on the matrix value, there is a knowledge of channel can obtain.

#### QUADRATURE PHASE SHIFT KEYING (QPSK)

One of the important modulation schemes in digital communication is QPSK. It is a phase shift keying which shifts the phase of the carrier signal with respect to two bits or symbols. So, the bandwidth required amount will decrease to half of the actual range. This method is based on the selecting phase among the 4

carrier phase shifts(0,  $\Pi/2$ ,  $\Pi$ ,and3 $\Pi/2$ ).

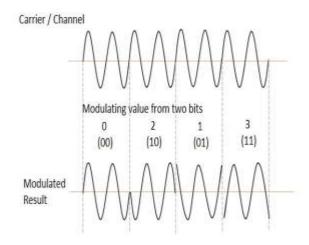


Fig (1): Modulated waveform of QPSK signal

From the basic, this has two carriers which are In-phase (I)carrier. Quadrature-phase (Q) carrier. Among these, the odd carrier will change the phase range from 0° to 180° and ° and the Even carrier will change from between 90° and 270°. This is used to indicate the four states of a 2-bit binary code. In this modulation, 2 bits are represented by four possible carriers with respect to their odd and even capability. Each possible carrier in the modulation is named symbol. These symbols are arranging in a circle form in a constellation diagram. practical QPSK modulation values are calculated, and the value is compared with theoretical value, based on the outcome, there is one thing is clear, this method is a good performance in communication. When Compared with BPSK, this method offers a double data rate and the bandwidth usage is very low. Hence, it is strong that the measured modulation system will offer the finest outcomes in mobile communication.

#### **QPSK Demodulator**

The QPSK demodulator placed at the receiver side, which demodulates the received QPSK modulated signal. Demodulation is the reverse process of modulation. The Receiver circuit consists of two product demodulator circuits with a local oscillator which is multiplied with the signal and bandpass filters, two integrator circuits, the final outcome is given to 2 bit P/S converter.

#### **CHANNEL ESTIMATION:**

When the signal is transmitted from sender to receiver, the channel state information is necessary to understand the channel parameters, ex, scattering, reflecting, multipath propagation, power decay, near-far problem, noise, error estimated in a noisy channel and etc. This method is called Channel estimation which can be supported in different ways (1) parametric model (2) frequency or time association belongings in the mobile channel (3) flexible or non-flexible approaches.

#### parametric model: -

The first method is detecting pilot signal using nonparametric methods which are used to estimate the frequency response without depending on a model of a specific channel. Equally, a channel is assumed for parametric estimation and with the help of this model's parameters are determined and finally, it reduces the amounts of interest.

#### frequency and/or time correlation:-

In order to improve the quality of the channel estimation, the time and frequency correlation are determined. This correlation has a unique property like coherent detection etc.

#### training pilots: -

Training pilots is a commonly used estimation technique in which a well-known signal is transmitted to understand the system property. It is mostly applicable to the wireless channel.

#### **Blind estimation: -**

This estimation is only based on the signal properties such as cyclostationarity etc. It cannot be used in real time OFDM systems.

#### Adaptive estimation methods: -

Adaptive estimation methods are usually applicable for channels with quick time disparity.

#### **GUARD INTERVAL:**

In communications, the guard intervals are used to confirm that different broadcasts do not affect each other. otherwise, it causes overlapping transmissions. If overlapping has occurred, then the retrieve of the original signal will be difficult or sometimes, it is not possible. The interfere is not a major problem in TDMA. The full bandwidth is allocated for different users at a different time and. In OFDM, the signal is transmitted with overlapping, but the receiver those who know the pseudo code, only retrieve the original signal. For the OFDM system, the original signal is guarded by a code. Each OFDM signal has a guard interval at the beginning. If the echoes occur within the guard interval, then the receiver can easily retrieve the signal but this not possible when it occurred at outside of the guard interval. The guard interval used for the OFDM system is  $0.8 \mu s$ . To increase the data rate in the OFDM system,  $0.4 \mu s$  guard interval is additionally added. This offers an 18% rise in data rate.

