Study on the Deformation of Water Droplets under a Time-Varying Electric Field

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Abstract: *Electric field enhancement due to the presence of water droplets on the insulation surface is one of the main factors reducing the flashover voltage of insulating material. To increase the electrical performance of an insulation system, the mechanisms of electric field intensification are important to study. This article presents the behavior of water droplets under a time-varying electric field. Water droplets under different voltage levels, from 0 to 3 kV, and various electrical stress directions, vertical and radial direction, were investigated. The deformation of water droplets was described by a system of equations, including Navier-Stokes equation and Poisson equation. In order to determine the interface between water and air, the level set method is chosen and applied to the models. According to the calculated results, the direction of the electric field has a significant effect on the deformation of the water droplet. Additionally, this research reveals that with higher input voltage, the degree of distortion is further increased from the equilibrium state; the shape of water droplet at zero-field. The field intensification of a deformed droplet is comparable between two different directions of electrical stress; vertical field and radial field respectively. However, these values are lower when compared with the results from a non-deformed droplet.*

Key words: *Deformation, electric field, finite element method, intensification, water droplet*

1. Introduction

Insulators are important components in the electrical power transmission system. They are used to provide electrical separation between high voltage and ground structure, and to support the mechanical load of such systems. Failure and degradation of these insulators due to surface contaminants, ultraviolet radiation, defective material, and surface discharge lead to reduced reliability and stability of the power system. Surface discharge is the most commonly found problem which can cause a reduction of the insulating properties or even permanent damage of the insulation material. For example, the discharge may create a conducting path on the insulator's surface, also known as surface tracking. Along this path, the water-repellent property of the surface decreases and the water tends to form a thin film rather than droplets. This distortion shortens the leakage distance of the insulator, and may lead to easier flashover of insulator at a lower voltage compared to the normal breakdown voltage [1-8].

Water droplets on the surface of an insulator enhance electric fields nearby, especially at the triple junction of water, air, and insulating material. Partial discharge may initiate at this region when the electric field is much higher than the dielectric strength of air. Additionally, the droplets will deform due to the force exerted by the electric field. The droplet always extends in the direction of the electric field. This means that the electric field intensification on the surface will change with the deformation of the droplets. From [9-20], the behavior of water droplets in electric fields was investigated by experiments, and the enhancement of electric field was explained with a static droplet model. However, variation in the contact angle and electric field stress on the droplet surface according to the time-varying electric field, especially at the contact line, did not show in their results. To control the electric field on the insulator, it is important to understand the time-dependence mechanisms of water droplet deformation under high voltage stress.

This work presents the deformation of water droplets under alternating sinusoidal voltage. The motion of droplets is governed by the Navier-Stokes equation, while Poisson's equation describes the distribution of electric field. The finite element method was used to solve the coupled problem. The purpose of this work is to clarify the behavior of water droplets under time-varying electric field. The enhancement of the electric field on the insulator surface due to the water droplet is investigated and compared between static and dynamic models, which differs from the previous works. Additionally, the direction of applied electric field is considered as an important factor that affects the shape of deformed water droplet, this has also not been explored in previous work. The obtained results from this work can be useful in basic design of high voltage apparatus to mitigate the partial discharge activity during rain and fog conditions.

2. The mathematical model

In general, the frequency of transmission and distribution systems is between 50 and 60 Hz. This means that the time scale is long compared to the relaxation time of charge. Therefore the electric field distribution between the electrodes can be calculated by electrostatic equations:

$$\nabla^2 V = -\frac{q_V}{\varepsilon_0 \varepsilon_r} \tag{1}$$

$$\boldsymbol{E} = -\nabla V \tag{2}$$

where V and q_V are electric potential (V) and charge density (C/m^3). The constants ε_0 and ε_r in Eq. (1) are permittivity of free space (F/m) and relative permittivity.

To describe the behavior of water droplets, the incompressible Navier-Stokes equations are used and shown in the following:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho (\boldsymbol{u} \cdot \nabla) \boldsymbol{u} = \nabla \cdot \left[-pI + \mu \left(\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T \right) \right] + \boldsymbol{F_{st}} + \rho \boldsymbol{g} + \boldsymbol{F} \tag{3}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{4}$$

where $\boldsymbol{u}$, $\boldsymbol{g}$, $\boldsymbol{F_{st}}$ and $\boldsymbol{F}$ are fluid velocity (m/s), gravitational acceleration (m/s^2), surface tension force (N/m^3), volume force (N/m^3) and ρ, p, μ and I are density (kg/m^3), pressure (N/m^2), dynamic viscosity (N·s/m^2) and identity matrix, respectively.

