Performance Analysis of Spiral Based Inductive Position Sensor using Equivalent Circuit Model

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Abstract

This paper presents a simple way to analyse the Resonant Inductive Position Sensor through a transfer function model. This is derived from equivalent circuit of both sine and cosine output spiral coils from the basics of magnetic coupling between excitation and resonator coils. In this model, the impedance of both sine and cosine output coils and the output voltage are predicted. This has been implemented in MATLAB environment through Graphical User Interface (GUI) along with its signal conditioning and algorithm to predict the linear position of the moving resonator which is attached with the target position to be measured. This design tool will reduce the design cycle time and expenses due to either lab experiment or Finite Element Analysis (FEA). The accuracy of the sensor is also predicted through an error calculation by comparing the measured results with actual position in ideal condition.

Keywords: *Spiral, Sine, Cosine, GUI, MATLAB, Transfer function, Inductive*

I. Introduction

The Resonant Inductive Position Sensor is one of the most popularly employed sensors for precise position applications. The total sensor comprises of spiral inductor coils of different geometries as the source of excitation of magnetic field. So the importance of predicting the inductance is very critical during the design process. Presently the design is carried out by using either lab experiment or simulation using FEA tools. To conduct lab experiments the designer has to make different inductor coil shapes. Prototypes should be made for every concept till the desired response is obtained with a required level of accuracy. The simulation using FEA tools takes lot of time.[1] Both these approaches will be time consuming and expensive. The aim is to develop a design solution for the spiral sensors by understanding the physics behind the inductor.

Inductive sensors are applicable for position detection or travel measurement in industrial applications. These sensors yield acceptance in many sectors of industry due to the advantages they offer - noncontact and wear-free sensing of the target, resistance to fouling, reliability and robustness and compact size. Sorin has discussed the technological innovations, recent implementations and current trends regarding the distance and travel sensing offered by contactless inductive sensors for industrial applications. Starting with the fundamentals of inductive sensing, the physical basics gained by modern analytic and simulation methods, as well as high-level integrated circuits for inductive sensors have been presented. [2]

Bernhard has presented a sensor working on similar principles as resolvers but consisting of an antenna PCB, a moveable passive LC resonant circuit and a signal processing unit. The mechanical transducer arrangement and the signal condition electronics design have also been presented. [3] A mathematical model and the equivalent circuit of this kind of sensor have been explained. Such sensors suffer from transmitter to receiver coil capacitive crosstalk, which results in a phase sensitive offset. This crosstalk has been analyzed by a mathematical model. [4]

Mohamad has realized a signal conditioner that includes an application specific integrated circuit (ASIC) and an external microcontroller for readout and control of non-contact high-frequency inductive position sensors. The signal conditioner architecture is universal and can be used for other sensor types such as LVDTs. [5]

Mohamad has implemented an application-specific integrated circuit (ASIC) for readout and control of a contactless inductive position sensor that contain transmitter and receiver coils on a fixed printed circuit board and a moving passive resonant target which is used to measure linear and angular positions. Such an inductive position sensor suffers from transmitter-to-receiver signal coupling, which can result in a phase-sensitive offset; hence, an error in the position measurement occurs. The integrated front-end design that reduces phase sensitive offsets has been described. The integration of the sensor signal conditioning circuits in an ASIC offers considerable advantage in terms of size and performance. [6-8]

The ironless inductive position sensor is a novel linear position-sensing device that should exhibit immunity to external magnetic fields while simultaneously guaranteeing high-precision measurements in harsh environments. Alessandro has focused the working principle of this sensor and its magnetic field immunity. [9]

Aschenbrenner has studied the inductive position sensor operating principle which is similar to the resolver operating principle. However, instead of a rotor winding and two stator windings there are the transmit coil and four receive coils on the same antenna PCB. Furthermore, the coils are planar on the PCB instead of a spatial arrangement like the resolver windings in a motor stator.[11]

Snezana has proposed a sensor that is composed of two inductive coils of spiral shape. One coil is a stationary coil with input terminals, whereas the other coil is a moving (short-circuited) coil. The moving coil physically moves with respect to the stationary coil in x–z and y–z planes. The input inductance changes when the moving coil moves with respect to the stationary coil and it is measured between the terminals of the stationary coil. The inductive operating principle of the sensor was tested using the prototypes fabricated in printed circuit board technology.[12-13]

