An Efficient Method for Smart Grid Fault Observation with Minimum PMUs

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Abstract— ***A power system may have buses that cannot be installed with PMUs for reasons such as non-availability of communication facility, inconvenience in installing PMUs and severe limitation on number of channels of PMUs. Such buses, when identified, are modeled as proposed in this paper. The only solution and partial set of solutions to the fault observability problem (FOP) may not be implemented when such buses are not already known. Therefore, it becomes necessary to determine complete set of solutions to the FOP. A simple yet robust method capable of delivering complete set of global optimal solutions and complete set of the best global optimal solutions to the FOP is proposed in this paper. The method is illustrated through a sample network and it is applied to a real-world transmission system. The illustration and the simulation results on the IEEE 14-Bus system and the IEEE 30-Bus system show that the proposed method is superior to the known methods. The real-world application shows practical importance of the proposed method.***

Index Terms—binary integer linear programming, fault observability, Italian 400 kV transmission system , optimization, global solution, PMU, smart grid, system observability redundancy index.

1. Introduction

Synchrophasors, also known as phasor measurement units (PMUs), are advanced monitoring devices that take measurements of voltage, current and frequency at a location on the electric transmission system. The PMUs are integral part of modern power grids called smart grids. The measurements, typically taken 30 times a second, are time-stamped with signals from global positioning system (GPS) satellites. PMUs provide us with time-stamped real-time phasors of voltage and currents measured at a substation 30 times a second [1] as opposed to the conventional Supervisory Control and Data Acquisition (SCADA) system that typically takes measurements every four second. The frequent PMU measurements show system changes that would not be evident with SCADA data. Thus, the Synchrophasors technology potentially changes the traditional state estimation to state measurement [2].

With the measurement capabilities of PMUs ([3]), a PMU installed at a bus measures the voltage phasors at the PMU bus and the current phasors in the entire branches incident to the bus. Voltage phasors of all the neighboring buses can be calculated by using the branch current measurement and the line parameters of the incident lines, unless a fault lies on an incident line. Power engineers always strive to determine the minimal number of PMUs and their strategic locations, which are greatly influenced by the intended application of PMUs, on a power system.

A bus when installed with a PMU becomes a PMU bus. We obtain the voltage phasor at the PMU bus and current phasors in all branches incident to it. Since currents in all braches incident to a PMU bus are known, therefore, the voltage phasors at buses linked to a PMU bus can be calculated. Therefore, a PMU placed at a bus observes the PMU bus and all buses connected to the PMU bus. This leads to the following definition for observability in the absence of a line fault. A bus is said to be observable if the voltage phasor at that bus is known and a power system is said to be observable when installed buses are observable. An associated placement problem in this case, called the PMU placement problem for normal observability, is to determine the optimal (or minimum) number of PMUs and strategic positions on the network to ensure normal observability of given power system. There has been a great interest in solving this problem for more than thirty years and fellow researchers have published comprehensive surveys on PMU placement on power systems ([4]-[11]).

A line is said to be observable during a line fault if the voltage at both the ends of the line, and the current at either end of the line are determinable [12]. The power system is said to be fault observable (FO) if every line is observable during a fault. With the knowledge about the voltage phasor at both the ends of a faulted line and the current phasor at either end of the faulted line, the location of the fault can be determined [12].An associated placement problem in this case is to determine the optimal (or minimum) number of PMUs and their strategic locations on power system so that the system may become fault observable. What is observed from existing surveys on optimal placement of PMUs ([4]-[11]) is that the PMU placement problem for fault observability has not been studied as much as the PMU placement problem for normal observability has been studied. A comprehensive survey on observing faults on power systems is given in [13].

The authors in [12] describe *'one-bus-spaced strategy'*, which means that every two PMUs are spaced by one bus, and propose a fault locating technique combined with fault-side selector which requires measurement of synchronized voltage phasors at both the terminals of a faulted line and fault current phasors fed from one of the line terminals. The Branch and Bound (B&B) method employed in [14] solves the FOP for a power system without zero injection buses (ZIBs). The binary integer linear programming (BILP) oriented methods reported in [15] and [16] solve the FOP for power systems (with and without ZIBs), a differential evolution (DE) framework in [17] delivers multiple solutions to the FOP for a power system (with and without ZIBs), and the genetic algorithm in [18] solves FOP for a power system. The observations about the known methods to solve the FOP are as follows.

1. It is argued in [15] that although the measurements seem to produce accurate fault locations, the PMU placement technique described in [12] is manual, cumbersome for large systems and unable to deal with ZIBs.
2. It is also argued there in [15] that the B&B method in [14] delivers incomplete and incorrect results.
3. However the work reported in [15] improves upon the work reported in [12] by covering presence of ZIBs but the limitation of the integer programming method proposed in [15] is that it delivers *only* solution to the FOP for a given power system.
4. A simple DE method proposed in [17] may deliver infeasible solutions and incomplete set of optimal solutions to the FOP for a power system. Common observations about the DE method are as follows.

