A SIMPLIFIED MODELING APPROACH FOR ACCOUNTING SKEWING EFFECT IN ROTOR BARS OF SQUIRREL CAGE INDUCTION MOTOR AND ITS APPLICATION IN MOTOR INDUCTANCE CALCULATION

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***Abstract*:** *Recently Multiple coupled circuit approach using 2D Modified Winding Function Theory (2D-MWFT) is used to model the squirrel cage induction motor for asymmetrical fault analysis. Simulation based characterization of inclined eccentricity fault in the machine can be done using this approach. In order to enhance correctness of model, it is preferred to include the skewing of rotor bars along with eccentricity while computing the mutual inductance between stator and rotor and also between rotor bars. In this paper, a new methodology is presented to calculate the mutual inductance between stator and skewed rotor winding along with eccentricity using 2D-MWFT and its impact on various machine parameters are discussed.*

Key words: dynamic model, eccentricity, induction motor, modified winding function theory, skewing

Nomenclature

g^{-1}	Inverse air gap function
ℓ	Stack length
m_s	Number of stator slots
n_r	Number of rotor bars
Ns	Number of stator turns in series
n	Turn function
N	Winding function
n_i	Turn function of winding 'i'
N_j	Winding function of winding 'j'
P	Permeance of air gap
r	Average radius
γ	Rotor skew angle
α_r	Angle between two rotor slots
θ	Rotor angle
Φ	Rotor circumferential angle

1.Introduction

Winding Function Theory (WFT) is used extensively to model three phase induction motor as it can accommodate asymmetrical fault conditions of the motor. In the past several researchers [1-9] have modeled the motor using this theory without considering the skewing effect of rotor bars.

In 1998, a model was proposed for synchronous machine suffering from dynamic eccentricity using modified winding function theory (MWFT)[10]. For inductance calculation, only turn functions of windings are used. The usage of MWFT to model induction motor is reported in paper [11]. It also reports that the mutual inductance between two windings i and j and its transpose remains the same, ($L_{ij}=L_{ji}$) for non-uniform air gap machines as long as the magnetic circuit is linear. Machine with tangential eccentricity is modeled using MWFT in papers [12-14].

A simplified MWFT mutual inductance calculation equation as given in (1) is reported in [15]

$$L_{ij} = 2\pi\mu_0 r\ell\left[\langle Pn_i n_j\rangle - \frac{\langle Pn_i\rangle\langle Pn_j\rangle}{\langle P\rangle}\right] - -(1)$$

Paper [16] reports about the inductance calculation along the axial length 'z' of the motor and considers the effect of skew and axial non uniformity. The mutual inductance between two windings i and j is calculated using 2D Modified Winding Function (2D-MWF) as given in the equation (2)

$$L_{ij}(\theta) = \mu_0 r\int_0^{2\pi}\int_0^{l} n_i(\phi,z,\theta)n_j(\phi,z,\theta)g^{-1}(\phi,z,\theta)dzd\phi - (2)$$

where $n(\Phi,z,\theta)$ is called the 2-D spatial winding distribution and represents the number of the winding turns enclosed by the closed path under consideration and this distribution depends on the geometry of the windings down the length, and n(Φ,z,θ) is obtained by the Equation (3).

$$\mathrm{n}(\phi,z,\theta)=\mathrm{N}(\phi,z,\theta)-\frac{1}{2\pi l\left\langle \mathrm{g}^{-1}(\phi,z,\theta)\right\rangle}\int_0^{2\pi}\int_0^{l}\mathrm{N}(\phi,z,\theta)\mathrm{g}^{-1}(\phi,z,\theta)dzd\phi-(3)$$