Max has presented a new inductive sensor using planar technology, which provides a major improvement over a conventional sensor. The new device presents high mechanical hardness, high merit factor and stability over a wide temperature range at operation conditions.[14] Manfred has presented non-contacting inductive sensors which are applicable on a large scale for position detection or travel measurement in industrial applications due to their wear-free sensing of the target (any metal object), reliability, robustness, resistance to fouling, water tightness and compact size. Mostly these sensors are realized as inductive proximity sensors (IPS), which convert the distance between the target and sensor active face into an electrical analog or binary signal.[16]

Weissinger has discussed the design of a novel position sensor using finite-element analysis (FEA) based on the effect of local magnetic saturation. This sensor is a position sensor, which enables distances to be measured with high resolution. The operating principle provides a high-precision linear output signal along with cheap production costs. The main aim of the presented work is to design a feasible finite-element model of the sensor, which is precise enough to represent the physical properties and effects, so that there is no need to build up new prototypes for every step of an optimisation or development.[17]

Nanying has investigated and designed a novel deep displacement sensor based on the electromagnetic induction theory, which can directly convert the varied sliding displacement and tilt angle at any depth within the landslide mass to the variation of mutual inductance.[18] Qifu has presented a sensor that makes use of planar sinusoidal-shape primary coils to generate magnetic field which has sinusoidal distribution between the stator and rotor. It was proved by the simulation that this sinusoidal layout offers good linearity between the phase of output signal and angular displacement.[29]

After this introductory part, section II describes the basic principle and overview of Resonant Inductive Position Sensor. Section III shows spiral inductor circuit model and system description. Section IV depicts the derivation of transfer function for the system output voltage. In section V computing the position from the spiral sensor is given. Section VI shows the analysis of spiral sensor through Graphical User Interface (GUI) using MATLAB. In section VII the sensor performance is shown, error is calculated and discussed. Finally, conclusions are drawn in section VIII.

II. Basic Principle and Overview of Resonant Inductive Position Sensor

Resonant Inductive Position Sensing is basically a non-contact type sensing technology. Resonant Inductive Position Sensor uses this technology and it detects linear or angular displacement. These are reliable, have high speed and long life and are directly compatible with other electronic circuits. It is more economical due to low-cost components and simple electronics with no onboard microprocessor. This is usually implemented in conventional Printed Circuit Board (PCB) as tracks.

The position of a moving resonator relative to fixed track of spiral coil is sensed. The position sensor has four spiral tracks. The target whose position is to be measured is attached with an inductively coupled moving resonator. The fixed track has one coil as excitation coil to power the resonator coil and two more sensor output tracks such as sine and cosine coils to find the signals provided by the resonator coil.

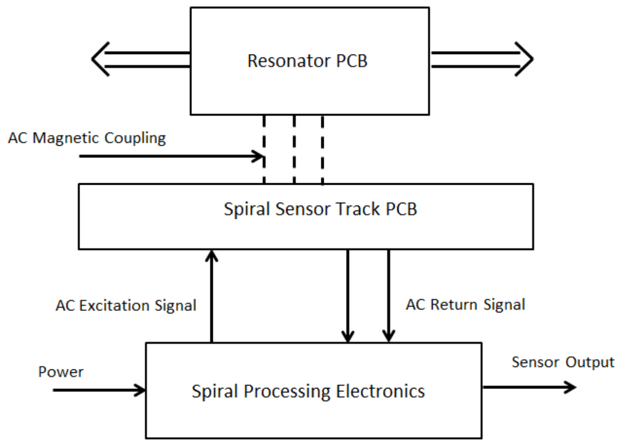

Fig.1 Lay out of Induction Position Sensor

The spiral track operates in conjunction with an electrical resonator, which comprises of a coil and a capacitor. The resonator is placed above the track and moves along with it, which is the motion that is sensed. An AC current is passed through the excitation coil whose frequency normally matches that of the resonator. This forces the resonator to resonate.