    a) It is the best evolutionary algorithm to find global optimal solution (GOS) to an optimization problem.
    b) The quality of solutions delivered by the DE method cannot be judged enough for DE being a heuristic method.
    c) It does not guarantee an optimal solution is ever found.
    d) DE converges prematurely causing the entire population to converge to a point that is not optimum ([19]). For this reason, the solution (B2, B4, B5, B6, B8, B9, B13, B14)(see [Table I, 17]) to the FOP for the IEEE 14-Bus system (with ZIB: B7) is infeasible solution and hence non-optimal solution. For the same reason, the solution (B2, B3, B5, B10, B11, B12, B13, B15, B16, B19, B24, B26, B28, B29) (see [Table II, 17]) to the FOP for the IEEE 30-Bus system (with ZIBs: B6, B9, B22, B25, B27, B28)is infeasible solution and hence non-optimal solution. These non-optimal solutions are not considered when the proposed method is compared with existing methods elsewhere in this paper.
    e) There may be a problem of stagnation in the population, where the population stops proceeding towards the global optimum though it allows new individuals to enter the population [20].

These observations and the fact that the only optimal solution and a partial set of solutions to the FOP may not be implemented for reasons given in the abstract justify the necessity of complete set of multiple GOSs to the FOP. It motivates this author to devise a method that delivers the complete set of GOSs and the complete set of *the best global optimal solutions* (BGOSs) to the FOP. This paper proposes a simple yet robust method consisting of two BILP-Driven algorithms – one delivering the complete set of global optimal solutions to the FOP and the other directly delivering the complete set of the best global optimal solutions to the FOP. The proposed method has an edge over the methods given [15] and [17] in the sense that it delivers the complete set of GOSs to the FOP and the complete set of the BGOSs to the FOP whereas the method given in [15] delivers only solution to the FOP and the method given in [17] may deliver partial set of solutions to the FOP.

The organization of remainder of this paper is as follows. Section II presents two mathematical formulations of the FOP. Section III presents a method that delivers multiple GOSs and multiple BGOSs to the FOP. Section IV illustrates the proposed method. Simulation results on the IEEE 14-Bus and the IEEE 30-Bus are given in Section V. Section VI presents a real-world case study. Finally, Section VII concludes the work embodied in this paper.

2. Mathematical Formulation

Let SG(n) be the smart grid with n buses. Define

- S : The set of indices of all buses.
- T : The set of indices of ZIBs.
- Tk : The set of indices of buses linked to the ZIB k, k $\in$T.
- Ψ : The set of indices of the terminal buses.
- x_i : i^{th}Binary decision variable showing presence (absence) of a PMU at the bus i such that $x_i = 1\ (= 0)$.

Objective function: The total number of PMUs on an n-bus power system is given by the linear function given by (1).

$$f(x_1, \dots, x_i, \dots, x_n) = \sum_{i \in s} x_i \tag{1}$$

Terminal-bus constraint: A bus linked with one and only one bus in the SG(n) is called a *terminal bus*. It is necessary to install a PMU on every terminal in order to observe in the event of fault in link [15]. It means we definitely need as many PMUs as the number of terminal buses. The *terminal-bus constraint* is given by equation (2).

$$\sum_{i \in \Psi} x_i = CARD(\Psi) \tag{2}$$

Here$CARD(\Psi)$, denoting the cardinality of the set Ψ, represents the number of terminal buses in the SG(n).

Fault observability constraints: Let bus i and bus j be any two connected buses in SG(n). In order to make SG(n) fault observable, the bus-i, the bus-j or both should have PMU(s). The voltage and current phasors are measured at a PMU bus and the bus at the other end of the connecting line could be a pseudo-measurement bus whose voltage may be calculated from other measurements. This gives rise to the *fault observability constraints* given by equation (3).

$$g(x_i, x_j) \equiv x_i + x_j \geq 1 \quad i \neq j; i, j \in \text{S}, \text{i,j} \notin \text{T} \tag{3}$$

ZIB constraints: The ZIB-constraint for the ZIB k is as follows ([15]).

$x_k + \sum_{i \in T_k} x_i$ ($\forall k \in T$) $\geq$ Number of buses linked with ZIB k minus number of cross links in buses directly linked to ZIB k. (4)

Restrict-Bus constraint: A bus that cannot be installed with a PMU is termed as a restricted bus. A power grid may not have any restricted buses. The data such as the number of restricted buses and their identifiers are necessary for constructing the *restrict-bus constraint*. Let RB be the set of indices of restricted buses. It becomes obvious that $x_i = 0 \; \forall i \in RB$. Eqn. (5) gives the restrict-bus constraint.

$$H(RB).(\textstyle\sum_{i \in RB} x_i = 0) \tag{5}$$

$$H(RB) = \begin{cases} 0, & if \; RB = \{\,\} \\ 1, & otherwise \end{cases} \tag{6}$$

The general mathematical formulation of the FOP for SG(n) is as given by P.

