

OPTIMIZATION CONTROL STATION-KEEPING BOX MANOEVRES FOR GEO SATELLITES USING ELECTRIC THRUSTERS (OCSKBOX)

¹Louardi Beroual , ²Djamel Benatia

^{1,2}Department of Electrical Engineering, Batna University, Algeria

¹louardi_b@yahoo.fr , ²dj_benatia@yahoo.fr

Abstract: *The study presented in this paper deals with an optimization control station keeping box manoeuvres for geostationary satellites equipped with electric propulsion.*

The station keeping box (SKBOX) represented the maximum permitted values of the excursion of the satellite in longitude and latitude. It can be represented as a pyramidal solid angle, whose vertex is at the centre of the earth, within which the satellite must remain at all times. In this work, the station keeping box is defined by the two half angles at the vertex, one within the plan of the equator (E-W width), and the other in the plan of the satellite meridian (N-S width).

A number of different techniques are available for the numerical solution of the station keeping box problem. In this work we will consider the direct method for solution of continuous optimal control problem. Simulation results have demonstrated that the spacecraft can be tightly controlled within station keeping box.

Keywords: *geostationary satellites, SKBOX, box-limit, electric propulsion, specific impulse, quadratic programming.*

1. Introduction

An astrodynamics orbital station-keeping is the orbital maneuvers made by thruster burns that are needed to keep a spacecraft in a particular assigned orbit.

For many Earth satellites the effects of the non-Keplerian forces, i.e. the deviations of the gravitational force of the Earth from that of a homogeneous sphere, gravitational forces from Sun/Moon, solar radiation pressure and air-drag must be counteracted.

The deviation of Earth's gravity field from that of a homogeneous sphere and gravitational forces from Sun/Moon will in general perturb the orbital plane [1]. For geostationary spacecraft the inclination change caused by the gravitational forces of Sun/Moon must be counteracted to a rather large expense of fuel, as the inclination should be kept sufficiently small for the spacecraft to be tracked by a non-steerable antenna [1,2].

Solar radiation pressure will in general perturb the eccentricity (i.e. the eccentricity vector) [3]. For some missions this must be actively counter-acted with manoeuvres, for geostationary spacecraft the eccentricity must be kept sufficiently small for a spacecraft to be tracked with a non-steerable antenna. Also for Earth observation spacecraft for which a very repetitive orbit with a fixed ground track is desirable, the eccentricity vector should be kept as fixed as possible. A large part of this compensation can be done by using a frozen orbit design, but for the fine control manoeuvres with thrusters are needed [4].

Electric propulsion engines are more efficient than chemical ones: they require significantly less propellant to produce the same overall effect, for example a specific increase in spacecraft velocity. The propellant is ejected up to 20 times faster than from chemically-based thrusters, and, thus the same propelling force is obtained with a log less propellant, the forces EP produces can be applied continuously for very long periods-months or even years.

To maintain the satellite within box, orbit corrections are achieved by applying velocity impulses to the satellite at a point in the orbit. These impulses are generated by activating the thrusters that are mounted on the satellite as part of the propulsion subsystem.

The tool developed in the frame of this paper is based on numerical optimization techniques and uses a thrusters-based model of the satellite to take directly into account the activity of each thruster used for the control of the satellite on the optimization process. This paper presents the tool design and the main principle of the optimization algorithm. Usually, control strategies consider satellites as a point. The present work includes the mathematical definition and the satellite model that allow considering it as a system. The results of some simulations and their practical applications are presented.

2. Mathematical Modeling

A. Coordinate Frames:

The coordinate system used in this work for describing the perturbing forces is the satellite based Radial Tangent Normal (RTN) coordinate system with orthonormal basis $\vec{R}\vec{T}\vec{N}$. The \vec{R} axis is defined as always pointing from the Earth's center along the radius vector toward the satellite, The \vec{N} axis is normal to the orbit plane with direction of the satellite angular momentum vector, The \vec{T} axis is perpendicular to \vec{R} in the orbit plane and with the direction toward the satellite movement. It completes, with the unit vectors \vec{R} and \vec{N} , a right-handed orthogonal basis (see Figure 1).

