A DIRECT APPROACH FOR OBTAINING STEADY-STATE CHARACTERISTICS OF A DOUBLY-FED INDUCTION MACHINE

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Abstract: *In this paper, direct and simple expressions for the doubly-fed induction machine (DFIM) characteristics such as current, power and torque, for different operating conditions, are obtained. Using these expressions, the performance of the DFIM machine can be easily obtained when the magnitude of the rotor voltage, rotor phase angle and speed are changing. To validate the presented expressions, the results obtained from them are compared with previously published results, and the two sets of results are found to be identical.*

Keywords: *Doubly-fed induction machines, steady-state analysis, closed-form expressions.*

1. Introduction

Generation of electrical energy from wind energy has grown in a higher rate around the world [1-2]. This is due to the economical and environmental benefits of this renewable source of energy. Doubly-fed induction machines (DFIMs) are used in wide range in wind plants to produce electrical power [3-5], because this machine can generate electrical power at variable speed, either with speed below (sub) or above (super) synchronous speed [6,7]. Also, among the merits of this machine, its stator can be connected directly to the grid without connecting a power electronics converter, while its rotor is connected to the grid via an inexpensive back-to-back converter because the rotor power is approximately not more than 30% of the stator power [8,9]. By this rotor converter the phasor of the rotor voltage and frequency can be regulated to control the flow of active and reactive power between the grid and the rotor circuit. This enables to maintain a constant grid voltage and to control the machine power factor [9,10]. However, the control of the DFIM is more complicated than the control of comparative machines; such as squirrel-cage induction generator and the synchronous generator [4,9].

Generally, depending on the rotor speed and the rotor voltage phasor, the DFIM has four quadrants of operation. In two quadrants, the DFIM operates as a motor at subsynchronous or supersynchronous speed, and in the other two quadrants the DFIM operates as a generator at subsynchronous or supersynchronous speed. The net power of the machine has a positive value when the machine operates as a motor, while it has a negative value when the machine operates as a generator [7].

Many investigations were carried out to investigate the dynamic performance of the DFIM during the variation of the rotor speed, or to investigate the transient performance of the DFIM during the variations of the grid voltage or the load power [10-16]. These investigations have dealt with the machine performance at certain operating points using different techniques to control the flow of active and reactive power of the machine.

The steady-state operation of the DFIM was dealt with in previous publications [17-21]. In these publications the machine characteristics, such as machine power and electromagnetic torque, are given in complex and indirect expressions.

In this paper, direct, simple and closed-form expressions are obtained for the characteristics of the DFIM for different operating conditions. These expressions include the current, active and reactive power, for both the stator and the rotor of the DFIM. Also, simple expressions for the input power, the net power and the electromagnetic torque of the machine are obtained. These expressions allow to study and investigate the effect of changing the speed of the machine, for certain magnitude and angle of the rotor voltage, on the machine characteristics easily. The results obtained from these expressions are compared with previously published results.

2. Method of Analysis

2.1 Machine Currents

Fig. 1 [9] shows a DFIM system connected to a grid. The stator is connected directly to the grid while the rotor is connected to the grid via two back-to-back converters. Hence, the stator voltage and frequency (V_s and f_s) are constant as long as the grid voltage and frequency are constant. But the rotor voltage and frequency (V_r and f_r) vary as the speed of the machine varies. The rotor frequency can be obtained from:

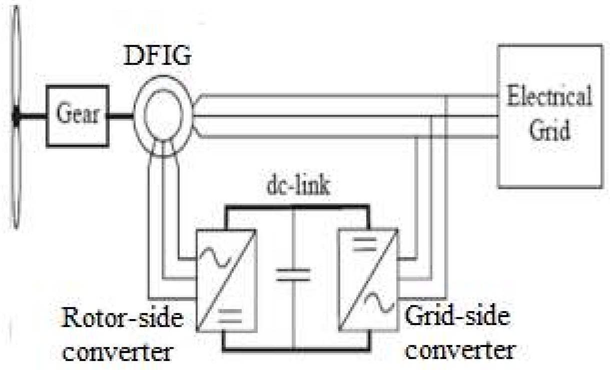

Fig. 1 Variable speed DFIG in Wind turbine

$$f_r = s f_s \quad (1)$$

where s is the slip of the machine; which is obtained from:

$$s = \frac{\omega_s - p\omega_m}{\omega_s} \quad (2)$$

where p is the number of pole pairs of the machine, ω_s and ω_m are the synchronous speed and the mechanical rotor speed, respectively, in rad/s.
The slip for subsynchronous speeds has positive values, while it has negative values for the operation with supersynchronous speeds.
In the case in which s = 0, i.e. at synchronous speed, the rotor voltage is a DC value, hence in this case the machine will operate as a synchronous generator.

Fig. 2 [8,19,20] shows the per-phase equivalent circuit of the DFIM at steady-state operation. The voltage equations describing the operation of the machine are:

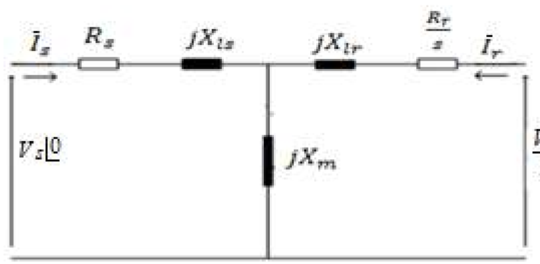
Fig. 2 Equivalent circuit of DFIG at steady-state operation

$$V_s\angle 0 = (R_s + jX_s)\bar{I}_s + jX_m\bar{I}_r \quad (3)$$

and

$$\frac{V_r}{s}\angle\alpha = \left(\frac{R_r}{s} + jX_r\right)\bar{I}_r + jX_m\bar{I}_s \quad (4)$$

where:

$X_s = X_m + X_{ls}$, $X_r = X_m + X_{lr}$

and V_s, V_r, α, R_s, R_r , X_{ls}, X_{lr} and X_m are, respectively, the stator voltage, the rotor voltage referred to the stator, the phase angle of the rotor voltage, the stator resistance, the rotor resistance referred to the stator, the stator leakage reactance, the rotor leakage reactance referred to the stator and the magnetization reactance.
From eqns. (3) and (4), the stator and rotor currents can be obtained as:

$$\bar{I}_s = \frac{(\frac{R_r}{s}V_s + X_m\frac{V_r}{s}\sin\alpha) + j(X_rV_s - X_m\frac{V_r}{s}\cos\alpha)}{(\frac{R_sR_r}{s} - X_sX_r + X_m^2) + j(\frac{R_rX_s}{s} + R_sX_r)} \quad (5)$$

and

$$\bar{I}_r = \{\frac{V_r}{s}(R_s\cos\alpha - X_s\sin\alpha) + j[\frac{V_r}{s}(R_s\sin\alpha + X_s\cos\alpha) - X_mV_s]\} / [(\frac{R_sR_r}{s} - X_sX_r + X_m^2) + j(\frac{R_rX_s}{s} + R_sX_r)] \quad (6)$$

The magnitude of the stator current and its angle can be obtained from eqn. (5) as:

$$|I_s| = \frac{N_{is}}{D} \quad (7)$$

and

$$\phi_s = \tan^{-1}\left(\frac{N\phi_s}{D\phi_s}\right) \quad (8)$$

where

$$N_{is} = \sqrt{\{V_s^2(R_r^2 + s^2X_r^2) + V_r^2X_m^2 + 2V_sV_r X_m(R_r\sin\alpha - sX_r\cos\alpha)\}}$$

$$D = \sqrt{\{sX_m^2[sX_m^2 + 2(R_sR_r - sX_sX_r)] + (R_s^2 + X_s^2)(R_r^2 + s^2X_r^2)\}}$$

$$N\phi_s = -V_rX_m\{[R_sR_r + s(X_m^2 - X_sX_r)]\cos\alpha + (sR_sX_r + R_rX_s)\sin\alpha\} - V_s[R_r^2X_s + s^2X_r(X_sX_r - X_m^2)]$$

and

$$D\phi_s = V_rX_m\{[R_sR_r + s(X_m^2 - X_sX_r)]\sin\alpha - (sR_sX_r + R_rX_s)\cos\alpha\} + V_s[R_r(R_sR_r + sX_m^2) + s^2R_sX_r^2]$$