The last term in the right hand side of Eq. (3) is an external body force that acts throughout the fluid volume and it can be calculated by taking the divergence of Maxwell stress tensor (σ):

$$\sigma = \begin{bmatrix} \varepsilon_0\varepsilon_r E_r^2 - \frac{1}{2}\varepsilon_0\varepsilon_r E^2 & \varepsilon_0\varepsilon_r E_r E_\theta & \varepsilon_0\varepsilon_r E_r E_z \\ \varepsilon_0\varepsilon_r E_\theta E_r & \varepsilon_0\varepsilon_r E_\theta^2 - \frac{1}{2}\varepsilon_0\varepsilon_r E^2 & \varepsilon_0\varepsilon_r E_\theta E_z \\ \varepsilon_0\varepsilon_r E_z E_r & \varepsilon_0\varepsilon_r E_z E_\theta & \varepsilon_0\varepsilon_r E_z^2 - \frac{1}{2}\varepsilon_0\varepsilon_r E^2 \end{bmatrix} \tag{5}$$

where the electric field strength is determined by $E = \sqrt{E_r^2 + E_\theta^2 + E_z^2}$

Additionally, the interface between the surrounding air and the water droplets is captured by the level set method as the following equation:

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u} \cdot \nabla \phi + \nabla \cdot \left[\phi (1-\phi) \boldsymbol{u_c} \right] = 0 \tag{6}$$

Here $\boldsymbol{u_c}$ is the compressed velocity (m/s^2) [21] and the interface between air and water is defined at the point where the value of ϕ equals 0.5.

3. The model configuration

Fig. 1 shows the configuration that is used in this work. A water droplet with volume of 4 µl was placed at the center of a silicone rubber plate. Due to the small size of the droplet [11], it is initially modeled as semicircle with contact angle of 90°. A no-slip boundary condition was enforced to the surface of plate where the water was in contact with silicone rubber. The electric field stress was applied in two different directions: E_z in the vertical direction and E_r in the radial direction. The relative permittivity of water and silicone rubber are 80 and 3, respectively [15], [19], [21]. A dielectric constant for the surrounding air is equal to 1.

From the Eq. (1) to (6), it is complicated to calculate the coupled electrical and fluid mechanical problem in the 3-dimensional model because it requires a lot of time and memory. Since the model is symmetric around the z-axis at the center of the droplet, this problem is reduced to 2-dimensional axisymmetry.

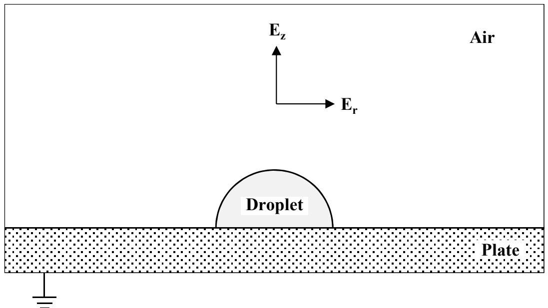


Fig. 1. Model configuration

The system of equations above was computed using finite element method (FEM) software, COMSOL Multiphysics [22]. An example of meshes used for calculation is illustrated in Fig. 2.

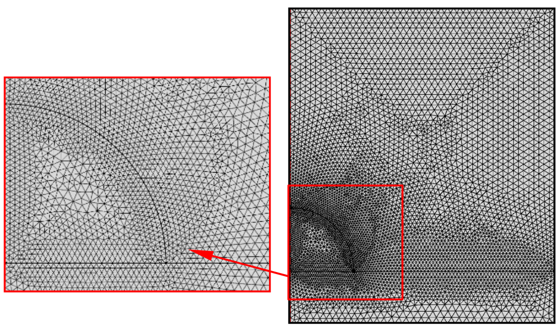


Fig. 2. Elements pattern used for calculation by the FEM

4. Results and discussion

4.1 Distortion of water droplet

As mentioned above, the models are simplified and simulated in the 2-dimensional r-z plane. To investigate the effects of the electric field on the deformation of water droplets, five models with different voltage levels and stress directions are used in the calculation as shown in Table 1. Model A is the model not subjected to high AC voltage, and is used as a reference. As a closed system model, there are no flow and no leak of water through the boundaries. Therefore, the total volume of the water droplet should be constant in time. To verify the computed results, the calculations of droplet volume are carried out and their results as a function of time are shown in the appendix A.