P: $\{ \text{Min}\, Z = \sum_{i \in S} x_i \; s.t. \; (2) - (5), x_i = 0 \; or \; 1 \quad \forall i \in S \}$

Mathematical program P may possess multiple optimal solutions depending upon the topology of a given power system. One of the algorithms of the proposed method delivers multiple GOSs to P. The quality of an optimal solution to P is assessed on the basis of value of a redundancy index employed by power engineers. The *bus observability index*(BOI) of a bus is defined as the number of PMUs observing the bus ([21]). The *bus observability redundancy index*($BORI$) ([22]) of a bus is equal to *BOI minus one*. The sum of $BOIs$ is known as the *system observability redundancy index*($SORI$) ([21]) and the sum of $BORIs$ is known as the *observability redundancy index* (ORI) ([22]).
We have,

$$BORI = BOI - 1$$
$$\Rightarrow \sum_{i \in S} BOI = \sum_{i \in S} BORI + \sum_{i \in S} 1$$
$$\Rightarrow SORI = ORI + n \tag{7}$$

Power engineers may employ either SORI or ORI to assess the quality of a solution to P, for *n* being a constant. Those yielding the highest $SORI$ value are regarded as the BGOSs to P. Determining the optimal solutions to P followed by assessing their quality becomes computationally burdensome. Therefore, we need to maximize the $SORI$ subject to the constraints ((2) – (5)) such that the optimal number of PMUs remains maintained. Corresponding general formulation is as given by Q.

Q: $\{Max \zeta = \sum_{i \in S} (\sum_{j \in S} a_{ij}) \, x_i \; s.t. (2) - (5), \; \sum_{i \in S} x_i = Z^o, x_i = 0 \; or \; 1 \quad \forall i \in S\}$

3. Method

This section presents a method that delivers multiple global optimal solutions to the FOP for a given power grid. The method can be called as Verma's method. In this, Algorithm.1 of the method delivers complete set of global optimal solutions to the FOP, in groups of different SORI values. Algorithm.2 of the method delivers complete set of the best global optimal solutions to the FOP for a given power grid. Define

I, J : Iteration counters.
S_I : Set of indices of PMU buses at the I^{th} iteration of Algorithm.1.
S_J : Set of indices of PMU buses at the J^{th} iteration of Algorithm.2.

Algorithm.1: COMPLETE SET OF GLOBAL OPTIMAL SOLUTIONS TO THE FOP

Step 1: Set $I = 1$ and $Z_0^o = n$.
Step 2: Solve P for its optimal solution. Let $S_I = \{ i \,|x_i^o = 1\}$.
Step 3: If solution to P does not exist or $Z_I^o > Z_{I-1}^o$, go to Step 5.
Step 4: Add the constraint $\sum_{i \in S_I} x_i \leq CARD(S_I) - 1$ to P, set $I = I + 1$ and go to Step 2.
Step 5: Exit.

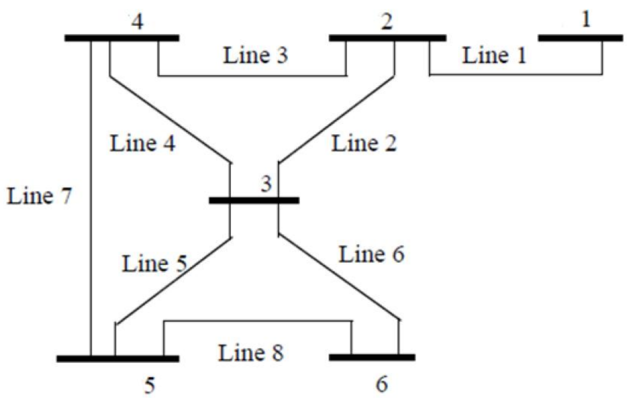
Algorithm.2: COMPLETE SET OF THE BEST GLOBAL OPTIMAL SOLUTIONS TO THE FOP

Step 1: Step 1 of Algorithm.1.
Step 2: Step 2 of Algorithm.1.
Step 3: Set $J = I$ and $\zeta_0^o = 0$.
Step 4: Solve Q for its optimal solution. Let $S_J = \{ i \,|x_i^o = 1\}$.
Step 5: If solution to Q does not exist or $\zeta_J^o < \zeta_{J-1}^o$, go to Step 7.
Step 6: Add the constraint $\sum_{i \in S_J} x_i \leq CARD(S_J) - 1$, set $J = J + 1$ and go to Step 4.
Step 7: Exit.

The binary cut $\sum_{i \in S_I} x_i \leq CARD(S_I) - 1$due to the solution to P at the I^{th} iteration. Algorithm.1 stops recurrence of a solution to P in subsequent iterations ([23]). Similarly, the binary cut $\sum_{i \in S_J} x_i \leq CARD(S_J) - 1$ due to the solution to Q at the J^{th} iteration rules out the possibility of recurrence of a solution in subsequent iterations ([23]).*This **author** and Zheng Zhao have successfully used this cut in [24] and [25], respectively, to obtain all GOSs to the problems falling within the realm of* *normal observability of power systems*. Non-commercial (integer) linear programming problem solver ([26]) solves $FOPs$ in Sections III, IV and V on a computing machine with configurations: 2.3 GHz Intel Core i7 CPU with 8 GB RAM.

4. Illustration

Consider the 6-bus sample network shown in Figure 1 (borrowed from [15]). Here $\Psi = \{1\}$, $S = \{1,2,3,4,5\}$, $RB = \{\}$and $SORI = 2x_1 + 4x_2 + 5x_3 + 4x_4 + 4x_5 + 3x_6$. There arise eight different cases: Case 1(No ZIB, $RB = \{\}$, Algorithm: Algorithm.1), Case2 (ZIBs: Bus3, $RB = \{\}$, Algorithm: Algorithm.1), Case 3 (No ZIB, $RB = \{\}$, Algorithm: Algorithm.2), Case 4 (ZIB: Bus 3, $RB = \{\}$, Algorithm: Algorithm.2), Case 5 (No ZIB, $RB = \{2,5\}$, Algorithm:Algorithm.1), Case 6 (ZIB: Bus3, $RB = \{2,5\}$, Algorithm: Algorithm.1), Case 7 (No ZIB, $RB = \{2,5\}$, Algorithm: Algorithm.2) and Case 8 (ZIB: Bus3, $RB = \{2,5\}$, Algorithm:Algorithm.2).