Similarly, the magnitude of the rotor current and its angle can be obtained from eqn. (6) as:

$$|I_r| = \frac{N_{ir}}{D} \quad (9)$$

and

$$\phi_r = \tan^{-1}\left(\frac{N\phi_r}{D\phi_r}\right) \quad (10)$$

where

$$N_{i_r} = \sqrt{\{V_r^2(R_s^2 + X_s^2) + s^2 V_s^2 X_m^2 - 2sV_sV_rX_m(R_s\sin\alpha + X_s\cos\alpha)\}}$$

$$N\phi_r = sV_sX_m[s(X_sX_r - X_m^2) - R_sR_r] + V_r\{[R_s(R_sR_r + sX_m^2) + R_rX_s^2]\sin\alpha + s[X_s(X_m^2 - X_sX_r) - R_s^2X_r]\cos\alpha\}$$

and

$$D\phi_r = -sV_sX_m(R_rX_s + sR_sX_r) + V_r\{[R_s(R_sR_r + sX_m^2) + R_rX_s^2]\cos\alpha - s[X_s(X_m^2 - X_sX_r) - R_s^2X_r]\sin\alpha\}$$

2.2 Machine Power

The active and reactive power of the stator of DFIM can be obtained, respectively from:

$$P_s = \Re(3\dot{V}_s I_s^*) \quad (11)$$

and

$$Q_s = I_{\mathfrak{M}}(3\dot{V}_s I_s^*) \quad (12)$$

where $\Re$ and $I_{\mathfrak{M}}$ are the real part and imaginary part respectively.

Using eqns. (7) and (8) of the magnitude and angle of the stator current, in eqns. (11) and (12), then the stator active and reactive power can be obtained as:

$$P_s = \frac{3V_sV_rN_{p_{s1}} + 3V_s^2N_{p_{s2}}}{D^2} \quad (13)$$

and

$$Q_s = \frac{3V_sV_rN_{q_{s1}} + 3V_s^2N_{q_{s2}}}{D^2} \quad (14)$$

where

$$N_{p_{s1}} = X_m\{[R_sR_r + s(X_m^2 - X_sX_r)]\sin\alpha - (sR_sX_r + R_rX_s)\cos\alpha\}$$

$$N_{p_{s2}} = R_r(R_sR_r + sX_m^2) + s^2R_sX_r^2$$

$$N_{q_{s1}} = X_m\{[R_sR_r + s(X_m^2 - X_sX_r)]\cos\alpha + (sR_sX_r + R_rX_s)\sin\alpha\}$$

and

$$N_{q_{s2}} = R_r^2X_s + s^2X_r(X_sX_r - X_m^2)$$

Similarly, the active and reactive power of the rotor can be obtained, respectively, from:

$$P_r = \Re(3V_r\angle\alpha\, \dot{I}_r^*) \quad (15)$$

and

$$Q_r = I_{\mathfrak{M}}(3V_r\angle\alpha\, \dot{I}_r^*) \quad (16)$$

Using eqns. (9) and (10) for the magnitude and angle of the rotor current, in eqns. (15) and (16), then, the active and reactive power of the rotor can be obtained as:

$$P_r = \frac{3V_sV_rN_{p_{r1}} + 3V_r^2N_{p_{r2}}}{D^2} \quad (17)$$

and

$$Q_r = \frac{-3sV_sV_rN_{q_{r1}} - 3sV_r^2N_{q_{r2}}}{D^2} \quad (18)$$

where

$$N_{p_{r1}} = X_m\{[s^2(X_sX_r - X_m^2) - sR_sR_r]\sin\alpha - s(R_rX_s + sR_sX_r)\cos\alpha\}$$

$$N_{p_{r2}} = R_s(R_sR_r + sX_m^2) + R_rX_s^2$$

$$N_{q_{r1}} = X_m\{(R_rX_s + sR_sX_r)\sin\alpha + [s(X_sX_r - X_m^2) - R_sR_r]\cos\alpha\}$$

and

$$N_{q_{r2}} = X_s(X_m^2 - X_sX_r) - R_s^2X_r$$

Thus, the total input active power of the DFIM can be obtained from:

$$P_{in} = P_s + P_r \quad (19)$$

Using eqns. (13) and (17) in eqn. (19), then P_{in} can be expressed as:

$$P_{in} = \frac{N_{pin}}{D^2} \quad 20)$$

where

$$N_{pin} = 3V_sV_rX_m\{(1-s)[R_sR_r + s(X_m^2 - X_sX_r)]\sin\alpha - (1+s)(sR_sX_r + R_rX_s)\cos\alpha\} + 3V_s^2(R_sR_r^2 + sR_rX_m^2 + s^2R_sX_r^2) + 3V_r^2(R_s^2R_r + sR_sX_m^2 + R_rX_s^2)$$

The net power of the machine can be obtained from:

$$P_e = P_{in} - 3I_s^2R_s - 3I_r^2R_r \quad (21)$$

Using eqn. (20) for P_{in}, eqn. (7) for $|I_s|$ and eqn.(9) for $|I_r|$, the net power can be expressed as:

$$P_e = \frac{3(1-s)X_m N_{pe}}{D^2} \tag{22}$$

where

$$N_{pe} = V_s V_r[(sX_m^2 - sX_s X_r - R_s R_r)\sin\alpha - (R_r X_s - sX_r R_s)\cos\alpha] + X_m(sR_r V_s^2 - R_s V_r^2)$$

If equation (22) gives s positive result this will indicate that the DFIM absorbs power from the grid (motoring), while for a negative sign it indicates that the DFIM delivers power to the grid (generating).

2.3 Electromagnetic Torque

The electromagnetic torque developed in the DFIM can be obtained from:

$$T_{em} = \frac{P_e}{\omega_m} \tag{23}$$

Therefore, from eqn. (22), T_{em} becomes:

$$T_{em} = \frac{3(1-s)X_m N_{pe}}{D^2 \omega_m} \tag{24}$$

The mechanical speed of the rotor can be obtained from eqn. (1) as:

$$\omega_m = (1-s)\frac{\omega_s}{p} \tag{25}$$

Substituting by the expression of ω_m in eqn. (24), then the electromagnetic torque can be expressed as:

$$T_{em} = \frac{3pX_m N_{pe}}{D^2 \omega_s} \tag{26}$$

3. Results

The expressions of the power and the electromagnetic torque obtained in subsections(2.2) and (2.3) are used to determine the performance of DFIM whose parameters are given in Appendix. The performance is obtained for the operation at subsynchronous speed or supersynchronous speed for different values of the magnitude or the angle of the rotor voltage. The results obtained for the active, reactive, total input power and the electromagnetic torque are compared with the results published in references [20] and [21].

3.1 Machine Power

In this subsection, the active and the reactive power of the machine are obtained at a rotor voltage of 0.2 p.u. The results are obtained at slip of 0.1, 0.15 and 0.2 for subsynchronous speed, and at slip of -0.1, -0.15 and -0.2 for supersynchronous speed. The results obtained are compared with the results published in [20].

Figs. 3 and 4 show the variation of the stator active and reactive power versus the phase angle of the rotor voltage, and Fig. 5 shows the active power of the rotor, at subsynchronous speeds corresponding to slips of 0.10, 0.15 and 0.20.