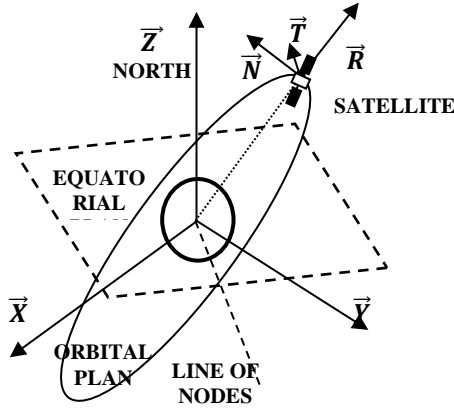


Fig. 1. Coordinate frames

In the following, a generic acceleration vector \vec{u} induced from propulsive force acting on the satellite will be expressed as

$$\vec{u} = u_R \vec{R} + u_T \vec{T} + u_N \vec{N} \quad (1)$$

Where u_R, u_T, u_N are the acceleration components along the radial, tangential and normal directions.

B. Orbit elements:

A total of six independent parameters are required to describe the motion of a satellite around the earth [3,4]. Two of these elements, semi-major axis a and eccentricity e describe the form of the orbit, one element the mean anomaly M defines the position of satellite along the orbit, the three others, the right ascension Ω , inclination i and argument of perigee ω define the orientation of the orbit in space. Given these six elements, it is always possible to uniquely calculate the position and velocity vector (see figure. 2).

In many application, satellite orbits are chosen to be near-circular, to provide a constant distance from

the surface of the Earth or a constant relative velocity. Typical examples are low-altitude remote sensing satellite or geostationary satellite.

While there is no inherent difficulty in calculating position and velocity from known orbital elements with e and i close to zero, the reverse task may cause practical and numerical problems. These problems are due to singularities arising from the definition of some of the classical orbital elements. The argument of perigee ω , for example, is not a meaningful orbital element for small eccentricities, since the perigee itself is not well defined for an almost circular orbit. Similar consideration apply to small inclination i where the line of node is no longer well defined and where the equations for Ω become singular. Several attempts have therefore been made to substitute other parameter for the classical keplerian elements. These elements are usually referred to as non-singular, regular or equinoctial elements [4].

The satellite orbit plane is defined thanks to the component of the inclination vectors (with modulus $\tan(i/2)$) direct along the line of nodes and pointing towards the ascending node.

$$\vec{i} = [p \ q]^T = 2 \tan\left(\frac{i}{2}\right) [\cos\Omega \ \sin\Omega]^T \cong [\cos\Omega \ \sin\Omega]^T, i \rightarrow 0 \quad (2)$$

The satellite trajectory on its orbit is defined to the semi major axis a , and supposing the parameters Ω and ω in the same plane, to the component of the eccentricity vector directed along the line of apsis and pointing towards the perigee.

$$\vec{e} = [h \ k]^T = e [\cos(\Omega + \omega) \ \sin(\Omega + \omega)]^T = e [\cos\tilde{\omega} \ \sin\tilde{\omega}]^T \quad (3)$$

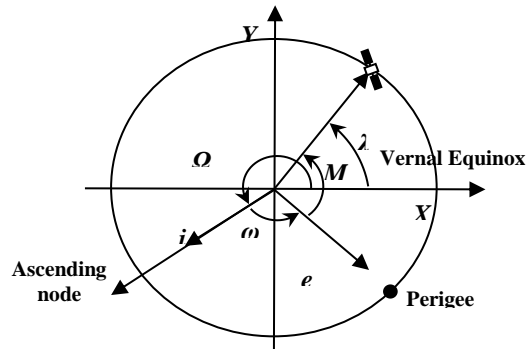


Fig. 2. Orbit elements

Finally, the position of the satellite along its orbit is represented by the mean longitude

$$l = \tilde{\omega} + M - \Theta(t) \quad (4)$$

Where Θ is the Greenwich sidereal angle.