The 2D-MWF allows consideration of skewing of rotor bars and rotor eccentricity effects in the inclined direction. In 2006, [17], authors have used 2D-MWF to calculate mutual inductance between stator and rotor of the machine having axial eccentricity and skewed rotor bars. Authors claim that the equation developed by them reduces the computational requirements significantly. The mutual inductance for any rotor angle is calculated using Equation (4)

$$\mathrm{L}_{\mathrm{ij}}(x_r)=2\pi\mathrm{r}\,\ell\mu_0\left[\left[\sum_{k=1}^{q}\sum_{m=1}^{p}\left\langle \mathrm{Pn}_{\mathrm{ik}}\,\mathrm{n}_{\mathrm{jm}}\right\rangle\right]-\frac{\left\langle \mathrm{Pn}_{\mathrm{i}}\right\rangle\left\langle \mathrm{Pn}_{\mathrm{j}}\right\rangle}{\left\langle \mathrm{P}\right\rangle}\right]-(4)$$

where q and p are the number of coils of winding i and j respectively. The techniques presented in papers [16,17] demands the knowledge of coil distribution of stator phases. Paper [18] proposes a method to model the machine in which definition of turn function of rotor bar is modified to include the skewing effect. In this paper, the skewed bar is divided into 8 direct bars of equal length (i=1,2….rotor length/8) in the axial direction and the turn function is defined as in Equation (5).

$$\begin{aligned}\mathrm{n}_{\mathrm{i}}(\phi)&=\left(\frac{1}{\alpha_{\mathrm{i}}}\right)\phi+(i-1) && (i-1)\,\alpha_{\mathrm{i}}<\phi<\mathrm{i}\alpha_{\mathrm{i}}\\ &=\left(-\frac{1}{\alpha_{\mathrm{i}}}\right)\phi+(i+1) && \mathrm{i}\alpha_{\mathrm{i}}<\phi<(i-1)\,\alpha_{\mathrm{i}}-(5)\end{aligned}$$

In Paper [19],authors have considered the skewing effect by incorporating a trigonometric function sin(hγ/2) while defining the turn function of rotor bars where h=1,2,3…..

In this paper, new method is proposed to define the turn function for the skewed rotor bar and mutual inductance between windings is calculated using 2D-MWFT. The equation (1) is extended for mutual inductance calculation between the stator windings and rotor bars and between rotor bars in the axial direction . All the earlier research works reported that there is no change in the mutual inductance magnitude between stator and rotor with and without skew. But our observation is that even though there is no effect of skewing on magnitude of mutual inductance between stator and rotor bar but it does affect the magnitude of mutual inductance between rotor bars and is illustrated in this paper with the help of simulation results.

Section II describes the procedure adopted to model the skewed rotor bar in detail. Advantage of this method is that mutual inductances are calculated in terms of rotor angle and it requires only the knowledge of turn functions of rotor bar and stator windings. MATLAB is used to calculate the mutual inductances between windings. The simulated waveforms of the machine with inclined static eccentricity for different skewing factors are presented in section III. In section IV, conclusion is drawn based on the discussions presented in section III.

2. Calculation of mutual inductance using 2D-MWFT

Consider the skewed rotor as shown in Figure 1. Axial length(z) of rotor is divided into 10 sections. For each section mutual inductance between this section of the rotor bar and stator needs to be calculated. Equation (2) is used to calculate mutual inductance between two windings 'i' and 'j' of the machine and can be rearranged as

$$\mathrm{L}_{\mathrm{ij}}(\theta)=\mu_0\mathrm{r}\int_0^{l}\left[\int_0^{2\pi}\mathrm{n}_{\mathrm{i}}(\phi,z,\theta)\mathrm{n}_{\mathrm{j}}(\phi,z,\theta)\mathrm{g}^{-1}(\phi,z,\theta)d\phi\right]dz-(6)$$

