EXACT ANALYTICAL SOLUTION OF THE CURRENTS OF A THREE-PHASE SIX-STEP INVERTER/THREE-PHASE INDUCTION MOTOR SYSTEM

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Abstract: *In this paper analytical solutions of the currents when a three-phase, six-step voltage-source inverter (VSI) is feeding an induction motor (IM), are presented. Explicit analytical expressions of the steady-state current of the motor and of the input current of the inverter and of its switching elements are derived instead of using numerical solution methods. To check the validity of the presented analytical solutions, the results obtained of the motor currents and of the currents of the switching elements of the inverter were compared with previously published results, and the two sets of results were found to be identical.*

Keywords: *Analytical solution, voltage-source inverter, induction motor, steady-state analysis.*

1. Introduction

Induction motor drives are widely used in industry because of their advantages; such as their simplicity, ruggedness, low cost and good self-starting capability. The speed of the induction motor can be controlled by applying variable-frequency, variable-magnitude AC voltage to the motor. Both voltage-source and current-source inverters are used to control the speed of the induction motor. A motor fed from a variable-frequency, six-step voltage-source inverter exhibits different behaviour compared to the behaviour when it is operating from a sinusoidal voltage source. When inverters are used, the supply current waveform is nonsinusoidal with harmonic content. These harmonics are noticeable at low speed and causes speed oscillations. By controlling the switching elements of the voltage-source inverter, the applied voltage to the induction motor can be controlled.

A lot of attention was given to voltage-source inverters, control of three-phase induction motors and voltage-source inverters /induction motor systems [1-16].

In previous investigations, numerical methods were usually used to obtain the performance of the three-phase voltage- source inverter when feeding an induction motor [3-15], and the steady-state performance of the system was obtained by passing through the transient performance first, and was not obtained directly [3,5,8,9] .

In this paper, direct analytical expressions at steady state for the currents of the motor which is fed from a six-step voltage-source inverter and for the currents of the switching elements of the inverter, and for the current of the supply feeding the six-step inverter are derived.

2. Method of analysis

At steady state and any speed, the equivalent circuit per phase of the induction motor is as shown in Fig. 1 [10]. If, from Fig. 1, the corresponding equivalent resistance of the motor is R_e, and the corresponding equivalent reactance is X_e, then:

$$R_e = R_1 + \frac{X_m^2 R_2^{/} s}{R_2^{/2} + s^2 (X_m + X_2^{'})^2} \quad (1)$$

and

$$X_e = X_1 + \frac{R_2^{/2} + s^2 X_2^{'} (X_m + X_2^{'})}{R_2^{/2} + s^2 (X_m + X_2^{'})^2} . X_m \quad (2)$$

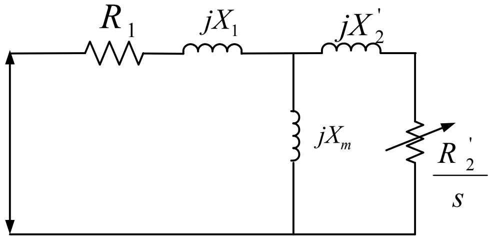


Fig. 1 Induction motor equivalent circuit.

If the phase angle of the motor at any slip, s, is assumed to be □, then:

$$\tan(\phi) = X_e / R_e \quad (3)$$

A three-phase, six- step inverter feeding a three-phase induction motor is shown in Fig. 2 in which thyristros are used as switching elements [2,3,5,8,12] which are shown as T_1, T_2, T_3,...T_6. With each thyristor an inverse parallel diode is connected. These diodes are shown in the figure as D_1, D_2, D_3,....D_6 .

U_d and i_d are the input supply DC voltage and current of the inverter.

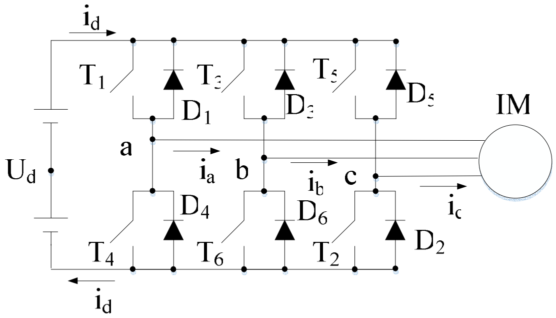


Fig. 2 Voltage-source inverter / induction motor system.

2.1 Motor currents

Fig. 3 shows typical steady-state waveforms of the output voltage and current of each phase of the six- step inverter when feeding an induction motor for phases (a), (b) and (c) of the motor [16].

The waveforms of the currents of phase (b) and phase (c) of the motor will be similar to that of phase (a) of Fig. 3, but shifted by 120° and 240° respectively from that of phase (a).

In Fig. 3 the angle ψ is the angle at which the current of phase (a) will reach zero from a negative value.

For a cycle of 360°, the conducting elements of diodes and thyristors of Fig. 2 will depend on the period under consideration. These are six periods each period will take 60°.

The conducting elements corresponding to this sequence are given in Tables 1, 2 and 3. In these tables either a diode or a thyristor will be conducting depending on the period under consideration with the angle ψ denoting the angle at which $i_a(\omega t)$ reaches zero starting from ωt= 0°,or starting from ωt= 180°. Table 1 shows the conducting elements corresponding to phase (a), Table 2 shows the conducting elements corresponding to phase (b), and Table 3 shows the conducting elements corresponding to phase (c).