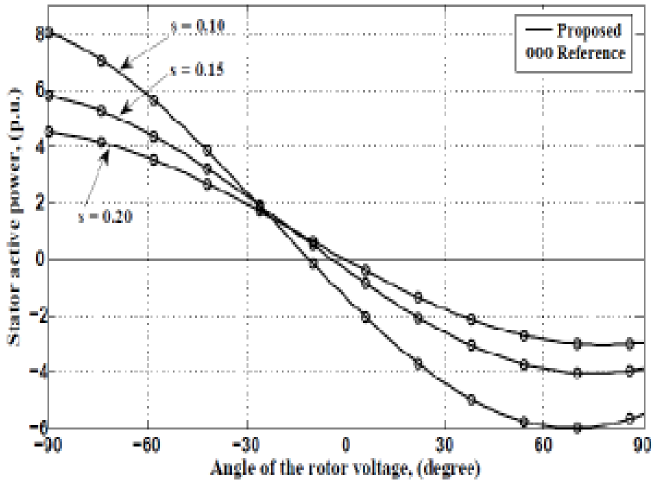

Fig. 3 Stator active power versus the rotor voltage angle

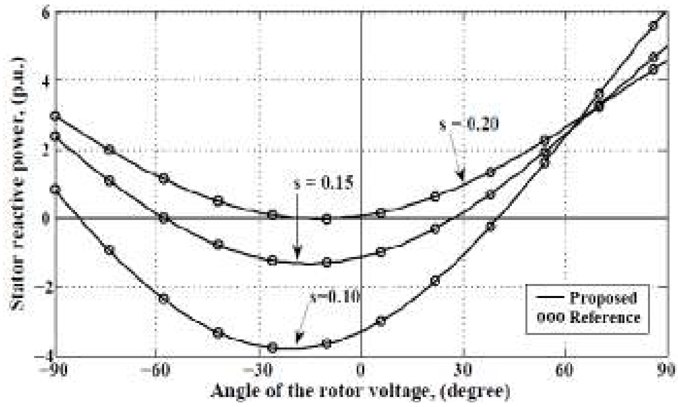

Fig. 4 Stator reactive power versus the rotor voltage angle

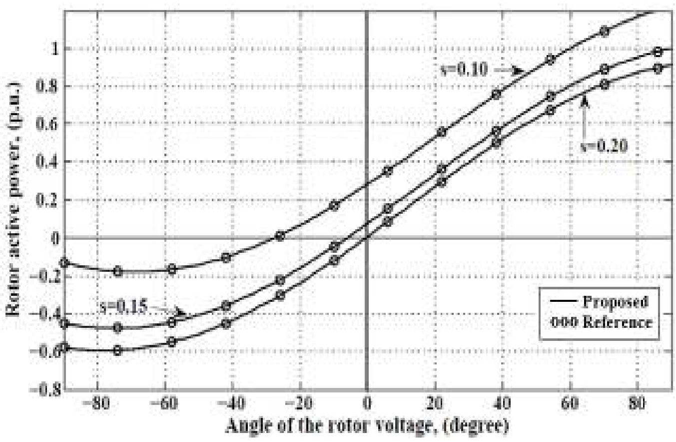

Fig. 5 Rotor active power versus the angle of the rotor voltage

At supersynchronous speeds corresponding to slips of -0.1, -.15 and -0.2, Figs. 6 and 7 show the variation of the stator active and reactive power versus the phase angle of the rotor voltage, and Fig. 8 shows the active power of the rotor versus the phase angle of the rotor voltage.