If the guard interval is very low, then the packet error rate will high. This will affect the performance of the mobile system. It is only possible when the data is transmitted over a long distance or spreader through long distance. But this shorter guard interval is a benefit for some low complexity devices and this communication depends on the high data rate signal that is achievable by the short guard interval.

# SEC (Stepwise Exponential Companding)

**SEC** stands for stepwise which exponential companding, proposed to achieve the advantages of the EC and PC system. It is a recent technique which is recently used in the OFDM system to reduce the Power value. By using PC function, the structure converts the numerical distribution of the minor and major signal into dissimilar ones, and the plan of the system are obtained based upon the evaluation. The major part of the EC work is, it can decrease the power of the input symbol by changing the insight of the amplitude of this signal into uniform dispersion. This strategy has a few good things about keeping a steady average power level within the nonlinear companding operation. By choosing the correct Pi and t value, we can get a different form of power level reducing techniques.

$$2P_i^{t+2} + (t+2)\sigma^2 P_i^d - (t+2)P_c^t P_i^2 + tP_c^{t+2} - (t+2)\sigma^2 P_c^d = 0 - - (1)$$

# STEPS INVOLVED IN MIMO QPSK DETECTION:

The following steps are involved in the detection of the MIMO system.

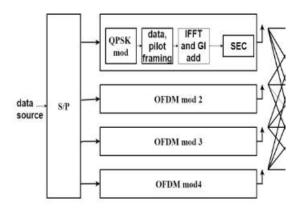


Fig (2a): Transmitter

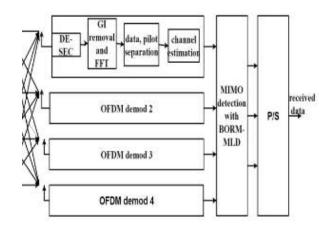


Fig (2b): Receiver

Fig (2): Proposed Block Diagram

#### **STEP 1: DATA SOURCE**

In wireless communication, Data source is the origin of the data which produces the information to be directed. At the Transmitter side, this data source is encoded to get a required format. Overall, it can generate a message in the form of words, bytes, code, symbols, bits, sound signal, image, video. The main function of the source is to produce wanted information which has to be transmitted. The information in the communication is called a symbol or data or bits.

### STEP 2: SERIAL TO PARALLEL CONVERTER:

The reason why the OFDM technique used parallel transmission over serial transmission. The answer is, for serial transmission multiple data are sent over the same channel which slows down the speed of the transmission. To avoids this kind of problem, parallel transmission is used.

Initially, data from the data source is given to the serial to parallel converter, here participate data stream is configured into the word size essential for transmission, e.g. 2 words shifted into an equivalent set-up by relegating each information word to one carrier within the communication.

The main advantage of serial to parallel converter is, the parallel transmission will be sent multiple data simultaneously in different channels. So, there is a speed up process in parallel transmission over the serial transmission. To perform this operation there are two shift registers are used.

#### **STEP 3: QPSK MODULATION:**

The data from the S/P converter is given to QPSK modulator, which modulates the signal with four phase Signal. Let consider QPSK signal with  $T_{sym}$  as follows,

$$m(t) = A\cos[2\pi f_c t + \theta n], 0 \le t$$
  
 $\le T_{sym}, n$   
 $= 1,2,3,4 - - - (2)$ 

In this method, data is divided into in-phase and quadrature phase signal from the transmitted signal. Each data stream is modulated with the carrier signal. IN BPSK, only one bit is modulated by a single carrier, but in this QPSK scheme, two bits are modulated by a single bit. These in-phase and quadrature bits are converted into Non-Return Zero format.  $T_{sym}=2T_{bsym}=2T_b$ .