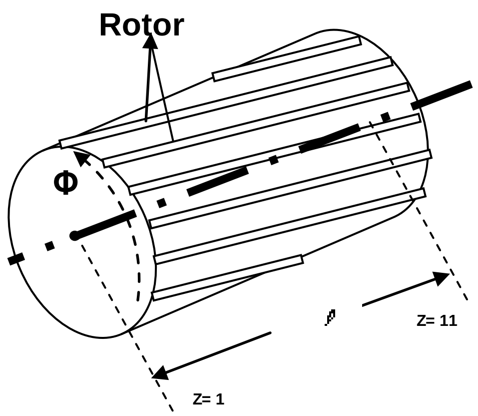


Fig1: A skewed rotor

For a given value of 'z', the Equation (6) can be rewritten as

$$\mathrm{L_{ij}}(\theta,z)=2\pi\mu_0\mathrm{r}\int_0^l\left[\langle \mathrm{Pn_i n_j}\rangle-\frac{\langle \mathrm{Pn_i}\rangle\langle \mathrm{Pn_j}\rangle}{\langle \mathrm{P}\rangle}\right]dz \quad --(7)$$

$$\mathrm{L_{ij}}(\theta)=\sum_{z=1}^{z=11}\mathrm{L_{ij}}(\theta,z) \quad --------------(8)$$

Equations (7) and (8) are used to calculate the mutual inductance between stator phase and skewed rotor bars of the machine having inclined static eccentricity. During modeling and simulation, it is noticed that increase in fragmentation of rotor bar in the axial direction, increases the computational time and memory requirement significantly. A trade off is made between fragmentation number and accuracy in derivative of mutual inductance and each rotor bar is fragmented into 10 equal parts down the axial length of rotor.

The stators phase A turn function and machine parameters considered for simulation are defined in the appendix. Turn function of a rotor bar, is defined considering the Figure 3

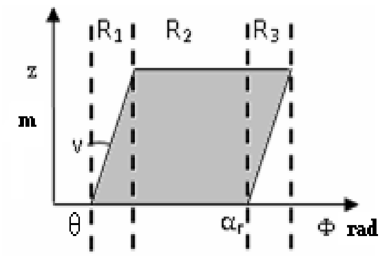


Fig2: A rotor bar with skewing angle γ

For Region $\mathrm{R_1}$ $\quad 0\le\phi\le l\tan(\gamma)$

$$\mathrm{n}(\phi,\mathrm{z},\theta)=1 \qquad 0\le \mathrm{z}\le\left(\frac{\phi(\mathrm{RR})}{\tan(\gamma)}\right)$$

$$=0 \qquad \left(\frac{\phi(\mathrm{RR})}{\tan(\gamma)}\right)<\mathrm{z}\le l$$

$$--(9)$$

For Region $\mathrm{R_2}$ $\quad l\tan(\gamma)\le\phi<\alpha_r$

$$\mathrm{n}(\phi,z,\theta)=1$$

For Region $\mathrm{R_3}$ $\quad \alpha_r\le\phi<\alpha_r+l\tan(\gamma)$

$$\mathrm{n}(\phi,\mathrm{z},\theta)=0 \qquad 0\le \mathrm{z}\le\left(\frac{\phi-\alpha_r}{\tan(\gamma)}\right)\mathrm{RR}$$

$$=1 \qquad \left(\frac{\phi-\alpha_r}{\tan(\gamma)}\right)\mathrm{RR}<\mathrm{z}\le l$$

Φ is in electrical radians.

RR=rotor radius+ air gap length/2

In the proposed methodology, each rotor bar is divided into three regions R_1, R_2 and R_3 as shown in Figure 2. Turn function for rotor bar is defined using Equation 9. Turn functions for other two phases B and C can be obtained by shifting the turn function of stator phase A by 60° and 120° respectively. The turn functions of other rotor bars are calculated by shifting the turn function of rotor bar 1 by (α_r*i) for i=2,3……n_r

3. Results and Observations

3D plots of the simulated mutual inductance between the stator phase A and rotor bar 1 along the axial length of rotor for skewing angles γ=0°,5°,9° are shown in Figure 3.