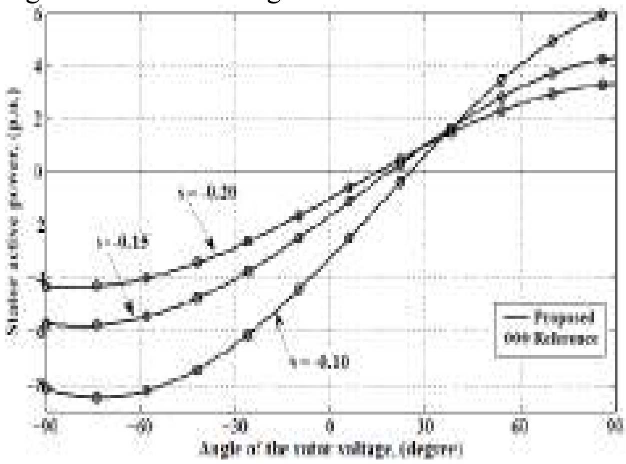

Fig. 6 Stator active power versus the angle of the rotor voltage

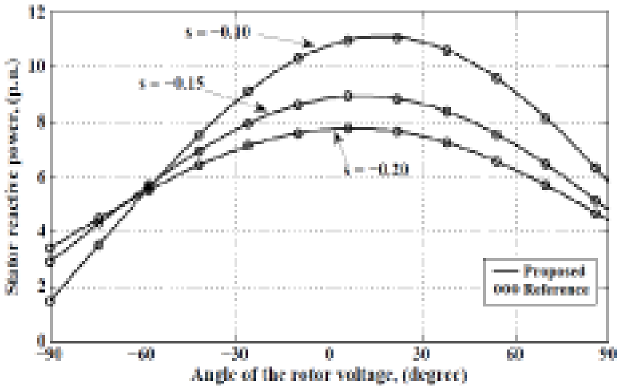

Fig. 7 Stator reactive power versus rotor voltage phase angle

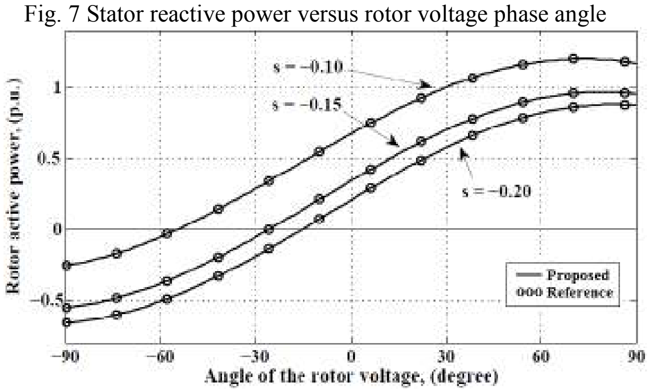

Fig. 8 Rotor active power versus rotor voltage phase angle

At subsynchronous speeds corresponding to slips of 0.10, 0.15 and 0.2, Figs. 9 and 10 show the variation of the total input active power and the total input reactive power of the machine versus the phase angle of the rotor voltage.