From this equation, we can say clearly that, the QPSK sends the data twice much as BPSK transmits. The signal at the in-phase side is then multiplied by  $\cos(2\pi f_c t)$  and the signal on the quadrature side is multiplied by  $-\sin(2\pi f_c t)$ . After completing all the process, there is one thing is clearly understood, that is, the data at I-arm and Q-arm are same as the BPSK signal with symbol duration  $2T_h$ . The original QPSK signals are obtained from adding both inphase and quadrature phase signals. This configuration should satisfy the Nyquist sampling theorem (fs\ge fc) and the oversampling rate is selected based on the equation below,

fc -> carrier frequency

fs -> sampling frequency

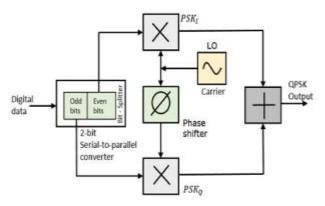


Fig (3): QPSK Modulation

The fig shows the modulation of the digital data signals. At the output side, both in-phase and quadrature signal are added and the original QPSK modulated signal is obtained for further processing.

#### **STEP4: PILOT FRAMING:**

Pilot sequence generation will be done by using Optimal pilot sequences based on the pilot design criterion. Let consider pilot arrangements  $y_i$ , i = 1, 2, ..., M. Pilot sequences are used for many multicell and single cell environments. For multicell, the pilot sequence is generated base on (CAZAC). Among these pilot sequences, Chu sequences placed an important role in generating correlation parameters for transmitting the pilot signal. Let  $y_{1m}$  Xlk be the pilot sequence spread by the  $m_{th}$  user at the 1 st cell, then  $y_{1m} = [x] 0 k,..., [x] n$  $k, ..., [x] \rho - 1 k$  where  $\rho$  is the length of pilot sequences. From the outcome, it is well known that increasing the pilot sequence will increase the accuracy of the channel, but the number of data to be transmitted is very low because it will more occupy the bandwidth. When the training period is larger than (Ns/2), the total rate will be significantly condensed,

then we assume that  $\tau \leqslant (Ns \cdot N \text{ smooth/2})$ . Moreover, with the aim of ensuring the orthogonality of pilot sequences, the length should not less than K. The pilot sequences placed in the lth cell can be demarcated as  $Xl = Xl \ 0 \ H$ ,  $Xl \ 1 \ H$ ,...,  $Xl \ K - 1 \ H \ \tau \times K$ .

## STEP 5: INVERSE FAST FOURIER TRANSFORM:

IFFT is a fast algorithm to determine the DFT and IDFT values and the modulation process of all subcarriers is done by the IFFT block. Basically, IFFT is used to transform the frequency domain signal into time domain signal and the multiplication complexity of IFFT reduces N/2 log2 N when compared with IDFT. Time domain refers to the examination of exact purposes, physical signals or time sequences data, with regard to time. Within the timespace, the flag or function's esteem is known for all actual numbers, for the case of persistent time, or at the various partitioned moments within the case of discrete time. An oscilloscope may be a tool normally utilized to imagine realworld signals within the time-space. A time-domain chart displays how a signal changes with time.

$$X(n) = \frac{1}{N} * \sum_{k=0}^{N-1} X(k) * e^{i*2*pi*n*\frac{K}{n}} - -$$

$$- - - (4)$$

By using this equation, we can find out the time domain signal representation.

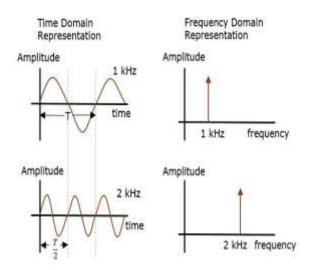


Fig 4: Time-domain and Frequency domain

Fig (4) shows the conversion of the signal from the frequency domain to the time domain.

Frequency

$$f = \frac{\text{how many cycles of an oscillation}}{\text{second}}$$

The period of a wave,

$$T = \frac{\text{amount of time it takes a wave to vibrate}}{\text{one full cycle.}}$$

These two terms are inversely relative to each other: f = 1/T, and T = 1/f.

#### **STEP 6: CHANNEL**

A channel is a separate path through which signals can flow. To create the best channel or known the best correction of the path, there must one possible solution, that is CSI. It is useful in OFDM to perform coherent recognition and diversity combining if numerous transmitter and receiver antenna is arranged. In practice, CSI can be

depended on evaluated at the collector by transmitting pilots along with information sequence. Afterward, the signal is transmitted over the channel. The lack of knowledge of channel statistics can make channel estimation much harder. So proper channel selection must need to get perfect communication in the OFDM scheme.