The resonator's oscillating magnetic field is detected by two further sets of coils called sine and cosine sensor coils. These coils are designed such a way that their coupling to the resonator varies sinusoidally with position and their outputs are in phase quadrature. The EMFs generated by the sensor coil are synchronously detected by the processing electronics.

III. Spiral Inductor Circuit Model and System Description

Spiral inductor circuit model consists of three coils, which are modelled as an inductor in series with resistor. The excitation coil is modelled as an inductor L_x in series with resistance R_x which is a transmitter. The moving element, which is resonator, is modeled as a RLC circuit with inductance L_{rso}, capacitance C_{rso} and resistance R_{rso}. The output coils (sine and cosine) are modeled as inductors 'L_{sine}' and 'L_{cos}' which is a receiver. All these inductors are mutually coupled.

As we know, the function of the excitation coil is to provide a uniform magnetic field along the track. K_x is the coupling coefficient between excitation coil and the resonator. The resonator circuit (an RLC combination) in turn induces voltages in the receiving coils and the magnitude and phase of these voltages vary as a function of position of the resonator. This function in these spiral sensors is sine and cosine. Krsine(phi) and Krcos(phi) are the coupling coefficients between the resonator and sine and cosine coils respectively which are the function of phi (angle which relates the position of the resonator on the board). There is also coupling between the excitation coil and the receiving coils. K_{xsine} and K_{xcos} are the coupling coefficients respectively.

The response of the sensor mainly depends on the inductance values of excitation coil, sine and cosine coils and the coupling coefficient between these coils. The variation of inductance of these coils leads to change in magnetic field, which in turn changes the response of sensor. So the prediction of inductance of these coils is crucial for the design process.

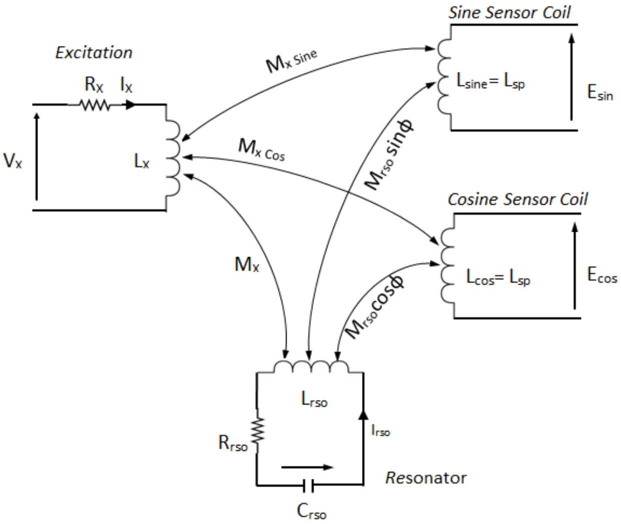

Fig.2 Spiral Arrangement of Coils

$$M_x = K_x\sqrt{L_{rso}L_x}$$
$$M_{Xsine} = K_{Xsine}\sqrt{L_xL_{sine}}$$
$$M_{Xcos} = K_{Xcos}\sqrt{L_xL_{cos}}$$
$$M_{rso}max = K_{rso}max\sqrt{L_xL_{sine}}$$

IV. Derivation of Transfer Function for the System Output Voltage

Transfer function is also known as the system function. It is a mathematical representation that describes the transfer characteristics of a system that is the relationship between the input and output of a system.

For sine coil

Let us only consider excitation, resonator and sine coil, for deriving transfer function of sine coil

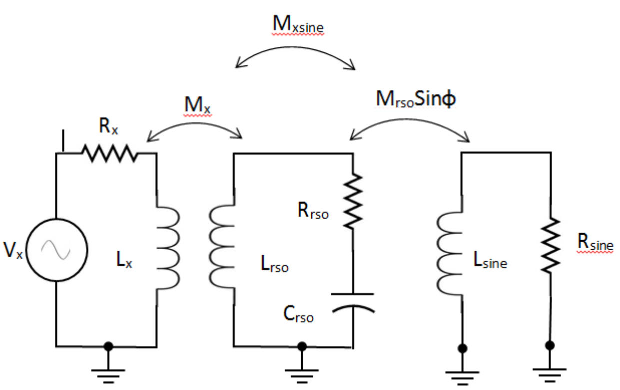

Fig.3Equivalent Circuit Model of Sine Coil

Let us apply Kirchoff's law in the above circuit and we get the following

$$V_x = I_xR_x + j\omega L_xI_x + j\omega M_xI_{rso} \quad (1)$$

$$0 = I_{rso}R_{rso} + j\omega L_{rso}I_{rso} - \frac{j}{\omega C_{rso}}I_{rso} + j\omega M_xI_x \quad (2)$$