Without applied voltage (model A) the shape of the water droplet distorts during the first period of the simulation. This can be explained by the imbalance between the gravitational force and the surface tension force at the droplet interface. In a very short duration, less than one cycle of input voltage, the distortion stops and the droplet reaches the equilibrium shape as shown in Fig. 3.

Table 1 Simulation setup of the model

Model	Voltage level (kV)	Direction of applied electric field
A	N/A	N/A
B	2	Vertical
C	3	
D	2	Radial
E	3	

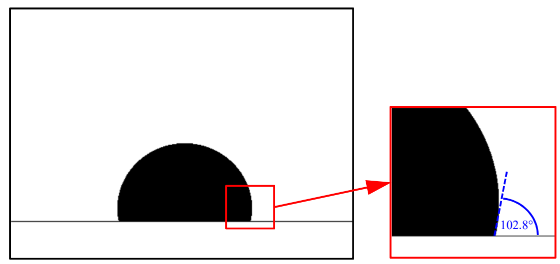


Fig. 3. Equilibrium shape of water droplet for model A

From Fig. 3, the shape of the water droplet is different from the initial state; it reshapes from a semicircle-shape to a mushroom-shape. The contact angle measured using drop analysis plug-in [23] of ImageJ is 102.8°.

To describe the deformation of the water droplet, the deformation factor (DF) is defined and the definition of this factor is shown in the Fig. 4 and Eq. (7), respectively.

$$DF = \frac{h}{D} \tag{7}$$

where h and D are the height and maximal diameter of the droplet, respectively.

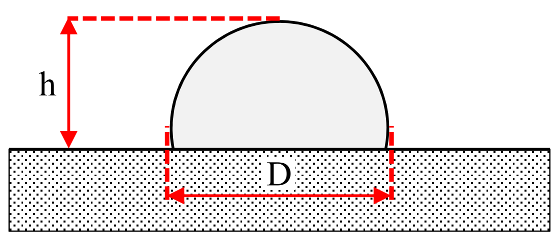


Fig. 4. Scheme of calculation of deformation factor

When high voltage is applied to the model, the water droplet deforms again. The second deformation comes from another force that acts on the droplet.

Table 2 Water droplet shape variation over time under an electric field

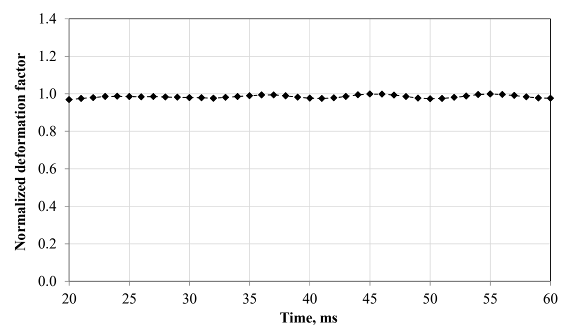
Model	Frame 1	Frame 2	Frame 3	Frame 4	Frame 5
B					
C					
D					
E					

* The interval between each frame is 5 ms.

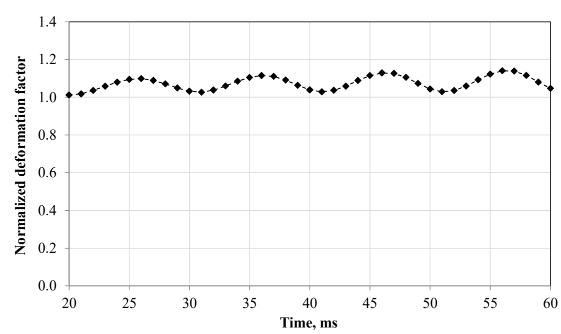
This force is an electrical force due to difference in the electrical stress at the water droplet interface. It is separated into two components with respect to the vertical axis and radial axis of the model. Therefore, the shape of the water droplet can deviate from the equilibrium shape in the two directions depending on the magnitude of total force along each axis. Table 2 shows the shape variation of the droplet after electrical stress was applied.

Fig. 5 and Fig. 6 illustrate the deformation factor of the water droplet when subjected to the electric field. Note that all values of DF are normalized with the DF of model A at the equilibrium state. In all models, this factor changes over time within a certain range. The period of the vibration is about 10 ms, that is twice the frequency of the input voltage. It is maximal when voltage reaches the peak value and minimal when voltage is equal to zero.

Additionally, vibration increases when the magnitude of input voltage increases. Compared to the vertical electric field, increasing the applied voltage has little effect on DF for the radial electric field. Similar to DF, the contact angle of the water droplet is also altered with the variation of the voltage. The waveform pattern is similar to that of DF. However, its value does not vary as much as in DF, as shown in Table 3.