#### **STEP 6: DE-SEC:**

At the receiver side, the inverse operation of SEC is performed to retrieve the original information hence the received data is expressed as follows,

$$m^{-1}(x) = \begin{cases} x & ; |x| < P_i \\ sgn(x) \sqrt{P_i^2 - \sigma^2 \ln \left( (|x|^t - P_c^t) / P_i^t - P_t^c \right)} \end{cases} ; |x|$$

$$\geq P_i - - - (5)$$

where  $sgn(\cdot)$  denotes the sign function.

If |x| < Pi, it will not do any companding operation. But if |x| > = Pi then, the

companding operation is performed by the equation,

$$\operatorname{sgn}(x)\sqrt{P_i^2 - \sigma^2} \ln \left( \frac{(|x|^t - P_c^t)}{P_i^t - P_c^t} \right) - (6)$$

To determine the de SEC function, we should need the parameter Pc, which can be decided by the predetermined value as  $Pc = \sigma \times 10(PAPR \text{ predetermined/20})$ . After finding out the Pc value and t value, the parameters Pi is expressed in equation (7),

$$2P_i^{t+2} + (t+2)\sigma^2 P_i^d - (t+2)P_C^t P_i^2 + tP_C^{t+2} - (t+2)\sigma^2 P_c^d$$

$$= 0 - (7)$$

After Pc and Pi are obtained, the decompanding function is performed. By adjusting, Pc, Pi, t various low power results can obtain.

#### **STEP 7: BOQRM MLD**

To detect the MIMO signal, let consider 4×4 MIMO system as follows,

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \end{bmatrix} - - - - - (8)$$

Where  $m_1$ ,  $m_2$ ,  $m_3$ ... are the transmitted signal from multiple transmitting antenna and n1, n2, n3... are the receiver noise present at the receiver side. Z11, z22, z33... are the medium response from ith communicating antenna to the jth receiving antenna. The short form of this equation is expressed, m = Z l + n In full MLD technique 4×4 MIMO system, the replicate of a signal is calculated as follows,

$$\begin{cases} \widehat{m}_1 = z_{11}l_1 + z_{12}l_2 + z_{13}l_3 + z_{14}l_4 \\ \widehat{m}_2 = z_{21}l_1 + z_{22}l_2 + z_{23}l_3 + z_{24}l_4 \\ \widehat{m}_3 = z_{31}l_1 + z_{32}l_2 + z_{33}l_3 + z_{34}l_4 \\ \widehat{m}_4 = z_{41}l_1 + z_{42}l_2 + z_{43}l_3 + z_{44}l_4 \\ - - - - (9) \end{cases}$$

In this method, QR decomposition is applied to channel matrix Z,

Hence it is replaced by a product of  $Q_d$  and  $T_d$ , Finally, the expression expressed as,

$$m = Q_d T_d l + n - - - - - - (10)$$

 $Q_d^H$  is multiplied on both sides, then the resultant equation,

$$Q_d^H m = T_d l + Q_d^H n - - (11)$$

Where  $Q_d^H$  is the complex conjugate transpose of Qd, so  $Q_d^H m$  replaces by  $\begin{bmatrix} m'_{1,} & m'_{2,} & m'_{3,} & \dots \end{bmatrix}^T$  And  $Q_d^H n$  is replaced by  $\begin{bmatrix} n'_{1,} & n'_{2,} & n'_{3,} & \dots \end{bmatrix}^T$ 

Where  $t_{ij}$  denotes the component of  $Q_d$  in the j-th row and i-the column. On the basis the next 2-step MLD is done.