C. Dynamics for a GEO satellite:

The motion of GEO satellite can be described by the rate of change of the equinoctial orbital parameters under the influence of the forces acting on the satellite. The geostationary dynamics results in the following nonlinear time varying system

$$\dot{x}(t) = \mathcal{K}(x(t)) + \mathcal{L}(t, x(t)) + \mathcal{G}(t, x(t))u(t) \quad (5)$$

Where

$$x = [a \ p \ q \ h \ k \ l]^T \quad (6)$$

$$u = [u_R \ u_T \ u_N]^T \quad (7)$$

And the functions, \mathcal{K} , \mathcal{L} and \mathcal{G} are the variation contribution to the equinoctial elements coming respectively from the Kipler's, Lagrange's and Gausse planetary equations [6] and [7].

\mathcal{K} : Describes the satellite motion under the effect of the gravitational attraction of the earth considered with takes into account the effect of the natural perturbing forces.

\mathcal{L} : Take into account the effect of the natural perturbing forces.

\mathcal{G} : Given by the acceleration by thrusts.

The translation of nonlinear model (equation 5) into the linear model we use the Taylor series up to first order around the nominal operating points

$$x_0 = [a_0 \ 0 \ 0 \ 0 \ 0 \ l_0]^T \quad (8)$$

$$u_0 = [0 \ 0 \ 0]^T \quad (9)$$

We obtain

$$\dot{x}(t) = \mathcal{A}(t)x(t) + \mathcal{B}(t)u(t) + \mathcal{D}(t) \quad (10)$$

Where

$$\dot{x} = x - x_0 \text{ and } u = u - u_0 \quad (11)$$

The $\mathcal{A}(t)$ and $\mathcal{D}(t)$ matrices turn out to be time varying because of the presence of periodic terms with periods equal to multiples of the periodicities of the earth, sun and Moon motion relative to the satellite.

The matrix $\mathcal{B}(t)$ is a periodic function with period equal to 24 hours.

$$\mathcal{B}(t) = \frac{1}{v_0} \begin{bmatrix} 0 & 2a_0 & 0 \\ 0 & 0 & \frac{1}{2}\sin\psi_0 \\ 0 & 0 & \frac{1}{2}\cos\psi_0 \\ -\cos\psi_0 & 2\sin\psi_0 & 0 \\ \sin\psi_0 & 2\cos\psi_0 & 0 \\ -2 & 0 & 0 \end{bmatrix} \quad (12)$$

Where

v_0 station keeping velocity equal to $\sqrt{\frac{\mu}{a_0}}$, μ is the earth gravitational coefficient.

And

$$\psi_0 = l_0 + \Theta(t) \quad (13)$$

The GEO orbits are characterized by very small values of eccentricity e and inclination i . the longitude and latitude can be defined

$$\lambda = l + 2h \sin(l + \Theta) - 2k \cos(l + \Theta) \quad (14)$$

$$\varphi = 2p \sin(\lambda + \Theta) - 2q \cos(\lambda + \Theta) \quad (15)$$

We denote y the spacecraft position vector, which can be considered as the output variable of the nonlinear model

$$y = f(x, t) \quad (16)$$

The output equation into its Taylor series up to the first order around x_0 [8], we get the output equation of the linear time varying system (eq.10)

$$y = C(t)x \quad (17)$$

Where

$$y = [\lambda - l_0 \ \varphi]^T \quad (18)$$

And

$$C(t) = \begin{bmatrix} 0 & -a_0 \cos\psi_0 & -a_0 \sin\psi_0 & 0 & 0 & 0 \\ 1 & -2\cos\psi_0 & 2\sin\psi_0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2\cos\psi_0 & 2\sin\psi_0 \end{bmatrix} \quad (19)$$

3. Station Keeping Box Problem Formulation

The station keeping box represents the maximum permitted values of the excursion of satellite in longitude λ and latitude φ . The SK box can be considered as pyramidal solid angle, whose vertex is at centre of the earth.

Is defined by the two half angles of vertex, one within plan of equator E - W width $2\lambda_{max}$ and other in the plan satellite meridian N - S width $2\varphi_{max}$ (see Figure. 3).