<Figure 1: 6 bus sample network>

A. ***Through Algorithm.1***

Case 1(No ZIB, $RB = \{\}$,Algorithm:Algorithm.1): In this case, the terminal bus constraint is given $x_1 = 1$, and the set of fault observability constraint is given by$C_1 = \{x_1 + x_2 \geq 1, x_2 + x_4 \geq 1, x_2 + x_3 \geq 1, x_3 + x_4 \geq 1, x_3 + x_5 \geq 1, x_3 + x_6 \geq 1, x_4 + x_5 \geq 1, x_5 + x_6 \geq 1\}$. The mathematical formulation of the FOP for this case is given by P.1.

$P.1$: $\{Min Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 s.t. x_1 = 1, C_1, x_i = 0 \ or\ 1 \forall (1 \leq i \leq 6)\}$

Let Z_i^obe the optimal value of $Z_i$at the optimal solution to $P.1$ at the i[th] iteration.

Step.1: Set$I = 1$.
Step 2: Solve $P.1$ for $(x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o) = (1, 1, 1, 0, 1, 0)$ atwhich $Z_1^o = 4$. $S_1 = \{1,2,3,5\}$.
Step 3: As solution to $P.1$ exists, go to Step 4.
Step 4: Add the constraint $x_1 + x_2 + x_3 + x_5 \leq 3$ to P.1, set $I = I + 1 (= 2)$ and go to Step 2.
Step 2: Solve the current version of $P.1$ for $(x_1^o, x_2^o,$ $x3o, x4o, x5o, x6o = 1, 0, 1, 1, 1, 0, Z2o=$ $4.S_2 = \{1, 3, 4, 5\}$.
Step 3: As the solution to $P.1$ exists, go to Step 4.
Step 4: Add the constraint $x_1 + x_3 + x_4 + x_5 \leq 3$ to the current$P.1$, set $I = I + 1 (= 3)$ and go to Step 2.
Step 2: Solve $P.1$for $(x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o) = (1, 0, 1, 1, 0, 1)$, $Z_3^o = 4. S_3 = \{1,3,4,6\}$.
Step 3: As the current version of P.1 possesses solution, go to Step 4.
Step 4: Add the constraint $x_1 + x_3 + x_4 + x_5 \leq 3$ to $P.1$, set $I = I + 1 (= 4)$ and go to Step 2.
Step 2: Solve the current $P.1$ for its optimal solution at which $Z_4^o = 5$.
Step 3: As $Z_4^o > Z_3^o$, go to Step 5.
Step 5: Exit.

GOSs: $\{1,2,3,5\}, \{1,3,4,5\}$*and* $\{1,3,4,6\}$. *Z^o = 4.*
BGOSs:$\{1,2,3,5\}$*and* $\{1,3,4,5\}$. *Z^o = 4 and SORI = 15.*

B. ***Through Algorithm.2***

Case 3 (No ZIB, $RB = \{\}$,Algorithm:Algorithm.2): Mathematical formulations of the FOP for this case are given by P.1 and P.3. According to the first iteration of Algorithm.1 on P.1, $Z^o = 4$ (see Case 1).

$P.3$: $\{Max\ \zeta = 2x_1 + 4x_2 + 5x_3 + 4x_4 + 4x_5 + 3x_6 s.t.\ C1,\ x_1 = 1 x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 4, x_i = 0\ or\ 1 \forall\ i\ (1 \leq i \leq 6)\}$

Let ζ_i^o be the optimal value of ζ_i at optimal solution to P.3 at the i[th] iteration.

Step 3: Set $J = 1$ and $\zeta_0^o = 0$.
Step 4: Solve $P.3$. Optimal solution to $P.3$ is (1, 0, 1, 1, 1, 0), $\zeta_1^o = 15$..$S_1 = \{1,3,4,5\}$.
Step 5: As the solution to $P.3$ exists and$\zeta_1^o > \zeta_0^o$, go to Step 6.
Step 6: Add the constraint $x_1 + x_3 + x_4 + x_5 \leq 3$ to $P.3$, set $J = J + 1$ and go to Step 4.
Step 4: Solve the current version of $P.3$ for $(1,1,1,0,1,0)$, $\zeta_2^o = 15$. $S_2 = \{1,2,3,5\}$.
Step 5: As the solution to $P.3$ exists and $\zeta_J^o = \zeta_{J-1}^o$, go to Step 6.
Step 6: Add the constraint $x_1 + x_2 + x_3 + x_5 \leq 3$ to $P.3$, set $J = J + 1$ and go to Step 4
Step 4: Solve the current version of P.3. Optimal solution is $(1,0,1,1,0,1)$, $\zeta_3^o = 14$.
Step 5: As $\zeta_3^o < \zeta_2^o$, go to Step 7.
Step 7: Exit.