#### [1] step:

Replica  $m'_3$  and  $m'_4$  as follows,

From this equation,  $(m_3, m_4)$  is calculated by graded in climbing order by the sum of a replica of the matrices  $(\widehat{m}_3', \widehat{m}_4')$ . In this method, the Manhattan-Chebyshev metric is used for selection and leads to low calculation.

$$\sum_{j=3,4} \left( |Re(m'_{j-}\widehat{m}'_{j})| + \left| Im(m'_{j} - \widehat{m}'_{j}) \right| + \max(\left| Re(m'_{j} - \widehat{m}'_{j}) \right|) - - - - (14) \right)$$

Where, Re  $(\cdot)$  -> Real part of the complex number

Max  $(\cdot)$ ->which returns the maximum of inputs.

#### [2] Step:

Replica  $m'_1$  and  $m'_2$  as follows,

 $(m_1, m_2)$  are calculated as the same steps followed in stage1.

To end the selected candidates calculated by a squared Euclidean metrics are combined among the receiving antennas. The Low-value replica candidate is selected for the detection purpose.

### STEP 7: PARALLEL TO SERIAL CONVERTER:

Conversion of a stream of multiple data elements into a single data element.

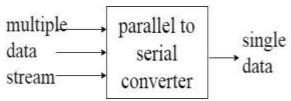


Fig (5): Parallel- Serial Converter

Finally, the original signal is available at the output of the parallel to serial converter. When, compared with other methods, this technique has an advantage. So, we can say that this method gives low complexity value and low BER and good detection performance.

#### **RESULT AND ANALYSIS:**

The performance of the system is analyzed based on the utilized slices and the number of addition and multiplication needed for this method. The proposed VLSI implementation is based on FPGA Vitex2 and everything is coded in MATLAB language. Among many VLSI implementation, FPGA is commonly used because of its low price and complexity, FPGA performance is similar to PROM, but FPGA has low complexity. When compared with other methods the running capability of FPGA is high, so it has great features for Real-Time applications.

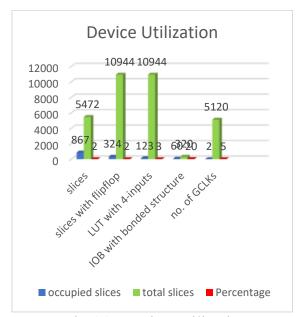


Fig (6): Device Utilization

Fig (6) shows the device utilization of the proposed method. To implement this technique, the FPGA virtex2 device is used. Virtex FPGAs are typically programmed in hardware description languages such as VHDL or Verilog, using the Xilinx ISE. The resultant output is shown in detail with the table. From this graph, we can say that number of used slices are high when compared with other technologies so we can say that this method utilizes more slices than other methods so, the price will reduce and the complexity is reduced based on the multiplication and addition. Used slices in the flipflop are 5% but other techniques this value is less than 2%. No of used Lookup Table is 3% and it will also improve the performance. In FPGA number of slices means a number of flipflops and LUT used. If the slices per increases, area are the design performance will improve and the speedup process will accompany the share required the use of an InterCONnect (ICON) block, which presented the resources to appropriate detectors at appropriate cycles. It can be seen that the ICON devours around 54% of slices, 45% of flip flops and almost 43% of LUTs. implementation The **FPGA** needs 0.0225MBytes of RAM in contradiction of an obtainable RAM of 8MB.

modulation		PSK		QPSK	
		Multiplication	Addition&	Multiplication	Addition&
			sub		sub
Full MLD		2,300	3,703	2758	2648
BORM- MLD	SEC & de- SEC	927	520	645	457
	IFFT	258	1283	447	11,28
	Total	1443	2403	1092	1585

Table (7): complexity comparison between PSK and QPSk

The following table shows the comparison between PSK and QPSK detection. When compared with PSK, this method only needs a smaller number of addition and multiplication of stages. So the system complexity ranges are reduced by 33% when compared with Full MLD.

#### **4.9 CONCLUSION:**

This paper concluded by telling that the complexity of the MIMO system is reduced by using the proposed Stepwise Exponential Companding technique. In this technique, MIMO detection is accomplished by combining techniques, which are IFFT and SEC. In IFFT the frequency is converted into the time domain and the complexity is reduced by 23.56% in multiplication and 34.05% in addition. Similarly, Stepwise Exponential Companding reduces the complexity ranges by 13% and Addition complexity in the range of 25%.

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