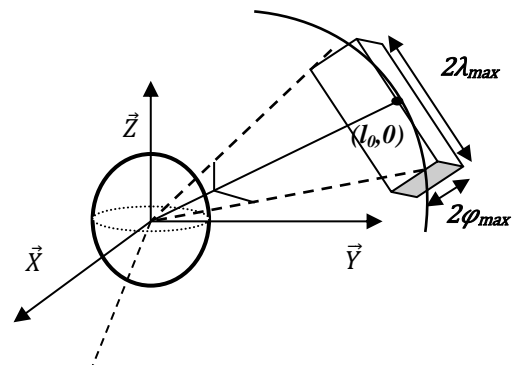


Fig. 3. Station keeping box

The station keeping problem can be formulated as constrained linear quadratic continuous time optimal control problem [9]. Given the linear model equation (10) and equation (17) with initial condition $\bar{\mathbf{X}}(t_i) = \bar{\mathbf{X}}_i$, the problem is to find the control optimal $\bar{\mathbf{u}}_{opt}(t)$ over a finite time horizon t_f-t_i the minimize the criterion

$$J = \frac{1}{2} \int_{t_i}^{t_f} (y^T(t) Q(t) y(t) + u^T(t) R(t) u(t)) dt \quad (20)$$

Subject to the conditions

$$-y_{max} \leq y \leq y_{max} \quad (21)$$

Where

$$y_{max} = [\lambda_{max} \quad \varphi_{max}] \quad (22)$$

The thruster accelerations are defined as control laws in the optimization problems. These control variables can thus take at any time any value comprised between zero and the maximum thruster acceleration, in general with j thruster, we can write the control vectors in RTN fram as

$$u_j = \frac{1}{m} \Gamma F_j \quad (23)$$

Where m is the spacecraft and Γ is the thruster system configuration matrix can be defined for a satellite equipped with four electric thrusters as

$$\Gamma = \begin{bmatrix} -\sin\gamma \cos\sigma & -\sin\gamma \cos\sigma & -\sin\gamma \cos\sigma & -\sin\gamma \cos\sigma \\ \sin\gamma \sin\sigma & \sin\gamma \sin\sigma & \sin\gamma \sin\sigma & \sin\gamma \sin\sigma \\ -\cos\gamma & -\cos\gamma & \cos\gamma & \cos\gamma \end{bmatrix} \quad (24)$$

And F is the thrust vector of the thruster system

$$0 \leq F \leq F_{max} \quad (25)$$

We defined the constraints on the control variable by

$$\frac{-F_{max}}{m} \leq \Gamma(\Gamma\Gamma^T)^{-1}u \leq \frac{F_{max}}{m} \quad (26)$$

4. Numerical solution of the problem

A number of different techniques are available for the numerical solution of the station keeping box problem. In this work we will consider the direct method for solution of continuous optimal control problem, the idea behind direct method is to discrete the control time history and/or stat variable history [10,11].

In this technique, the control inputs have to be written explicitly as function of the stat and its rate of change so that bounds on the control variables have translated in bounds on the attainable rates of change of the state variable [12].

The linear model (equation 10) can be written in different form, characterized by matrix B . to this

purpose, we can use the Lyapunov transformation [9,13], in the stat space defined as

$$\tilde{\mathbf{x}} = \mathcal{L}(t)\mathbf{x} \quad (27)$$

Where

$$\mathcal{L}(t) = \begin{bmatrix} 0 & 0 & 0 & 2 \cos \psi_0 & -2 \sin \psi_0 & -1 \\ 0 & 0 & 0 & 2 \cos \psi_0 & -2 \sin \psi_0 & -\frac{3}{2} \\ -\frac{3}{2a_0} & 0 & 0 & 2 \sin \psi_0 & 2 \cos \psi_0 & 0 \\ 0 & 2 \cos \psi_0 & -2 \sin \psi_0 & 0 & 0 & 0 \\ 0 & 2 \sin \psi_0 & 2 \cos \psi_0 & 0 & 0 & -2 \sin \psi_0 \\ -\frac{2}{a_0} & 0 & 0 & 2 \sin \psi_0 & 2 \cos \psi_0 & 0 \end{bmatrix} \quad (28)$$

The linear system (equation 10) can be written in the new state variables $\tilde{\mathbf{x}}$ as

$$\dot{\tilde{\mathbf{x}}}(t) = \tilde{A}(t)\tilde{\mathbf{x}}(t) + \tilde{B}(t)u(t) + \tilde{D}(t) \quad (29)$$