BGOSs:$\{1,3,4,5\}$ *and* $\{1,2,3,5\}$.*Z^o = 4 and SORI = 15.*

Similarly, Case2 (ZIBs: Bus3, $RB = \{\}$, Algorithm: Algorithm.1) is solved for *GOSs:*$\{1,2,5\}$, $\{1,4,5\}$*and*

$\{1,4,6\}$ *such that* $Z^o = 3$ *and BGOSs:* $\{1,2,5\}$ *and* $\{1,4,5\}$ *at whcih* $.Z^o = 3$ *and SORI = 10, Case 4 (ZIB: Bus 3,* $RB = \{\}$, *Algorithm:Algorithm.2) is solved for* BGOSs: $\{1,4,5\}$and $\{1,2,5\}$*at which* $Z^o = 3$ and $\zeta^o = 10$, *Case 5 (No ZIB,RB* $= \{2,5\}$, *Algorithm:Algorithm.1) is solved for GOS* and *BGOS*: $\{1,3,4,6\}$ *at which* $Z^o = 4$ and $SORI = 14$, *Case 6 (ZIB: Bus3,RB* $= \{2,5\}$,*Algorithm:Algorithm.1) is solved for GOS, BGOS*:$\{1,4,6\}$ at which Z^o= 3 and SORI = 9, *Case 7 (No ZIB, RB* $= \{2,5\}$, *Algorithm:Algorithm.2) is solved for BGOS*: $\{1,3,4,6\}$ at which $Z^o = 4$ and $\zeta^o = 15$ and *Case 8 (ZIB:Bus3,RB* $= \{2,5\}$, *Algorithm:Algorithm.2) is solved for BGOS*: $\{1,4,6\}$ at which Z^o =3 and $\zeta^o = 10$.

TABLE 1 summarizes the results of all the eight cases dealt with in this section. **The boldfaced entries in TABLE 1 show that the proposed method delivers multiple GOSs and multiple BGOSs to the FOP.**The dash (-) in TABLE 1 shows that the corresponding case has not been dealt with in the corresponding reference.

TABLE 1: SUMMARY OF RESULTS

Cases	Ref.	PMUs	Number of Solutions	Solutions	SORI	Best SORI	Best Solutions Count
Case 1	[15]	4	1	{1,2,3,5}	15	15	1
	This Paper	**4**	**3**	**{1,2,3,5}**	**15**	**15**	**2**
				{1,3,4,5}	**15**		
				{1,3,4,6}	14		
Case 2	[15]	3	1	{1,4,5}	10	10	1
	This Paper	**3**	**3**	**{1, 2, 5}**	**10**	**10**	**2**
				{1, 4, 5}	**10**		
				{1,4,6}	9		
Case 3	[15]	-	-	-	-	-	-
	This paper	**2**	**2**	**{1, 3, 4, 5}**	**15**	**15**	**2**
				{1, 2, 3, 5}	**15**		
Case 4	[15]	-	-	-	-	-	-
	This paper	**2**	**2**	**{1,4,5}**	**10**	**10**	**2**
				{1,2,5}	**10**		
Case 5	[15]	-	-	-	-	-	-
	This paper	**4**	**1**	**{1,3,4,6}**	**14**	**14**	**1**
Case 6	[15]	-	-	-	-	-	-
	This paper	**3**	**1**	**{1,4,6}**	**9**	**9**	**1**
Case 7	[15]	-	-	-	-	-	-
	This paper	**4**	**1**	**{1,3,4,6}**	**15**	**15**	**1**
Case 8	[15]	-	-	-	-	-	-
	This Paper	**3**	**1**	**{1,4,6}**	**9**	**9**	**1**

5. Simulation Results

To stop the paper from being unwieldy, this section reports simulation results only on the IEEE 14-Bus system and the IEEE 30-Bus system ([27]).Commonly known Single-line diagrams of these systems ([27]) are not re-produced in this paper for shortage of space. This section does not consider non-optimal solution reported in [15] and [17], for comparing the proposed method vis-à-vis the existing methods.

TABLE 2: SOLUTIONS TO FOP FOR IEEE 14-BUS (NO ZIB)

$(x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o, SORI^o)$
(1,1,0,1,0,1,0,1,1,1,0,0,1,0,8,33)
(1,1,0,1,0,1,0,1,1,0,1,0,1,0,8,33)
(0,1,0,1,1,0,0,1,1,0,1,1,1,0,8,33)
(0,1,0,1,1,1,0,1,1,1,0,0,1,0,8,35)
(0,1,0,1,1,1,0,1,1,0,1,0,1,0,8,35)

TABLE 3: FREQUENCY DISTRIBUTION OF SORI

SORI:	33	**35**
f:	3	**2**

TABLE 4: PERFORMANCE COMPARISON

System	Ref.	PMUs	Number of Solutions	Best SORI	Number of the best solutions
IEEE 14-Bus	[15]	8	1	33	-
	[17]	8	4	33	-
	This Paper	**8**	**5**	**35**	**2**

TABLE 5: SOLUTIONS TO FOP FOR IEEE 14-BUS (ZIB: BUS 7)