Induced EMF on the sine and cosine coil are

$$E_{sine} = j\omega M_{Xsin} I_x + j\omega M_{rso} \sin\varphi \, I_{rso} \quad (3)$$

$$E_{cos} = j\omega M_{Xcos} I_x + j\omega M_{rso} \cos\varphi \, I_{rso} \quad (4)$$

$$\begin{bmatrix} V_x \\ 0 \\ E_{sine} \end{bmatrix} = \begin{bmatrix} R_x + j\omega L_x & j\omega M_x & K_1 \\ j\omega M_x & R_{rso} + j(\omega L_{rso} - \frac{1}{\omega C_{res}}) & K_2 \\ j\omega M_{Xsine} & j\omega M_{rso} \sin\varphi & K_3 \end{bmatrix} \begin{bmatrix} I_x \\ I_{rso} \\ I_{sine} = 0 \end{bmatrix}$$

Impedance of the resonator coil

$$Z_{rso} = R_{rso} + sL_{rso} + \frac{1}{sC_{rso}}$$

$$\begin{bmatrix} V_x \\ 0 \\ E_{sine} \end{bmatrix} = \begin{bmatrix} R_x + sL_x & sM_x & K_1 \\ sM_x & Z_{rso} & K_2 \\ sM_{Xsine} & sM_{rso} \sin\varphi & K_3 \end{bmatrix} \begin{bmatrix} I_x \\ I_{rso} \\ I_{sine} = 0 \end{bmatrix}$$

From equation 2, calculate $\frac{I_{rso}}{I_x}$

$$\frac{I_{rso}}{I_x} = \frac{-s^2 M_x C_{rso}}{s^2(L_{rso}C_{rso}) + sC_{rso}R_{rso} + 1} \quad (5)$$

$$Let\ T_{f1} = \frac{-s^2 M_x C_{rso}}{s^2(L_{rso}C_{rso}) + sC_{rso}R_{rso} + 1} \quad (6)$$

$$I_{rso} = T_{f1}(I_x) \quad (7)$$

From equation (3)

$$E_{sine} = sM_{Xsine} I_x + (T_{f1}(I_x)).\, sM_{rso} \sin\varphi$$

Solving we get,

$$\frac{E_{sine}}{I_x} = \frac{s^3(M_{Xsine}L_{rso}C_{rso} - M_{rso}M_xC_{rso}) + s^2 M_{Xsine}R_{rso}C_{rso} + sM_{Xsine}}{s^2(L_{rso}C_{rso}) + sC_{rso}R_{rso} + 1}$$

Input impedance of sine coil

$$\frac{V_x}{I_x} = \frac{s^3\left(L_{rso}L_x - {M_x}^2\right) + s^2 C_{rso}(R_xL_{rso} + R_{rso}L_x) + s(L_x + R_{rso}R_xC_{rso}) + R_x}{s^2(L_{rso}C_{rso}) + sC_{rso}R_{rso} + 1}$$

Transfer function of sine coil

$$\frac{E_{sine}}{V_x} = \frac{s^3(M_{Xsine}L_{rso}C_{rso} - M_{rso}M_xC_{rso}) + s^2 M_{Xsine}R_{rso}C_{rso} + sM_{Xsine}}{s^3\left(L_{rso}L_x - {M_x}^2\right) + s^2 C_{rso}(R_xL_{rso} + R_{rso}L_x) + s(L_x + R_{rso}R_xC_{rso}) + R_x}$$