2.1.1 Period 0° ≤ ωt ≤ 60°

During this period, for phase (a), D_1 will be conducting for (0° ≤ωt ≤ ψ), and the switching element T_1 will be conducting during (ψ ≤ ωt ≤ 60°), Table 1, and the instantaneous motor current of this phase is related to its phase voltage by the equation:

$$u_a(\omega t) = R_e i_a(\omega t) + L_e \frac{di_a(\omega t)}{dt} \quad (4)$$

The currents (i_a, i_b and i_c) in this period, (0° ≤ωt ≤ 60°),will be denoted by i_{a1}, i_{b1} and i_{c1}.

$u_a(\omega t)$ in Eq. (4) during this period (0° ≤ωt ≤ 60°), Fig. 3(a), is:

$$u_a(\omega t) = \frac{U_d}{3}$$

Substituting this value of $u_a(\omega t)$ in Eq.(4) with $i_a(\omega t)$ replaced by $i_{a1}(\omega t)$, and solving the resulting equation , the following equation is obtained:

$$i_{a1}(\omega t) = \frac{U_d}{3R_e} + [i_{a0} - \frac{U_d}{3R_e}]e^{-\omega t \cot\phi}$$

or

$$i_{a1}(\omega t) = k_1 + [i_{a0} - k_1]e^{-\omega t \cot\phi} \quad (5)$$

where $k_1 = \frac{U_d}{3R_e}$

To get the angle at which $i_{a1}(\omega t)$ reaches zero i.e. the angle ψ, the following equation should be solved numerically:

$$0 = k_1 + [i_{a0} - k_1]e^{-\psi \cot\varphi}$$

i_{ao} in Eq. (5) is the initial value of the phase (a) current at ωt = 0, and its expression will be derived later.

During the same period , (0° ≤ωt ≤ 60°) , for phase (b) the switching element T_6 will be conducting as shown in Table 2, and an equation similar to Eq. (4) can be obtained as :

$$u_b(\omega t) = R_e i_b(\omega t) + L_e \frac{di_b(\omega t)}{dt} \quad (6)$$

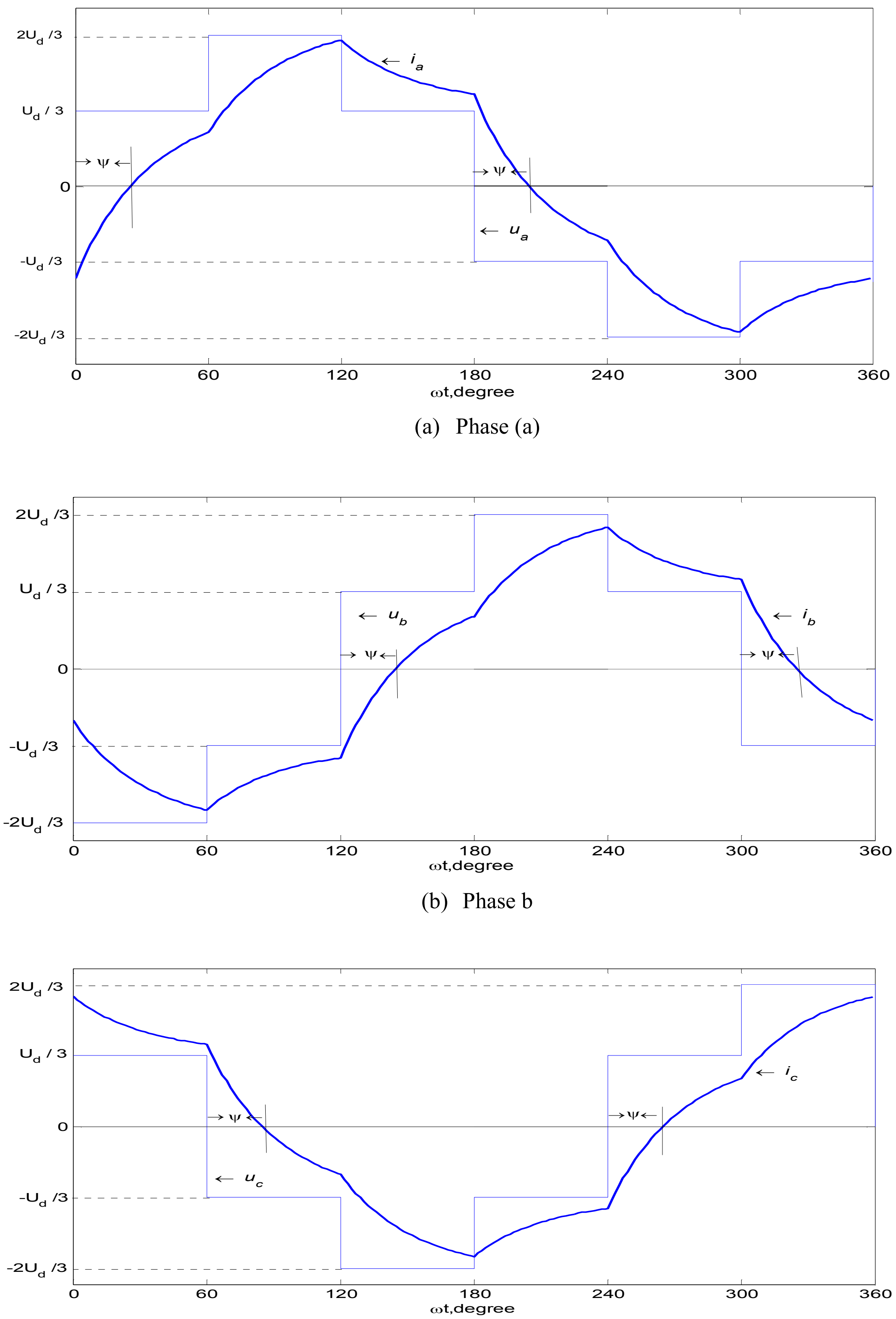
From Fig. 3(b), during this period:

$$u_b(\omega t) = \frac{-2U_d}{3}$$

Thus, solution of Eq. (6) is obtained as:

$$i_{b1}(\omega t) = -2k_1 + [i_{b0} + 2k_1]e^{-\omega t \cot\phi} \quad (7)$$

i_{b0} in Eq. (7) is the initial value of the current of phase (b) at ωt = 0, and its expression will be derived later.



(a) Phase (a)

(b) Phase b

(c) Phase (c)

Fig.(3) Voltage-source inverter output phase voltages and the corresponding motor phase currents.

Period	0^o-ψ	ψ-60^o	60^o-120^o	120^o-180^o	180^o-(ψ+180^o)	(ψ+180^o)-240^o	240^o-300^o	300^o-360^o
Firing Sequence	561	561	612	123	234	234	345	456
Conducting Diodes	D_1	_	_	_	D_4	_	_	_
Conducting Thyristors	_	T_1	T_1	T_1	_	T_4	T_4	T_4

Table 1 The conducting elements corresponding to phase (a) during a cycle.

Period	0^o-60^o	60^o-120^o	120^o- (ψ+ 120^o)	(ψ+120^o) -180^o	180^o-240^o	240^o-300^o	300^o-(ψ+ 300^o)	(ψ+300^o) -360^o
Firing Sequence	561	612	123	123	234	345	456	456
Conducting Diodes	_	_	D_3	_	_	_	D_6	_
Conducting Thyristors	T_6	T_6	_	T_3	T_3	T_3	_	T_6

Table 2 The conducting elements corresponding to phase (b) during a cycle.

Period	0^o-60^o	(60^o-ψ +60^o)	(ψ+60^o - 120^o)	120^o-180^o	180^o-240^o	240^o-(ψ+240^o)	(ψ+240^o) -300^o	300^o-360^o
Firing Sequence	561	612	612	123	234	345	345	456
Conducting Diodes	_	D_2	_	_	_	D_5	_	_
Conducting Thyristors	T_5	_	T_2	T_2	T_2	_	T_5	T_5

Table 3 The conducting elements corresponding to phase(c) during a cycle.

For phase (c) during the same period, ($0^o \le \omega t \le 60^o$), the switching element T_5, Table 3, will be conducting.

Thus, for phase (c) of the motor, the following equation is obtained:

$$u_c(\omega t) = R_e i_c(\omega t) + L_e \frac{di_c(\omega t)}{dt} \tag{8}$$

where, referring to Fig. 3(c):

$$u_c(\omega t) = \frac{U_d}{3}$$

Thus solution of Eq. (8) can be obtained as:

$$i_{c1}(\omega t) = k_1 + [i_{co} - k_1]e^{-\omega t \cot\phi} \tag{9}$$

i_{c0} is the initial value of the current of phase (c) at $\omega t=0$, and its expression will be derived later.

2.1.2 Period $60^o \le \omega t \le 120^o$

During this period, for phase (a), the switching element T_1 will be conducting for **($60^o \le \omega t \le 120^o$)**, Table 1, and the instantaneous motor phase current is related to its phase voltage by an equation similar to Eq. (4).

The currents of phases (i_a, i_b and i_c) during this period will be denoted by i_{a2}, i_{b2} and i_{c2}.

The value of $u_a(\omega t)$ during this period ,Fig. 3 (a), is:

$$u_a(\omega t) = \frac{2U_d}{3}$$

Thus, the expression of the current $i_a(\omega t)$ during this period, which is $i_{a2}(\omega t)$, can be derived by substituting this value of $u_a(\omega t)$ in Eq. (4), with $i_a(\omega t)$ replaced by $i_{a2}(\omega t)$ to get the corresponding equation which when solved will give:

$$i_{a2}(\omega t) = 2k_1 + (i_{a1}(\pi/3) - 2k_1)e^{-(\omega t - \pi/3)\cot\phi} \tag{10}$$

where $i_{a1}(\pi/3)$ in this equation can be obtained from Eq. (5) as:

$$i_{a1}(\pi/3) = k_1 + [i_{a0} - k_1]e^{(-\pi/3)\cot\phi}$$

Therefore, Eq. (10) becomes:

$$i_{a2}(\omega t) = 2k_1 + [(i_{a0} - k_1)e^{-(\pi/3)\cot\phi} - k_1]e^{-(\omega t - \pi/3)\cot\phi} \tag{11}$$

For phase (b), the switching element T_6, Table 2, will be conducting during this period.

The equation of $i_b(\omega t)$ will be similar to Eq. (6), with $u_b(\omega t) = \frac{-U_d}{3}$ from Fig. 3 (b).