$(x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, Z^o, SORI^o)$
(0,1,1,0,1,0,0,1,1,0,1,1,1,0,8,28)
(1,1,0,1,0,1,0,1,0,1,0,1,0,1,8,28)
(1,1,0,1,0,1,0,1,0,1,0,0,1,1,8,29)
(0,1,0,1,1,1,0,1,0,1,0,1,0,1,8,30)
(1,1,0,1,0,1,0,1,1,1,0,0,1,0,8,30)
(0,1,1,0,1,1,0,1,1,0,1,0,1,0,8,30)
(1,1,0,1,0,1,0,1,1,0,1,0,1,0,8,30)
(0,1,1,0,1,1,0,1,1,1,0,0,1,0,8,30)
(0,1,0,1,1,0,0,1,1,0,1,1,1,0,8,30)
(0,1,0,1,1,1,0,1,0,1,0,0,1,1,8,31)
(0,1,0,1,1,1,0,1,1,0,1,0,1,0,8,32)
(0,1,0,1,1,1,0,1,1,1,0,0,1,0,8,32)

TABLE 6: FREQUENCY DISTRIBUTION OF SORI

SORI:	28	29	30	31	**32**
f:	2	1	6	1	**2**

TABLE7: PERFORMANCE COMPARISON
(CONSIDERING IEEE 14-BUS SYSTEM (ZIB: 7))

System	Ref.	PMUs	Solutions Count	Best SORI	Number of the Best solutions
IEEE 14-Bus	[15]	8	1	30	-
	[17]	8	2	32	1
	This Paper	**8**	**12**	**32**	**2**

TABLE 8: SOLUTIONS TO FOP FOR IEEE 30-BUS (NO ZIB)

$(x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, x_{15}^o, x_{16}^o, x_{17}^o, x_{18}^o, x_{19}^o, x_{20}^o, x_{21}^o, x_{22}^o, x_{23}^o, x_{24}^o, x_{25}^o, x_{26}^o, x_{27}^o, x_{28}^o, x_{29}^o, x_{30}^o, Z^o, SORI^o)$
(1,0,0,1,1,1,0,1,0,1,1,0,1,1,1,1,0,0,1,0,1,0,0,1,0,1,1,0,0,1,17,64)
(1,0,0,1,1,1,0,1,0,1,1,0,1,1,1,1,0,0,1,0,1,0,0,1,0,1,1,0,1,0,17,64)
(1,0,0,1,1,1,0,1,0,1,1,0,1,1,1,1,0,0,1,0,0,1,0,1,0,1,1,0,0,1,17,65)
(1,0,0,1,1,1,0,0,0,1,1,0,1,1,1,1,0,0,1,0,1,0,0,1,0,1,1,1,1,0,17,65)
(1,0,0,1,1,1,0,1,0,1,1,0,1,1,1,1,0,0,1,0,0,1,0,1,0,1,1,0,1,0,17,65)
(1,0,0,1,1,1,0,0,0,1,1,0,1,1,1,1,0,0,1,0,1,0,0,1,0,1,1,1,0,1,17,65)
(1,0,0,1,1,1,0,0,0,1,1,0,1,1,1,1,0,0,1,0,0,1,0,1,0,1,1,1,0,1,17,66)
(1,0,0,1,1,1,0,0,0,1,1,0,1,1,1,1,0,0,1,0,0,1,0,1,0,1,1,1,1,0,17,66)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,0,1,0,17,67)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,0,1,0,17,67)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,0,0,1,17,67)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,0,0,1,17,67)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,0,1,0,17,67)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,0,1,0,17,67)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,0,0,1,17,67)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,0,0,1,17,67)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,0,1,0,17,67)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,0,0,1,17,67)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,0,0,1,17,67)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,0,1,0,17,67)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,1,0,1,17,68)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,0,0,1,17,68)

(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,0,0,1,17,68)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,1,0,1,17,68)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,0,1,0,17,68)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,0,1,0,17,68)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,1,0,1,17,68)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,0,0,1,17,68)
(1,0,0,1,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,0,0,1,17,68)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,1,0,1,17,68)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,0,1,0,17,68)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,0,1,0,17,68)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,1,1,0,17,68)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,1,1,0,17,68)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,0,0,1,17,68)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,1,1,0,17,68)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,0,1,0,17,68)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,1,0,1,17,68)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,1,0,0,1,0,1,0,0,1,0,1,1,1,1,0,17,68)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,1,1,0,17,68)
(0,1,1,0,0,1,1,1,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,0,0,1,17,68)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,1,0,1,17,68)
(0,1,1,0,1,1,0,1,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,0,1,0,17,68)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,1,0,0,1,0,1,1,1,1,0,17,68)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,1,0,1,17,69)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,1,0,1,17,69)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,1,0,1,17,69)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,1,0,1,17,69)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,1,1,0,17,69)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,1,1,0,17,69)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,1,1,0,17,69)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,1,0,1,17,69)
(0,1,1,0,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,1,1,0,17,69)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,1,1,0,17,69)
(0,1,1,0,0,1,1,0,0,1,1,1,1,0,1,1,0,0,1,0,0,1,0,1,0,1,1,1,0,1,17,69)
(1,0,0,1,1,1,0,0,0,1,1,1,1,0,1,0,1,0,1,0,0,1,0,1,0,1,1,1,1,0,17,69)

TABLE 9: SORI DISTRIBUTION (IEEE 30-BUS SYSTEM (NO ZIB))