Similarly for cosine coil, substituting equation (7) in equation (4) we get,

$$E_{cos} = sM_{Xcos} I_x + (T_{f1}(I_x)).\, sM_{rso} \cos\varphi$$

Solving we get

$$\frac{E_{cos}}{I_x} = \frac{s^3(M_{Xcos}L_{rso}C_{rso} - M_{rso}M_xC_{rso}) + s^2 M_{Xcos}R_{rso}C_{rso} + sM_{Xcos}}{s^2(L_{rso}C_{rso}) + sC_{rso}R_{rso} + 1}$$

Input impedance of cosine coil

$$\frac{V_x}{I_x} = \frac{s^3(L_{rso}L_x - M_x{}^2) + s^2C_{rso}(R_xL_{rso} + R_{rso}L_x) + s(L_x + R_{rso}R_xC_{rso}) + R_x}{s^2(L_{rso}C_{rso}) + sC_{rso}R_{rso} + 1}$$

Transfer function of cosine coil

$$\frac{E_{cos}}{V_x} = \frac{s^3(M_{Xcos}L_{rso}C_{rso} - M_{rso}M_xC_{rso}) + s^2M_{Xcos}R_{rso}C_{rso} + sM_{Xcos}}{s^3(L_{rso}L_x - M_x{}^2) + s^2C_{rso}(R_xL_{rso} + R_{rso}L_x) + s(L_x + R_{rso}R_xC_{rso}) + R_x}$$

Frequency of Oscillations:

Equate the denominator of the equation to zero

$$s^2(L_{res}C_{res}) + sC_{res}R_{res} + 1 = 0$$

$$s = \frac{-R_{res}C_{res} \pm \sqrt{R^2{}_{res}C^2{}_{res} - 4C_{res}L_{res}}}{2C_{res}L_{res}}$$

$$\omega = \frac{1}{\sqrt{C_{res}L_{res}}} - \frac{R_{res}}{L_{res}}$$

V. Computing the Position from the Spiral Sensor

The induced voltages of sine and cosine spiral coils are computed using the transfer function derived relating the output and input voltages in earlier section. These outputs are further processed using the processing electronics and finally the position is computed using the following formula

$$position = pitchlength * \frac{atan2\,(Vcos, Vsin)}{2\pi}$$

The obtained position plot with respect to the reference position is triangular in nature. In order to get the error plot, the output waveform must be linear and should be compared with ideal or actual position. Hence the triangular waveform is converted into linear waveform through an algorithm. The logic used here is