(a) Model B, 2 kV input voltage



(b) Model C, 3 kV input voltage

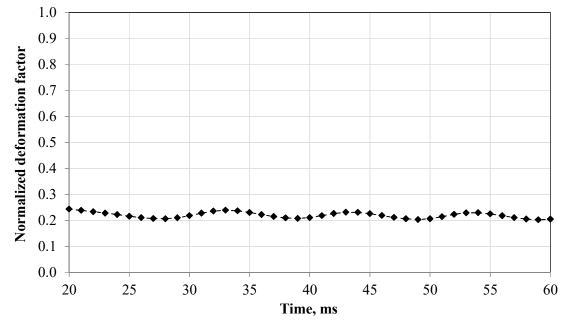
Fig. 5. Deformation factor for vertical electric field

Table 3 Contact angle of water droplet under high AC voltage

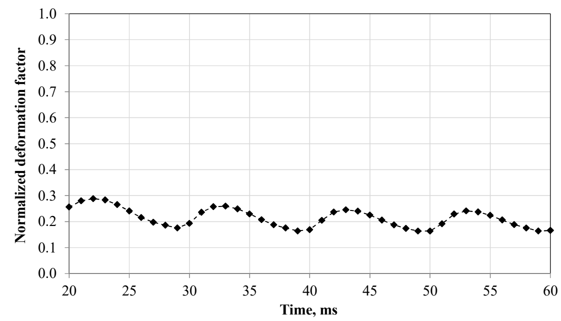
Model A at equilibrium state	102.8°	
	Minimum (°)	Maximum (°)
Model B	100.1	103.0
Model C	104.4	110.0
Model D	28.8	33.9
Model E	24.5	37.6

4.2 Electrical field intensification

To investigate the effect of the water droplet on the electric field enhancement, comparisons of potential distribution of the models with and without a water droplet placed on a plate are presented in Fig. 7. The potential is uniformly distributed when the droplet is not present, otherwise potential distribution is non-uniform. It can be clearly seen that the equipotential lines are compressed at the top of the water droplet. Therefore, the electrical stress is very high in this region. However, the magnitude of maximum electric field is comparable between the top and bottom positions of the water droplet for the radial electric field. The radial component of electrical force exerted at the interface of the water droplet is greater in the radial field than in the vertical field. This is the reason why the droplet extended in the radial direction for model D and model E.

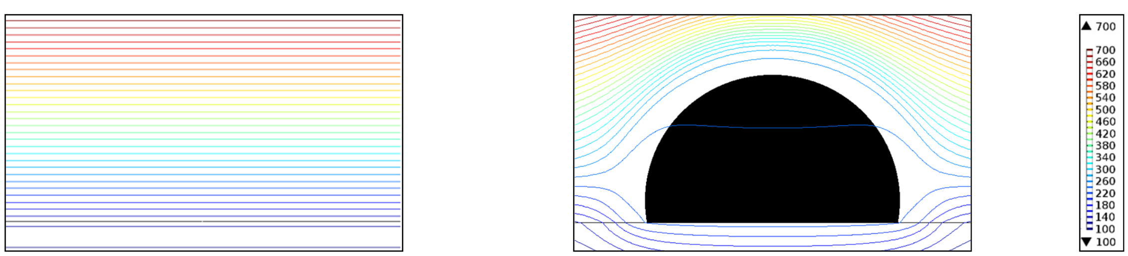


(a) Model D, 2 kV input voltage

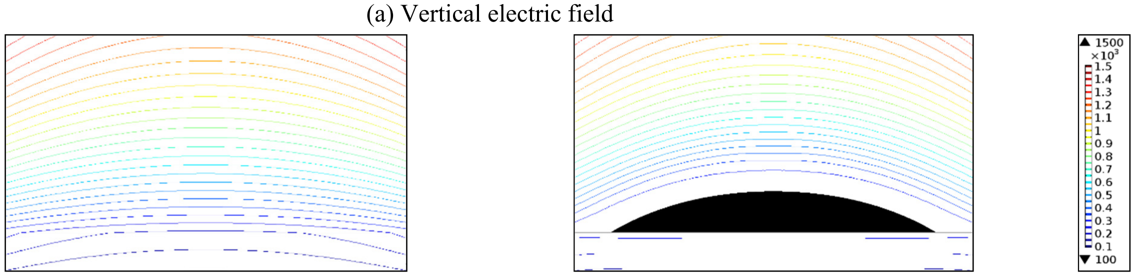


(b) Model E, 3 kV input voltage

Fig. 6. Deformation factor for radial electric field



(a) Vertical electric field



(b) Radial electric field

Figure 7. Distribution of electric field of model with and without water droplet for 2 kV input voltage and t = 25 ms

For all models under electrical stress, the results of maximum electric field show similar characteristics. Fig. 8 shows how the maximum electric field for model D varied with time. The values on the vertical axis are presented in a per-unit system. The maximum electric field shows a periodic pattern equal to that of the deformation factor of the water droplet, see Fig. 6.