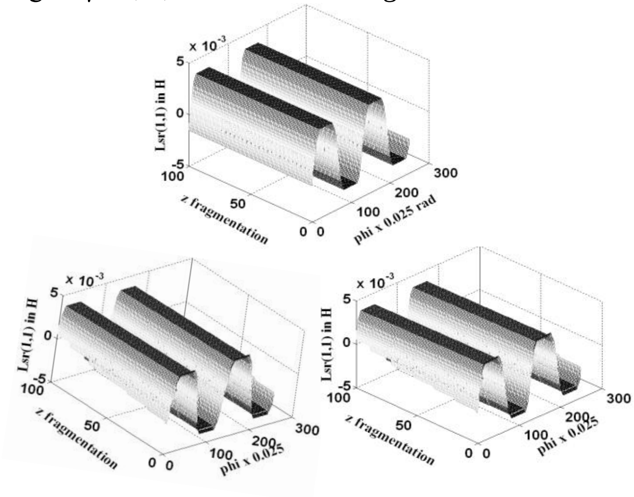


Fig:3. Mutual inductance between stator phase A and rotor bar 1 in the axial direction with skew=0°, 5°, 9°

The mutual inductance between stator phase A and rotor bar1 are simulated with and without skewing effect for one complete cycle and its variation over the rotor angle for a healthy machine is as shown in Figure 4.

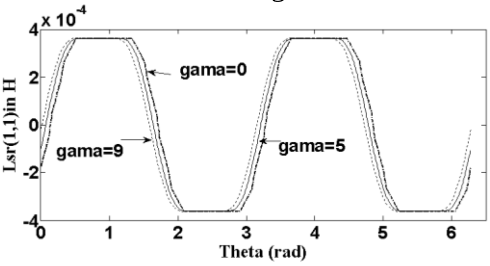


Fig:4 Mutual inductance between stator phase A and rotor bar 1 for different skewing angles γ=0, 5°, 9°

From Figure 4, it is inferred that the mutual inductance magnitude between stator phase A and rotor bar1 does not change with skewing of rotor bars, however the slope becomes smoother with the increase in skewing angles. Mutual inductance between stator phase A and rotor bar1 and its derivative for skewing angles γ=0°,5°,9° are simulated and are as shown in Figures 5. From Figure 5, it is observed that with the increase in skewing, the magnitude of derivative of mutual inductance between stator and rotor decreases.

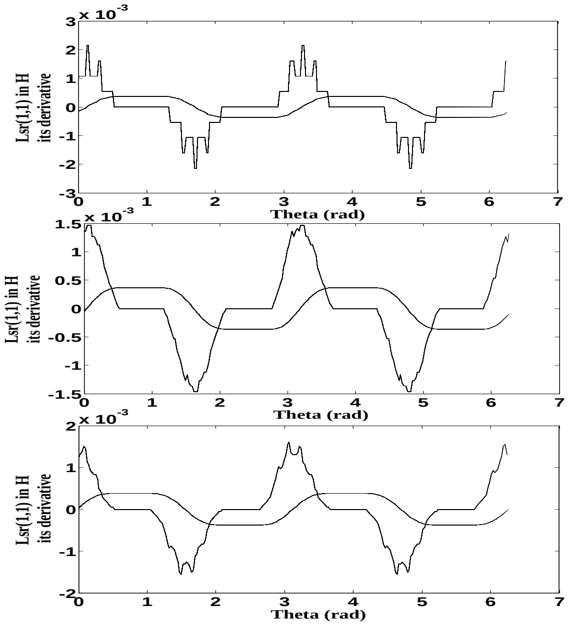


Fig:5 Mutual inductance between stator phase A and rotor bar 1 and its derivative with skew=0º. 5º, 9°

Inclined static air gap eccentricity of 20% in one end and -20% on the other end of rotor is created in the program and the program is run for skewing angles 0º. 5º, 9°. The simulated mutual inductance between stator phase A and rotor bar1 waveforms are shown in Figure 6-7. From Figure 6, it is observed that even with inclined static eccentricity, the magnitude of the mutual inductance remains the same for different skewing angles.