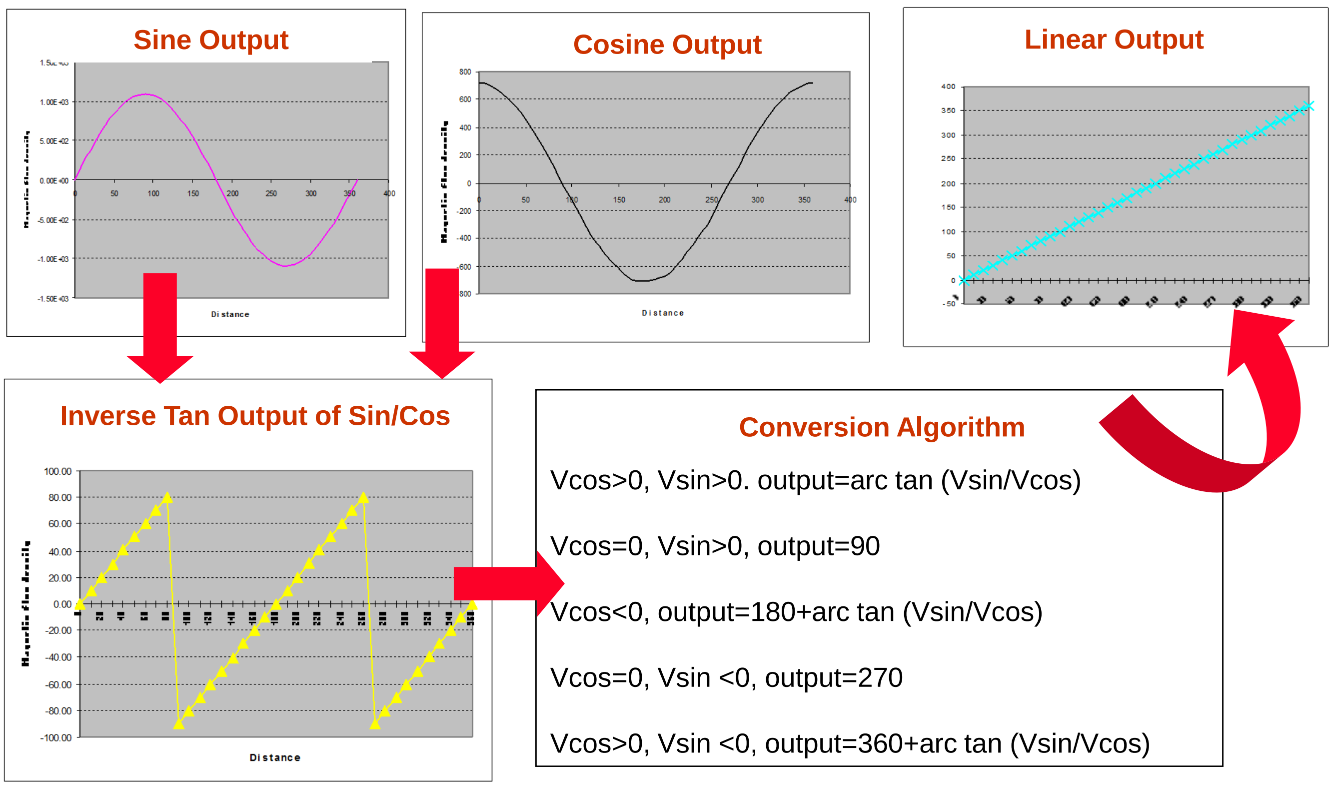

Fig.4 Flow Diagram of Position Calculation

Error Computation

Error values are obtained by subtracting the actual values from the values obtained upon linearization of the output curve

$$error = linear - actual$$

The obtained values of the error are plot against the reference position and the actual position predicted.

VI. MATLAB GUI for Analysis of Spiral Sensor

The analysis of the resonant inductive position sensor starts with the calculation of inductance values of the spiral coils implemented in the PCB. The spiral coils of different geometries are used as excitation coil, resonator coil and output coil. The magnetic field produced from the excitation coil links with output coils at resonant condition and produces output voltages. This magnetic field strength of coupling mainly depends on the inductance value of the spiral coils. The method of prediction of inductance and coupling coefficients are implemented in Matlab (refer our paper published). Once the values are predicted, these values are used as input to the tool. The input values given to the tool are shown in the table below, which consists of the individual values of spiral resistances, inductances and coupling coefficients, input voltage, frequency.

Table1 Sensor Parameters

Input	**Excitation Coil**	**Resonator Coil**	**Output Coils**	**Coupling Coefficients**
$V = 8.56\,V$	$L_x = 3.899\mu H$	$R_{rso} = 1.07\Omega$	$L_{sine} = 0.236\mu H$	$K_x = 0.1636$
$f = 3.4MHz$	$R_x = 4.88\Omega$	$L_{rso} = 1.787\mu H$	$L_{cos} = 0.223\mu H$	$K_{Xsine} = 0.0545$
				$K_{Xcos} = 0.0257$
				$K_{rso} = 0.16$

The prediction of sensor performance characteristics such as output voltages, positional accuracy with respect to ideal positions, frequency and phase response is done. The snapshot of the GUI of the tool is shown below.