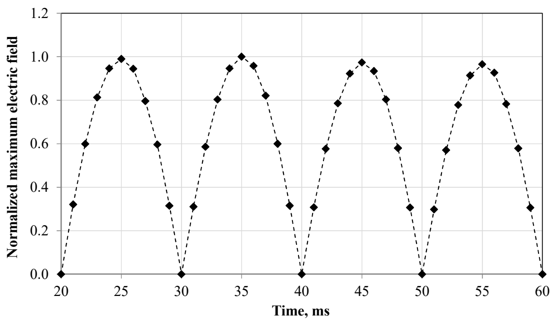


Fig. 8. The maximum electric field pattern of water droplet under high voltage AC

The electric field intensification (EFI) is a ratio obtained by dividing the maximum electric field ($E_{\max}$) due to water droplet by the average field (E_0) through the same line in the case of uniform electric field as given in Eq. (8),

$$EFI = \frac{E_{\max}}{E_0} \quad (8)$$

The values of EFI are divided into two groups, deformed and not deformed water droplet, as shown in Table 4.

Table 4 The enhancement of electric field by water droplet

Model	*EFI*	
	Non-deformed droplet	Deformed droplet
B	2.67	2.06
C	2.67	1.98
D	4.26	2.17
E	4.28	2.21

When the droplet is not deformed, the enhancement of the electric field is higher in the case of radial electrical stress. But there is no significant difference in EFI between the directions of applied electric field when the water droplet is deformed. It is also seen from the Table 4 that the intensified electric field is lower for all deformed droplets. This implies that the intensification of electric field is remarkably dependent on the geometric shape of the water droplet, particularly under high electrical stress. Additionally, the results show that EFI is not dependent on the level of input voltage.

The increase in the electric field caused by the water droplet on the insulator can reduce partial discharge inception voltage (PDIV) of the insulator. Partial discharge is usually generated in the regions of highest field strength. That means the discharge will occur on the top of the water droplet and maybe on the line where the droplet contacts the plate for the radial electric field. These results are in agreement with the results of [11], [24].

Furthermore, the deformation of the water droplet has an effect on the leakage current. The variation of the droplet's bottom area causes a change in the surface resistance. The distortion of leakage current waveform is increased according to the bottom surface area of droplet as reported in [25].

5. Conclusion

From the results of the calculations that were performed in this research, these conclusions can be drawn:

(1) The system of equations used in this research can be used to explain the deformation of a water droplet under a time-varying electric field.

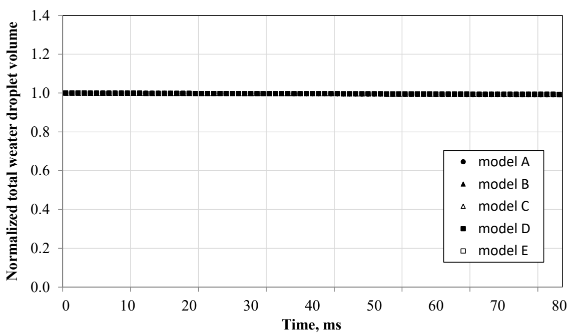
(2) The direction of applied electrical stress has an effect on the distortion of the water droplet. Additionally, the degree of distortion increases when the magnitude of the input voltage increases.

(3) The presence of a water droplet on the surface of the insulator will intensify the electric field on the insulating surface, about two to four times higher than normal operation. However, this enhancement is reduced when the geometry of the droplet deforms under time-dependence electric field.

(4) The decrement of PDIV and distortion of leakage current can be described by the variation of contact area between the water droplet and the insulating surface.

Appendix A
The total volume of the water droplet

To investigate the quality of the calculation results, the total volume of water droplet of each model was checked. The obtained results show that the volume loss during the simulation was less than 1%, as illustrated in App. Fig. 1. All values on the vertical axis are normalized by the droplet volume at time t = 0 ms.



App. Fig. 1. Total volume of the water droplet as a function of time

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