Thus the expression of $i_{b2}(\omega t)$, can be derived as:

$$i_{b2}(\omega t) = -k_1 + (i_{b1}(\pi/3) + k_1)e^{-(\omega t - \pi/3)\cot\phi} \tag{12}$$

where $i_{b1}(\pi/3)$ in the above equation can be obtained from Eq. (7) as:

$$i_{b1}(\pi/3) = -2k_1 + [i_{b0} + 2k_1]e^{-(\pi/3)\cot\phi}$$

Thus Eq. (12) becomes:

$$i_{b2}(\omega t) = -k_1 + [(i_{b0} + 2k_1)e^{-(\pi/3)\cot\phi} - k_1]e^{-(\omega t - \pi/3)\cot\phi} \tag{13}$$

For phase (c), during this period, the diode D_2 will be conducting during the period **($60^o \le \omega t \le \psi + 60^o$),** and the thyristor T_2 will be conducting during the period ($\psi + 60^o \le \omega t \le 120^o$) as given in Table 3.

Thus, for phase (c) with $u_c(\omega t) = \frac{-U_d}{3}$, Fig. 3 (c), and using an equation similar to Eq. (8) the expression of $i_c(\omega t)$ in this period, ($60^o \le \omega t \le 120^o$), which is $i_{c2}(\omega t)$, can be derived as:

$$i_{c2}(\omega t) = -k_1 + (i_{c1}(\pi/3) + k_1)e^{-(\omega t - \pi/3)\cot\phi} \tag{14}$$

where $i_{c1}(\pi/3)$ in the above equation is obtained from Eq. (9) as:

$$i_{c1}(\pi/3) = k_1 + [i_{co} - k_1]e^{-(\pi/3)\cot\phi}$$

Therefore Eq. (14) becomes:

$$i_{c2}(\omega t) = -k_1 + [(i_{co} - k_1)e^{-(\pi/3)\cot\phi} + 2k_1]e^{-(\omega t - \pi/3)\cot\phi} \tag{15}$$

2.1.3 Period $120^o \le \omega t \le 180^o$

During this period, for phase (a), the thyristor T_1 will be conducting for ($120^o \le \omega t \le 180^o$) and the instantaneous motor current is related to its phase voltage by an equation similar to Eq. (4).

During this period, ($120^o \le \omega t \le 180^o$), the phase currents ($i_a$, i_b and i_c) will be denoted by i_{a3}, i_{b3} and i_{c3}.

The value of $u_a(\omega t)$ during this period ,Fig. 3 (a), is:

$$u_a(\omega t) = \frac{U_d}{3}$$

Thus, the expression of $i_{a3}(\omega t)$ can be derived by substituting this value of $u_a(\omega t)$, with $i_a(\omega t)$ replaced by i_{a3} (ωt) in Eq. (4). Solving the resulting equation the following equation is obtained:

$$i_{a3}(\omega t) = k_1 + [i_{a2}(2\pi/3) - k_1]e^{-(\omega t-(2\pi/3))\cot\phi} \quad (16)$$

where i_{a2} (2π/3) is obtained from Eq.(11)

$$i_{a2}(2\pi/3) = 2k_1 + [(i_{a0} - k_1)e^{-(\pi/3)\cot\phi} - k_1]e^{-(\pi/3)\cot\phi}$$

Therefore, Eq. (16) becomes:

$$i_{a3}(\omega t) = k_1 + \{[(i_{a0} - k_1)e^{-(\pi/3)\cot\phi} - k_1]e^{-(\pi/3)\cot\phi} + k_1\}e^{-(\omega t-(2\pi/3))\cot\phi} \quad (17)$$

For phase (b), the diode D_3 will be conducting during the period ($120^o \le \omega t \le \psi + 120^o$), and the thyristor T_3 will be conducting during the period ($\psi + 120^o \le \omega t \le 180^o$).

The value of $u_b(\omega t)$ during this period is:

$$u_b(\omega t) = \frac{U_d}{3}$$

Using an equation similar to Eq. (6) the current of phase (b) can be derived as:

$$i_{b3}(\omega t) = k_1 + [i_{b2}(2\pi/3) - k_1]e^{-(\omega t-(2\pi/3))\cot\phi} \quad (18)$$

where i_{b2}(2π/3) can be obtained from Eq. (13) as:

$$i_{b2}(2\pi/3) = -k_1 + [(i_{b0} + 2k_1)e^{-(\pi/3)\cot\phi} - k_1]e^{-(\pi/3)\cot\phi}$$

Therefore, i_{b3}(ωt) is obtained as:

$$i_{b3}(\omega t) = k_1 + \{[(i_{b0} + 2k_1)e^{-(\pi/3)\cot\phi} - k_1]e^{-(\pi/3)\cot\phi} - 2k_1\}e^{-(\omega t-(2\pi/3))\cot\phi} \quad (19)$$

For phase (c) the thyristor T_2 during this period, ($120^o \le \omega t \le 180^o$), will be conducting.