SORI:	64	65	66	67	68	**69**
f:	2	4	2	12	24	**12**

TABLE 10: PERFORMANCE COMPARISON (IEEE 30-BUS SYSTEM (NO ZIB))

System	Ref.	PMUs	Number of Solutions	BEST SORI	Number of the Best Solutions
IEEE 30-Bus	[15]	17	1	68	-
	[17]	17	2	68	-
	This Paper	**17**	**56**	**69**	**12**

TABLE 11: THE SOLUTIONS TO FOP FOR IEEE 30-BUS (ZIBs: 6, 9, 22, 25, 27, 28)

$(x_1^o, x_2^o, x_3^o, x_4^o, x_5^o, x_6^o, x_7^o, x_8^o, x_9^o, x_{10}^o, x_{11}^o, x_{12}^o, x_{13}^o, x_{14}^o, x_{15}^o, x_{16}^o, x_{17}^o, x_{18}^o, x_{19}^o, x_{20}^o, x_{21}^o, x_{22}^o, x_{23}^o, x_{24}^o, x_{25}^o, x_{26}^o, x_{27}^o, x_{28}^o, x_{29}^o, x_{30}^o, Z^o, SORI^o)$
(0,1,1,0,0,0,1,0,0,1,1,1,1,0,1,0,1,0,1,0,0,0,0,1,0,1,1,1,0,1,15,57)
(0,1,1,0,0,0,1,0,0,1,1,1,1,0,1,0,1,0,1,0,0,0,0,1,0,1,1,1,1,0,15,57)
(0,1,1,0,0,0,1,0,0,1,1,1,1,0,1,1,0,0,1,0,0,0,0,1,0,1,1,1,0,1,15,57)
(0,1,1,0,0,0,1,0,0,1,1,1,1,0,1,1,0,0,1,0,0,0,0,1,0,1,1,1,1,0,15,57)

TABLE 12: PERFORMANCE COMPARISON (IEEE 30-BUS SYSTEM (ZIBs: 6, 9, 22, 25, 27, 28))

System	Ref.	PMUs	Number of Solutions	BEST SORI	Number of the Best Solutions
IEEE 30-Bus	[15]*	-	-	-	-
	[17]*	-	-	-	-
	This Paper	**17**	**56**	**69**	**12**

(*Refs. [15], [17] report non-optimal solutions)

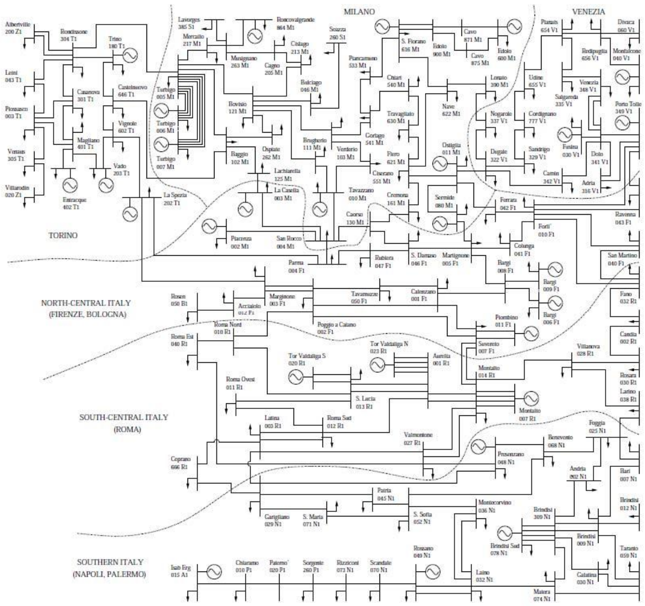
Results and Discussion:

1. The proposed method delivers ***five*** GOSs to the FOP for the IEEE 14-Bus system (see TABLE 2) whereas the DE method given in [17] could deliver only ***two*** GOSs to the problem (Cfm. TABLE I in [17]). Only two of the five solutions delivered by the proposed method are the BGOSs to the FOP such that SORI = 35. This SORI value (35) given by the proposed method is superior to the highest SORI value (31) given by solutions delivered by DE method in [17]. **This shows that the proposed method delivers better and more number of the best global solutions to the FOP.**
2. When bus 7 is considered as a ZIB in the IEEEB14-Bus then the proposed method delivers ***12*** solutions out of which only ***two*** solutions at which SORI=32 are the BGOSs to the FOP. The DE method [17] delivers ***one*** BGOs to the FOP. **This shows that the proposed method delivers more number of the best global optimal solutions to the FOP.**
3. The proposed method delivers ***56*** GOSs to the FOP for the IEEE 30-Bus system (see TABLE 8) whereas the DE method [17] could deliver ***one*** solution to the problem (Cfm. TABLE I in [17]). Out of 56 solutions, 12 solutions are BGOSs such that SORI = 69. It may be noted that the DE method [17] could not produce any best global optimal solution. **This comparison shows that the proposed method delivers much more number of global optimal solutions and much more number of the best global optimal solutions to the FOP.**
4. When buses numbered 6, 9, 22, 25, 27 and 28 are considered as ZIBs in the IEEE 30-Bus system, the proposed method delivers ***four*** best global solutions such that SORI = 57.One may easily verify that the only solution to the FOP for the IEEE 30-Bus system (with buses numbered 6, 9, 22, 25, 27, and 28 as ZIBs) reported in [17] is infeasible and hence not optimal. **This shows that the proposed method always produce optimal solutions.**

The foregoing discussion shows that the proposed method always delivers multiple global optimal solutions and multiple best global optimal solutions to the FOP for a given power system.