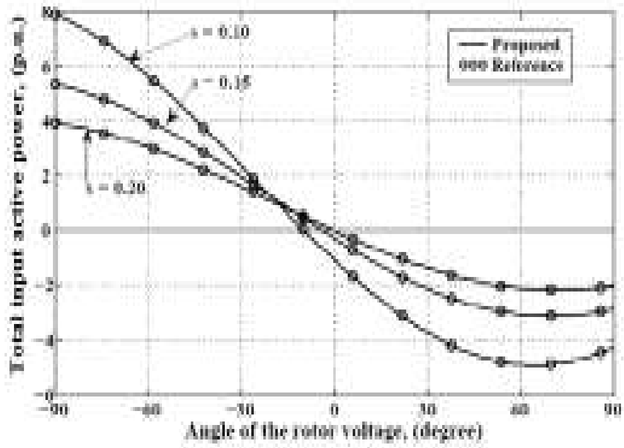

Fig. 9 Total input active power versus the angle of the rotor voltage

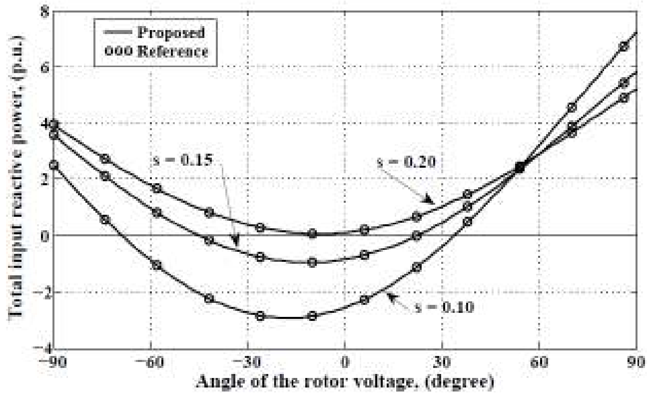

Fig. 10 Total input reactive power versus the angle of the rotor voltage

Fig. 11 shows the variation of the total input active power versus the angle of the rotor voltage at rotor voltage of 60V, 70V and 81.5V.

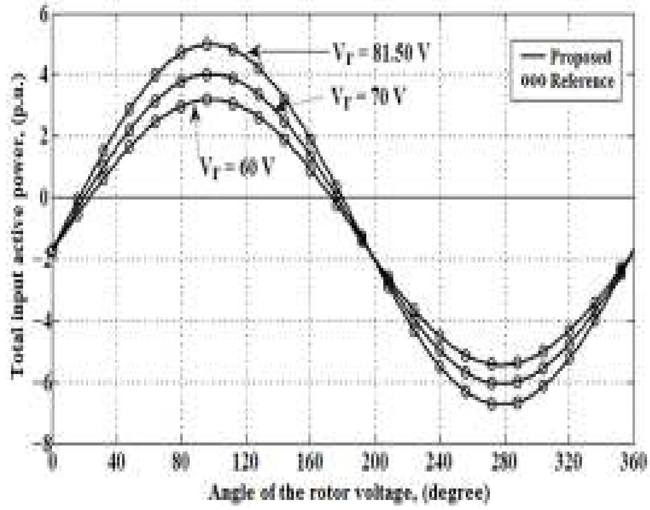

Fig. 11 Total input active power versus the angle of the rotor voltage

It is evident from Figs. 3 to 10 that the results obtained from the proposed expressions are identical with those published in reference [20], and from Fig. 11 the results obtained are identical with those published in reference [21].

3.2 Electromagnetic Torque

Fig. 12 shows the variation of the electromagnetic torque versus the machine slip at different angles of the rotor voltage (α=0^o, 90^o, 180^o and 270^o), the rotor voltage is kept constant at 0.1 p.u.

It is evident from the figure that the results of the torque obtained from the proposed expression are identical with those published in reference [21].

4. Conclusions

In this paper, simple, direct and closed-form analytical expressions for the characteristics of the DFIM operates at subsynchronous or supersynchronous speed were presented. The characteristics include, for both the stator and the rotor, the current, active power and reactive power. Also, expressions for the machine input power, the net power and the electromagnetic torque were derived.

The results obtained, for the machine power and torque, from these expressions were compared with those previously published results, and the two sets of results were found to be identical.

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Appendix

Machine parameters (M1) [20]:

Stator: V_s = 400 V/ph, I_s = 1900A, R_s = 2.2 mΩ, L_{ls} = 0.12 mH, L_m = 2.9 mH.
Rotor: R_r =1.8 mΩ, $L_{r\sigma}$ = 0.05 mH
Number of poles = 4

Machine parameters (M2) [21]:

The machine is a 3-phase, Y-connected, slip ring induction machine with the following data:
Stator: V_s = 220 V/ph, 50 Hz, I_s = 14A, R_s = 0.9 Ω, $L_{s\sigma}$= 11 mH, L_m = 398 mH
Rotor: V_r = 140 V/ph, I_r =17 A, R_r = 1.25 Ω, $L_{r\sigma}$ = 10.6 mH
Number of poles = 4 s_{rated} = -0.04