Using an equation similar to Eq. (8) with

$$u_c(\omega t) = \frac{-2U_d}{3}$$

The current for phase (c) during this period, ($120^o \le \omega t \le 180^o$), is obtained as:

$$i_{c3}(\omega t) = -2k_1 + [i_{c2}(2\pi/3) + 2k_1]e^{-(\omega t-(2\pi/3))\cot\phi} \quad (20)$$

where i_{c2}(2π/3) in the above equation can be obtained from Eq. (15) .

$$i_{c2}(2\pi/3) = -k_1 + [(i_{co} - k_1)e^{-(\pi/3)\cot\phi} + 2k_1]e^{-(\pi/3)\cot\phi}$$

Thus, the expression of i_{c3}(ωt) is obtained as:

$$i_{c3}(\omega t) = -2k_1 + \{[(i_{c0} - k_1)e^{(-\pi/3)\cot\varphi} + 2k_1]e^{(-\pi/3)\cot\varphi} + k_1\}e^{-(\omega t-(2\pi/3))\cot\phi} \quad (21)$$

It should be noticed that the currents obtained for the period $0 \le \omega t \le 180^o$ are identical in magnitude with the current for the period from $180^o \le \omega t \le 360^o$ but with a reversed sign.

2.2 Currents of switching elements and of supply to the inverter

Referring to Fig. 2 the following is obtained.

2.2.1 Period ($0 \le \omega t \le \psi$)

During this period the switching elements D_1, T_5 and T_6 are conducting while the switching elements D_2, D_3, D_4, D_5, D_6, T_1, T_2, T_3 and T_4 are not conducting.

Thus, in this case, Fig. 2:

$$i_{D1} = -i_a = -i_{a1}$$

$$i_{T5} = i_c = i_{c1}$$

$$i_{T6} = -i_b = -i_{b1}$$

Thus, from Eq. (5):

$$i_{D1} = -i_{a1} = -k_1 - [i_{a0} - k_1]e^{-\omega t \cot\phi} \quad (22)$$

and from Eq. (9):

$$i_{T5} = i_{c1} = k_1 + [i_{co} - k_1]e^{-\omega t \cot\phi} \quad (23)$$

and from Eq. (7):

$$i_{T6} = -i_{b1} = 2k_1 - [i_{b0} + 2k_1]e^{-\omega t \cot\phi} \quad (24)$$

The supply current of the inverter, i_d in Fig. 2, which will be denoted by i_{d1} during this period, is obtained as :

$$i_{d1} = i_{T6} = -i_b$$

Therefore:

$$i_{d1} = -i_{b1} = 2k_1 - [i_{b0} + 2k_1]e^{-\omega t \cot\phi} \quad (25)$$

2.2.2 Period ($\psi \le \omega t \le 60^o$)

During this period the thyristors T_1, T_5 and T_6 are conducting while the switching elements D_1, D_2, D_3, D_4, D_5, D_6, T_2, T_3 and T_4 are not conducting.

In this case, Fig. 2:

$i_{T1} = i_a = i_{a1}$

$i_{T5} = i_c = i_{c1}$

$i_{T6} = -i_b = -i_{b1}$

Therefore, from Eq. (5) :

$$i_{T1} = i_{a1} = k_1 + [i_{a0} - k_1]\mathrm{e}^{-\omega t \cot\phi} \quad (26)$$

and from Eq. (9):

$$i_{T5} = i_{c1} = k_1 + [i_{co} - k_1]\mathrm{e}^{-\omega t \cot\phi} \quad (27)$$

and from Eq. (7):

$$i_{T6} = -i_{b1} = 2k_1 - [i_{b0} + 2k_1]\mathrm{e}^{-\omega t \cot\phi} \quad (28)$$

The supply current of the inverter during this period, i_{d1}, is obtained from:

$i_{d1} = i_{T6} = -i_b$

Therefore:

$$i_{d1} = -i_{b1} = 2k_1 - [i_{b0} + 2k_1]\mathrm{e}^{-\omega t \cot\phi} \quad (29)$$

2.2.3 Period ($60^o \le \omega t \le \psi + 60^0$)

During this period the switching elements T_1, D_2 and T_6 are conducting while the switching elements D_1, D_3, D_4, D_5, D_6, T_2, T_3 , T_4 and T_5 are not conducting.

Thus, in this case, Fig. 2:

$i_{T1} = i_a = i_{a2}$

$i_{D2} = i_c = i_{c2}$

$i_{T6} = -i_b = -i_{b2}$

Therefore, from Eq. (11):

$$i_{T1} = i_{a2} = 2k_1 + [(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\omega \mathrm{t} - \pi/3)\cot\phi} \quad (30)$$

and from Eq. (15):

$$i_{D2} = i_{c2} = -k_1 + [(i_{co} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} + 2k_1]\mathrm{e}^{-(\omega \mathrm{t} - \pi/3)\cot\phi} \quad (31)$$

and from Eq. (13):

$$i_{T6} = -i_{b2} = k_1 - [(i_{b0} + 2k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\omega t - \pi/3)\cot\phi} \quad (32)$$

The supply current of the inverter during this period, which will be denoted by i_{d2}, is obtained from:

$i_{d2} = i_{T1} = i_{a2}$

Therefore:

$$i_{d2} = 2k_1 + [(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\omega \mathrm{t} - \pi/3)\cot\phi} \quad (33)$$

2.2.4 Period ($\psi + 60^o \le \omega t \le 120^o$)

During this period the thyristors T_1, T_2 and T_6 are conducting while the switching elements D_1, D_2, D_3, D_4, D_5, D_6, T_3, T_4 and T_5 are not conducting.