5. Case Study

The Italian 400 kV transmission system [28] shown in Figure 2 may be represented by the set GRID of ordered pairs such that the first element and the second element, respectively, in an ordered pair represent a bus number and the place where the bus exists. This section does not report all solutions to the FOP for shortage of space. The grid does not have any ZIB, nor any restricted buses. First iteration of Algorithm.1 delivers Z^o = 75. In the absence of restricted buses, every $BGOS$ to the FOP for the transmission system is an acceptable and implementable solution. Therefore, it is sufficient to obtain the first solution to Q for the Italian transmission system. Accordingly, those boldfaced in the set GRID are the best optimal strategic sites for 75 PMUs such that $SORI = 273$.



< **Figure 2. One-line diagram of the Italian 400 kV transmission system (most of this information is publicly available at the GRTN web site www.grtn.it [28])** >

GRID = {**(1, Albertville)**, (2, Leini), **(3, Piosasco)**, (4, Venaus), **(5, Villarodin)**, **(6, Rondissone)**, (7, Casanova), **(8, Magliano)**, **(9, Entracque)**, (10, Trino), **(11, Castelnuovo), (12, Vignole)**, (13, Vado), (14, Turbigo 005 M1), (15, Turbigo 006 M1), **(16, Turbigo 006 M1), (17, Lavorges), (18, Mercallo)**, (19, Musigrano), **(20, Roncovalgrande), (21, Casgo)**, (22, Cislago), (**23, Bovisio), (24, Baggio)**, (25, Ospiate), (26, Bulciago), **(27, Soazza), (28, Verderio)**, (29, Brugherio), (30,Lachiarella), **(31, Tavazzano), (32, Piancampuno)**, (33, Gorlago), (34, Ciserano), (35, San Rocco), (36, S. Franco), **(37, Chiari)**, (38, Travagliato), **(39, Flero)**, (40,), **(41, Cremona)**, **(42, Nave), (43, Ostiglia),** (44, Semide), **(45, Lonato),** (46, Cavo), **(47, Edolo)**, (48, Nogarole), **(49, Dugale)**, (50, Udine), **(51, Cordigrano)**, (52, Sandrigo), **(53, Planais), (54, Salgareda)**, (55, Camin), (56, Redipuglia), (57, Veneria), **(58, Fusira), (59, Dolo)**, (60, Adria), **(61, Divaca), (62, Monfalcone), (63, Parto Tolle), (64, La Casella), (65, Piacenza), (66, Rosen), (67, Marginone)**, (68, Acciaiolo), **(69, Parma), (70, Caorso)**, (71, Rubiera), (72, Tavarnuzze), **(73, S. Darnaso), (74, Calenzano), (75, Martignone)**, (76, Suvereto), (77, Colunga), (78, Bargi 008 F1), **(79, Piombino), (80, Forli), (81, Bargi009 F1), (82, Bargi 006 F1)**, (83, Ravenra), (84, San Martiro), **(85, Roma Est)**, (86, Ceprano), (87, Roma Nord), (88, Roma Ovest), **(89, Latina), (90, Tor Valdaliga S), (91, Roma Sud), (92, S. Lucia), (93, Tor Valdaliga N)**, (94, Valmontone), (95, Aurelia), **(96, Montalto 014 R1), (97, Montalto 007 R1), (98, Villanova), (99, Fano)**, (100, Candia), **(101, Rosara)**, (102, Larino), **(103, Garigliano)**, (104, S. Maria), (105, Patina), **(106, S. Sofia), (107, Presenznano)**, (108, Montecorvino), (109, Benevento), **(110, Foggia)**, (111, Andria), (112, Bari), **(113, BrindisiSud), (114, Brindisi 309 N1), (115, Brindisi 009 N1), (116, Brindisi 012 N1)**, (117, Galatira), **(118, Taranto)**, (119, Matera), **(120, Laino)**, **(121, Rossano)**, (122, Scandale), **(123, Rizziconi)**, (124, Sorgente), **(125, Paterno)**, (126, Chiararno), **(127, Isab Erg), (128, Turbigo 007 M1)**, (129, Cavo), (130, Ferrara), **(131, Poggio a Caiano)**}.

6. Conclusion

Mathematical formulation of the fault observability problem has been upgraded, keeping in mind that utilities might identify the buses that cannot be installed with PMUs for various reasons. A simple yet robust method, known as Verma's method, capable of delivering the complete set of the global optimal solutions and the complete set of the best global optimal solutions to the fault observability problem has been proposed. The method has successfully been illustrated through a sample network. Simulation results on the IEEE 14-Bus system and the IEEE 30-Bus system have been reported. Comparison among the proposed method, a known binary integer linear programming based method that delivers only solution and a known differential evolution method shows that the proposed method always delivers the complete set of the global optimal solutions and the complete set of the best global optimal solutions to the fault observability problem. Application of the method to a real-world power transmission system shows practical use of the proposed method.

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