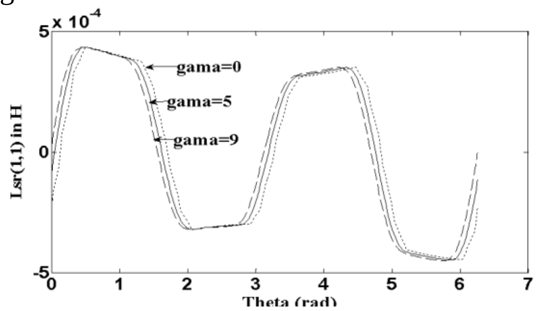


Fig:6 Mutual inductance between stator phase A and rotor bar 1 for different skewing angles γ=0, 5º, 9º with static eccentricity

The variation of self inductance between rotor bar1 with rotor bar1 and mutual inductance between rotor bar1 and rotor bar2 for different rotor angular position are shown in Figures 8-9 respectively.

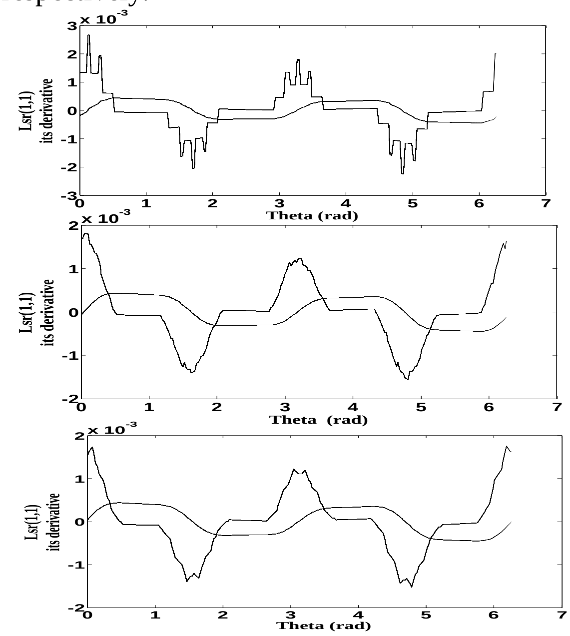


Fig:7 Mutual inductance between stator phase A and rotor bar 1 and its derivative with skew=0°,5°,9° and static eccentricity

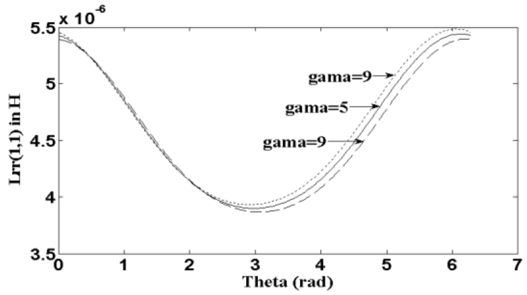


Fig:8 Self inductance between rotor bar 1 and rotor bar 1 with skew=0º,5º,9º with eccentricity

From Figure 9, it is observed that the magnitude of mutual inductance between rotor bar1 and rotor bar2 decrease with increase in skewing angles.

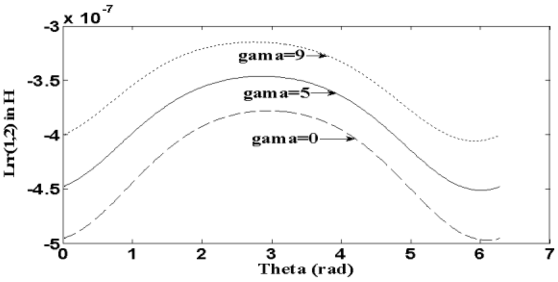


Fig:9 Mutual inductance between rotor bar 1 and rotor bar2 with skew=0º,5º,9º with eccentricity

The dynamic model of induction motor with inclined static eccentricity and skewed rotor is developed on SIMULINK/Matlab platform. Time taken by model to reach steady state value and final steady state speed values for different skewing factors are given in Table 1

Table1: Comparison of parameters for different skewing angles

Skewing factor	γ=0	γ=5	γ=9
Time taken to reach steady state in secs	0.28	0.42	0.69
Final speed rad/sec	188.5	188.5	188.5

From the table, it is inferred that simulation time to reach final steady speed increase with skewing whereas the final speed remains unaffected.