Thus, in this case, Fig. 2:

$i_{T1} = i_a = i_{a2}$

$i_{T2} = -i_c = -i_{c2}$

$i_{T6} = -i_b = -i_{b2}$

Therefore, from Eq. (11):

$$i_{T1} = i_{a2} = 2k_1 + [(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\omega \mathrm{t} - \pi/3)\cot\phi} \quad (34)$$

and from Eq. (15):

$$i_{T2} = -i_{c2} = k_1 - [(i_{co} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - 2k_1]\mathrm{e}^{-(\omega \mathrm{t} - \pi/3)\cot\phi} \quad (35)$$

and from Eq. (13):

$$i_{T6} = -i_{b2} = k_1 - [(i_{b0} + 2k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\omega t - \pi/3)\cot\phi} \quad (36)$$

The supply current of the inverter during this period, i_{d2}, is obtained from:

$i_{d2} = i_{T1} = i_{a2}$

Therefore:

$$i_{d2} = 2k_1 + [(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\omega \mathrm{t} - \pi/3)\cot\phi} \quad (37)$$

2.2.5 Period ($120^o \le \omega t \le \psi + 120^o$)

During this period the switching elements T_1, T_2 and D_3 are conducting while the switching elements D_1, D_2, D_4, D_5, D_6, T_3, T_4 , T_5 and T_6 are not conducting.

Thus, in this case, Fig. 2:

$i_{T1} = i_a = i_{a3}$

$i_{T2} = -i_c = -i_{c3}$

$i_{D3} = -i_b = -i_{b3}$

Thus, from Eq. (17):

$$i_{T1} = i_{a3} = k_1 + \{[(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\pi/3)\cot\phi} + k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (38)$$

and from Eq. (21)

$$i_{T2} = -i_{c3} = 2k_1 - \{[(\mathrm{i}_{c0} - k_1)\mathrm{e}^{(-\pi/3)\cot\varphi} + 2k_1]\mathrm{e}^{(-\pi/3)\cot\varphi} + k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (39)$$

and from Eq. (19):

$$i_{D3} = -i_{b3} = -k_1 - \{[(i_{b0} + 2k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\pi/3)\cot\phi} - 2k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (40)$$

The supply current of the inverter during this period, i_{d3}, is obtained from:

$i_{d3} = i_{T2} = -i_{c3}$

Therefore:

$$i_{d3} = 2k_1 - \{[(\mathrm{i}_{c0} - k_1)\mathrm{e}^{(-\pi/3)\cot\varphi} + 2k_1]\mathrm{e}^{(-\pi/3)\cot\varphi} + k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (41)$$

2.2.6 Period (ψ +120° ≤ ωt ≤ 180°)

During this period the thyristors T_1, T_2 and T_3 are conducting while the switching elements D_1, D_2, D_3, D_4, D_5, D_6, T_4, T_5 and T_6 are not conducting.

Thus, in this case, Fig. 2:

$i_{T1} = i_a = i_{a3}$

$i_{T2} = -i_c = -i_{c3}$

$i_{T3} = i_b = i_{b3}$

Thus, from Eq. (17):

$$i_{T1} = i_{a3} = k_1 + \{[(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\pi/3)\cot\phi} + k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (42)$$

and from Eq. (21)

$$i_{T2} = -i_{c3} = 2k_1 - \{[(\mathrm{i}_{c0} - k_1)\mathrm{e}^{(-\pi/3)\cot\varphi} + 2k_1]\mathrm{e}^{(-\pi/3)\cot\varphi} + k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (43)$$

and from Eq. (19):

$$i_{T3} = i_{b3} = k_1 + \{[(i_{b0} + 2k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\pi/3)\cot\phi} - 2k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (44)$$

The supply current of the inverter during this period, i_{d3}, is obtained from:

$i_{d3} = i_{T2} = -i_{c3}$

Therefore:

$$i_{d3} = 2k_1 - \{[(\mathrm{i}_{c0} - k_1)\mathrm{e}^{(-\pi/3)\cot\varphi} + 2k_1]\mathrm{e}^{(-\pi/3)\cot\varphi} + k_1\}\mathrm{e}^{-(\omega t - (2\pi/3))\cot\phi} \quad (45)$$

If the same procedure followed above is used the supply current to the inverter will be:

for period 180° ≤ ωt ≤ 240°, $i_{d4} = i_{T3}$

for period 240° ≤ ωt ≤ 300°, $i_{d5} = i_{T4}$

for period 300° ≤ ωt ≤ 360°, $i_{d6} = i_{T5}$

2.3 Determination of initial currents

The initial current of phase (a) of the motor is i_{a0}, thus, from Fig. 3(a), and since $i_a(0) = i_{a0}$ and $i_a(\pi) = -i_{a0}$, then:

$i_{a1}(0) = i_{a0}$ and $i_{a3}(\pi) = -i_{a0}$

Thus from Eq. (17) of $i_{a3}(\omega t)$:

$$i_{a3}(\pi) = k_1 + \{[(i_{a0} - k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\pi/3)\cot\phi} + k_1\}\mathrm{e}^{(-\pi/3)\cot\phi} = -i_{a0} \quad (46)$$

Therefore the expression of the initial value of the motor current for phase (a) (i_{a0}) is obtained from (46) as:

$$i_{a0} = \frac{k_1(e^{-\pi\cot\phi} + e^{-(2\pi/3)\cot\phi} - e^{-(\pi/3)\cot\phi} - 1)}{1 + e^{-\pi\cot\phi}} \quad (47)$$