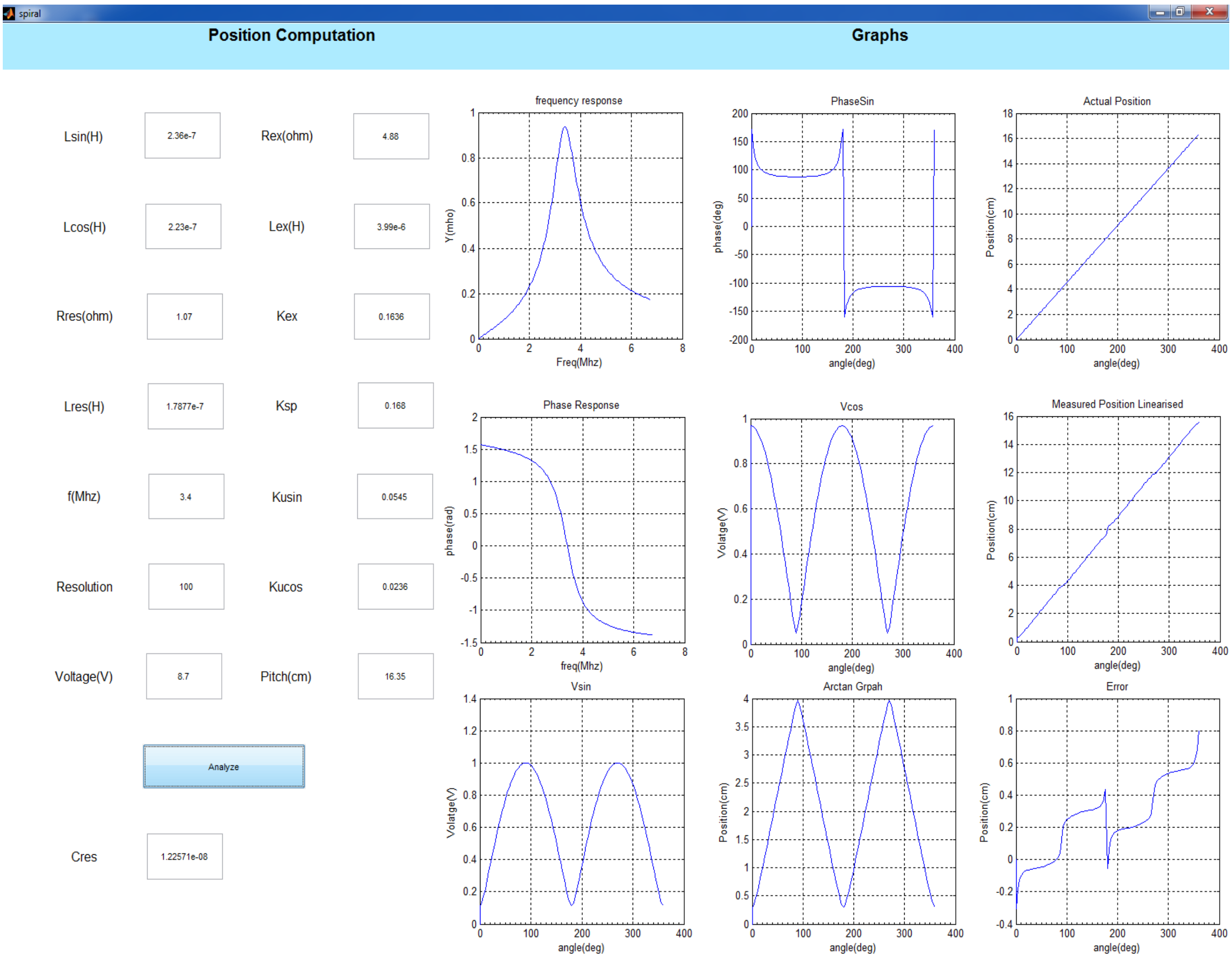

Fig.5 Snap Shot of the GUI

VII. Results and Discussion

The model predicts two aspects of sensor performance. Those are the response with position and the response with frequency.

1. Position Response Characteristics

   The coupling between the excitation and resonator coil along with its spatial phase (phi) produces the output on the sine and cosine coils.

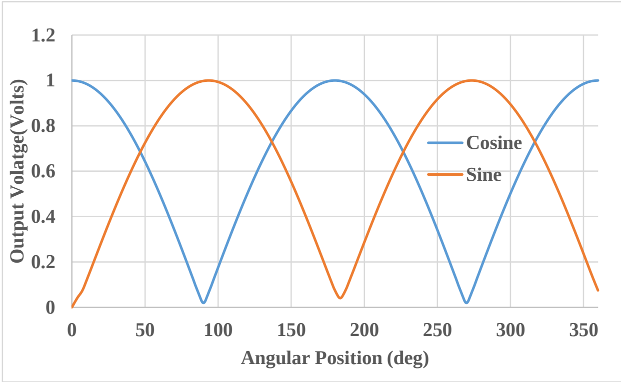

Fig.6 Output Voltage from Sine and Cosine coil before Processing Electronics

The above outputs shown in the plot are normalized. The actual output values are then processed using the electronics circuits shown in the block diagram.