The simulated waveforms of Speed ω versus time, dynamic torque Te versus time, Stator phase flux and current versus time, rotor flux and current versus time for the machine operating under no load are as shown in figure 10-12.

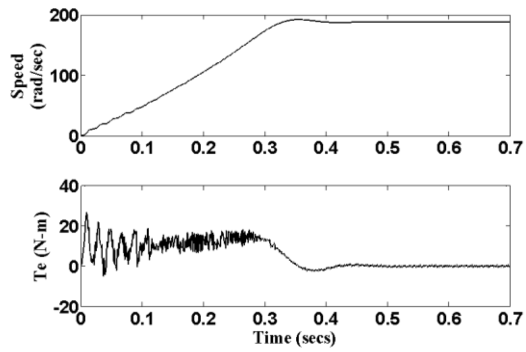


Fig10. Speed ω dynamic torque Te, of the machine with skewing factor=5 and static eccentricity

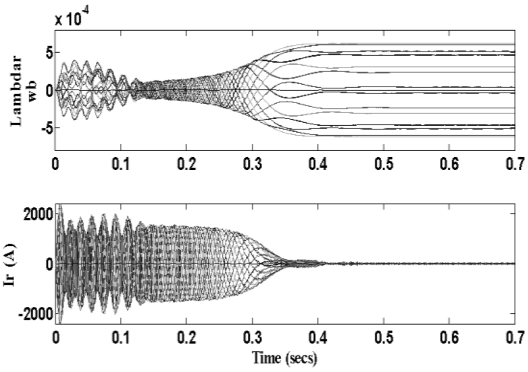
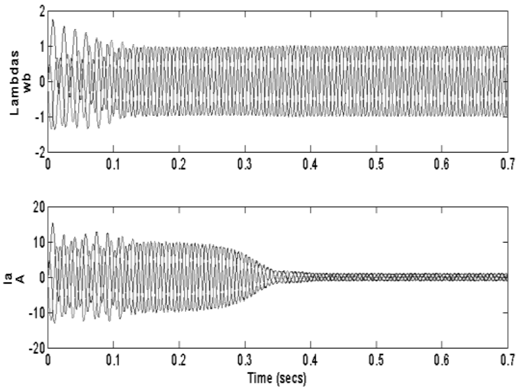


Fig11 Stator flux, Stator phase current Is of the machine with skewing factor=5 and static eccentricity



. Fig12 Rotor flux, Rotor bar currents Ir of the machine with skewing factor=5 and static eccentricity

4. Conclusion

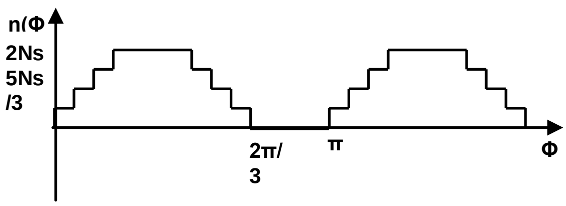
The mutual inductance between stator phase and skewed rotor bar varies down the axial length of rotor as seen in the 3D plot (Figure3). From the simulated waveforms, it is concluded that skewing of rotor bars in the machine is not affecting the magnitude of mutual inductances between stator phase and rotor bar even in a machine which has developed inclined static eccentricity. Magnitude of the derivative of the mutual inductance decreases with skewing and it is attributed to smooth slope in the mutual inductance waveforms with skewing. However the magnitude of mutual inductance between two rotor bars reduces with increase in skewing angle. It is observed that with the increase in skewing angles of rotor bars, the time taken by the machine to reach steady state speed increases. It is attributed to the existence of non uniform fluxes along the rotor length due to skewing of rotor bars.

5. Appendix

A. Machine parameters: [2,3]

5.5 kW, 60 Hz, 460V, 4 pole, m_s=36;
n_r=28; 2turns/phase; Ns=90; ℓ =102.4128mm;
ge=0.456438mm; r= 63.2968mm;

B. Turn function and winding function of stator phase A [2,3]



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