If the above procedure is applied to phase (b), and using Eq. (19), the following is obtained:

$$i_{b3}(\pi) = k_1 + \{[(i_{b0} + 2k_1)\mathrm{e}^{-(\pi/3)\cot\phi} - k_1]\mathrm{e}^{-(\pi/3)\cot\phi} - 2k_1\}\mathrm{e}^{-\pi/3\cot\phi} = -i_{b0}$$

Thus, the initial value of the motor current for phase (b) can be is obtained as:

$$i_{b0} = \frac{k_1(-2e^{-\pi\cot\phi} + e^{-(2\pi/3)\cot\phi} + 2e^{-(\pi/3)\cot\phi} - 1)}{1 + e^{-\pi\cot\phi}} \quad (48)$$

For phase (c), and using Eq. (21), the following is obtained:

$$i_{c3}(\pi) = -2k_1 + \{[(\mathrm{i}_{c0} - k_1)\mathrm{e}^{(-\pi/3)\cot\varphi} + 2k_1]\mathrm{e}^{(-\pi/3)\cot\varphi} + k_1\}\mathrm{e}^{(-\pi/3))\cot\phi} = -i_{c0}$$

Thus, the initial value of the motor current for phase (c), $i_{c0,}$ can be is obtained as:

$$i_{c0} = \frac{k_1(e^{-\pi \cot\phi} - 2e^{-(2\pi/3)\cot\phi} - e^{-(\pi/3)\cot\phi} + 2)}{1 + e^{-\pi \cot\phi}}$$

(49)

3. Results

To validate the performance equations derived in Section (2) for the system composed of a three-phase, six–step voltage source inverter and a three-phase induction motor, whose parameters are given in Appendix (A), the three- phase motor currents and the currents of diodes and thyristors were computed from the analytical expressions derived. The results obtained are compared with corresponding results given in reference [16].

In the following results the motor phase angle is considered to be =30^0.

Fig. 4 shows the results of the steady-state current waveform of phase (a) of the induction motor, [Eqs. (5), (11) and (17)] together with corresponding results in reference [16].

Fig. 5 shows the comparisons between results obtained from the analytical expressions presented, and the results obtained from reference [16] for the steady-state waveforms of the currents of diodes D_1 and D_4 shown in Fig. 2.

Fig. 6 shows the comparison between the currents of thyristors T_1 and T_4, shown in Fig. 2.

It should be noted that the waveforms of currents of diodes D_3 and D_5 will be identical to the current of diode D_1 shown in Fig. 5 but shifted from the current of diode D_1 by 120^0 and 240^0 respectively. The same shift applies to currents of thyristors T_3 and T_5 with respect to the current of thyristor T_1.

The waveforms of currents of diodes D_6 and D_2 will be identical to the current of diode D_4 shown in Fig. 5 but shifted from the current of diode D_4 by 120^o and 240^o respectively, and the same shift applies to thyristors T_6 and T_2 with respect to thyristor T_4,

It is evident from Figs. (4),(5) and (6) that the results obtained from the proposed analytical expressions are identical with the corresponding results in reference [16]. Therefore, the instantaneous supply current to the inverter, [Eq. (25), (33) and (41)] with the motor at a slip s, that will give a phase angle of =30^o for the motor , and an input impedance of $Z_m = 10\ \Omega$ is shown in Fig. (7).

4. Conclusions

In this paper, explicit closed-form analytical solutions for the currents of the system composed of a three-phase six-step voltage-source inverter and a three-phase induction motor are presented.

The obtained expressions are at steady-state for the currents of the motor, of the switching elements of the inverter and of its input current.

These expressions can be used to obtain directly the performance characteristics of the system under consideration.

5. List of symbols

R_1, R_2	Induction motor stator and rotor resistances per phase, respectively, referred to stator.
X_1, X_2	Induction motor stator and rotor leakage reactance per phase, respectively, referred to stator.
X_m	Induction motor magnetizing reactance per phase.
U	Induction motor terminal voltage per phase.
s	Induction motor slip.
ω	Fundamental angular frequency of the supply

6. Appendix (A)

The induction motor used in the system is a three-phase, 7500-W, 400-V, 50-Hz, 16-A, 4-pole induction motor with the following parameters:

$R_1 = 0.6\ \Omega$, $X_1 = 0.9425\ \Omega$, $R_2' = 0.4\ \Omega$, $X_2' = 2.325\ \Omega$ and $X_m = 37.7\ \Omega$.

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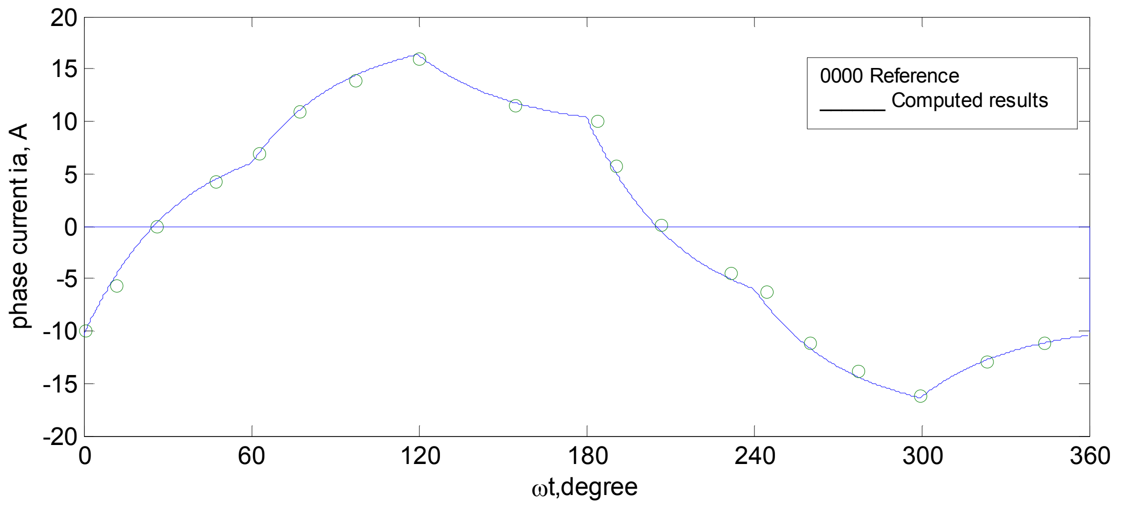