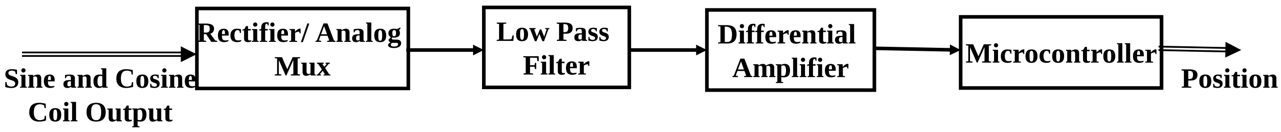

Fig.7 Functional Block Diagram for Position Calculation

Finally, the position output is converted into linear output using the algorithm which is shown in earlier section and the error is predicted.

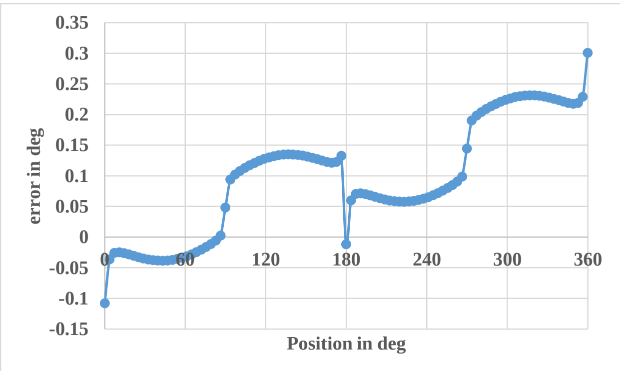

Fig.8 Position Error Plot

It is observed from the above result that at every 90 degree, the error values are high which can be reduced further by using a calibration method to predict the calibration coefficients to compensate the end effects.

2. Frequency Response

   The frequency response of the spiral sensor operating at 3.4 MHz is shown in the figure. These responses are normalized.

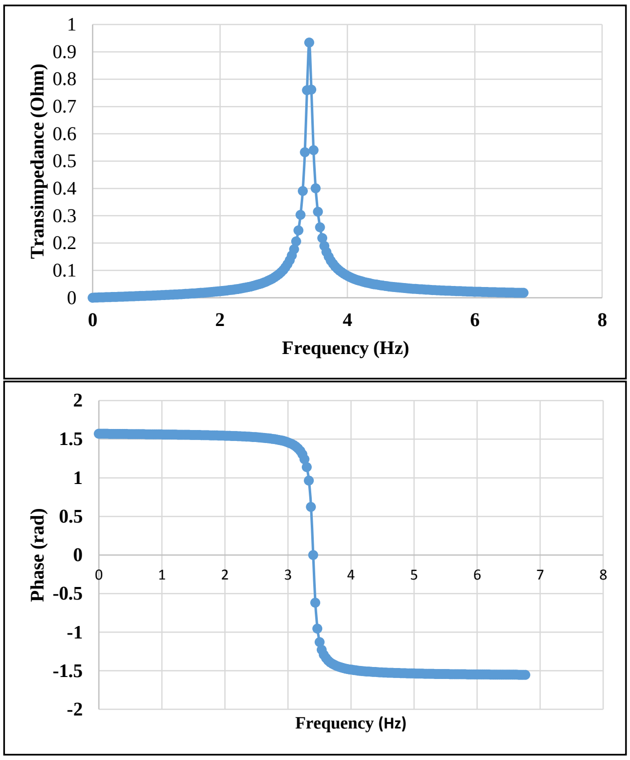

Fig.9 Frequency Response

It is noticed from the above responses that the resonator position is set at 90 deg. So, the resonator signal in the sine coil is maximum and the signal in the cosine coil is zero.

VIII. Conclusion

A transfer function model has been derived from equivalent circuit of spiral coils. This model has been implemented through Graphical User Interface (GUI) in MATLAB environment to predict the output of the position sensor from the sine and cosine spiral output coils. Also, the accuracy of the model has been evaluated through an error calculation by comparing the measured results with actual position in ideal condition. This design tool will reduce the design cycle time and expenses.

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