Fig. 4. Comparison between the steady-state waveforms of the current of phase (a) of the motor, □ = 30^0.

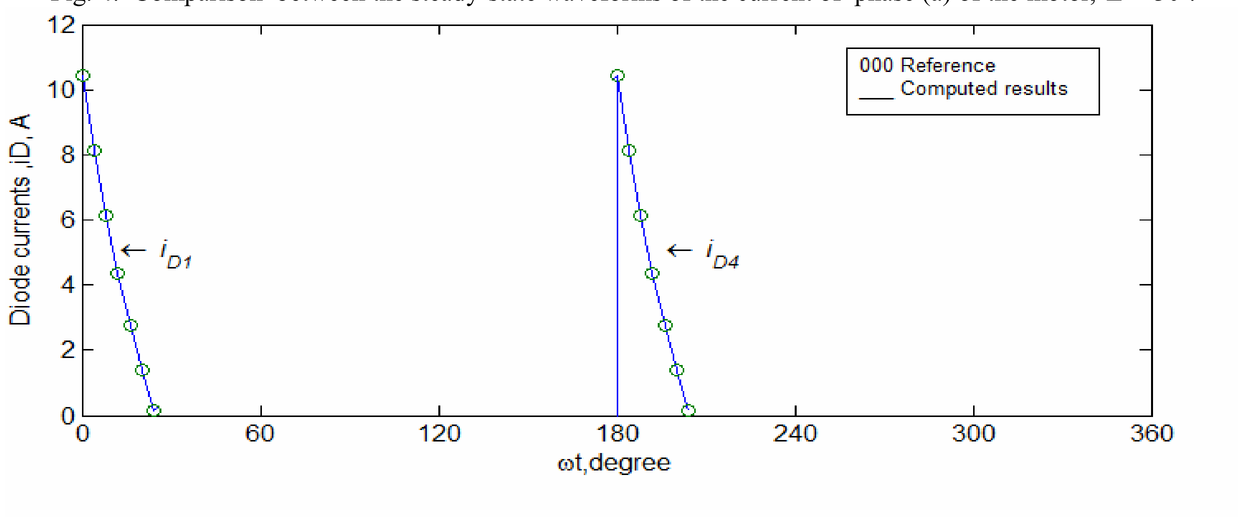

Fig. 5. Comparison between analytical results and results of reference [16] of the steady-state waveforms of the currents of diodes $D_1$and D_4, □ =30^0.

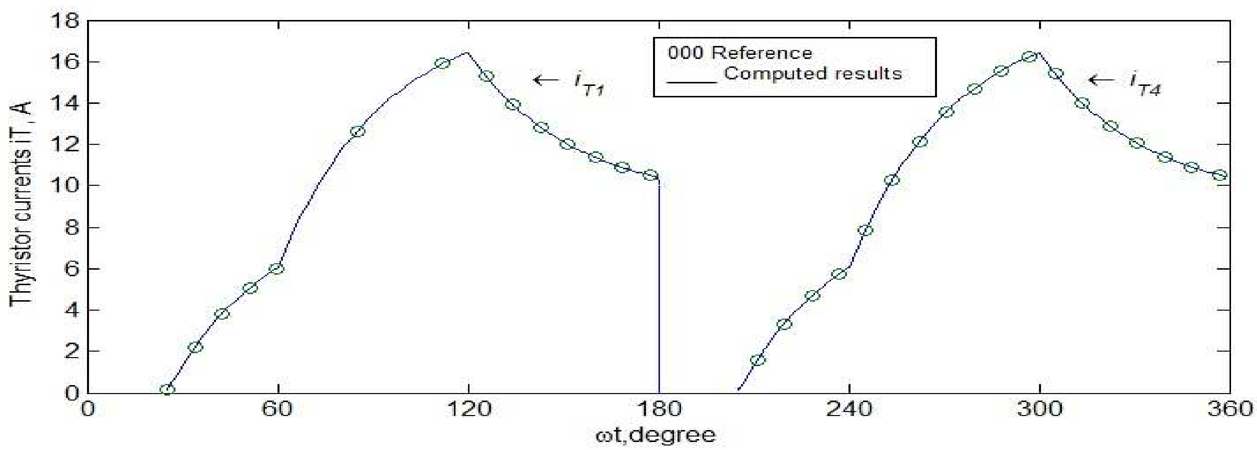

Fig. 6. Comparison between analytical results and results of reference [16] of the steady-state waveforms of the currents of thyristors $T_1$and T_4, □ = 30^0.

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