And we can write the control variable of the linear dynamics as a function of the state variable, so

$$u(t) = \tilde{B}^{-1}\dot{\tilde{\mathbf{x}}}(t) - \tilde{B}^{-1}\tilde{A}(t)\tilde{\mathbf{x}}(t) - \tilde{B}^{-1}\tilde{D}(t) \quad (30)$$

Where

$$\tilde{B}^{-1} = [\mathcal{L}(t)B(t)]^{-1} \quad (31)$$

We obtained

$$u(t) = M\dot{\tilde{\mathbf{x}}}(t) - M\tilde{A}(t)\tilde{\mathbf{x}}(t) - M\tilde{D}(t) \quad (32)$$

Where

$$M = \tilde{B}^{-1} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad (33)$$

And can be write the state variable as

$$\tilde{\mathbf{x}}(t) = \tilde{A}(t)\tilde{\mathbf{x}}(t) + \tilde{D}(t) \quad (34)$$

And the output equation given by

$$y(t) = \tilde{C}(t)\tilde{\mathbf{x}}(t) \quad (35)$$

Where

$$\tilde{C}(t) = C(t)\mathcal{L}^{-1}(t) \quad (36)$$

The station keeping problem formulated in the previous section as a constrained continuous time optimal control problem can be translated in a quadratic programming problem with constraint only on the state variables.

Above a finite time horizon t_f-t_i discretized in N intervals of length equal h each. The control optimal $u_k^{opt}(t)$ is taken constant equal to

$$u_k^{opt} \quad \text{With } k=0,1,\dots,N-1 \quad (37)$$

The problem is consist in finding the optimal sequence $\tilde{\mathbf{x}}_k^{opt}$ that minimized the criterion

$$J = \frac{1}{2} \sum_{k=1}^N y_k^T Q_k y_k + \frac{1}{2} \sum_{k=0}^{N-1} u_k^T R_k u_k \quad (38)$$

With

$$y_k = \tilde{C}_k \tilde{\mathbf{x}}_k \quad (39)$$

$$u_k = M \frac{\tilde{\mathbf{x}}_{k+1} - \tilde{\mathbf{x}}_k}{h} - M\tilde{A}_k \frac{\tilde{\mathbf{x}}_{k+1} - \tilde{\mathbf{x}}_k}{2} - M\tilde{D}_k \quad (40)$$

Subject

- The output variable $y(t)$

$$-y_{max} \leq y_k \leq y_{max} \quad (41)$$

- The control variable $u(t)$

$$\frac{-F_{max}}{m} \leq \Gamma(\Gamma\Gamma^T)^{-1}u_k \leq \frac{F_{max}}{m} \quad (42)$$

- The auxiliary state variable $\tilde{x}(t)$

$$\frac{\tilde{x}_{k+1} - \tilde{x}_k}{h} = \tilde{A}_k \frac{\tilde{x}_{k+1} + \tilde{x}_k}{2} + \tilde{D}_k \quad (43)$$

A step-by-step walkthrough of the algorithm is as follows:

Step1: Formulation of SKBOX problem

- Fixed the finite time horizon $t_f - t_i = 1$ day.
- The weighting matrices are equal to: $R = I_{2 \times 2}$ and $Q = I_{2 \times 2}$.
- Fixed the initial orbital elements vector $x(t_i)$.

Step2: Finding the optimal solution with minimized the criterion J with a discretization step of length $h = 0.01$ day.

Step3: Obtained the optimal control with equation (40).

Step4: Finding the output variable with equation (39).

Step5: Repeat the previous steps for 1 year.

5. Numerical simulation

The initial orbital elements for this simulation can be found in Table 1.

TABLE I
INITIAL ORBITAL ELEMENTS

Orbital parameter	Value
Semi major axis(km)	42166.279
Right ascension of A.N (°)	82
Eccentricity	0.0000778
Inclination (°)	0.002044
Argument of perigee (°)	315.67725
Mean anomaly (°)	324.34109

The satellite is considered in this simulation is equipped with four thrusters mounted on its anti nadir face (see Figure 4).the configuration of the force vector is

$$F = [F_1 \ F_2 \ F_3 \ F_4] \quad (42)$$

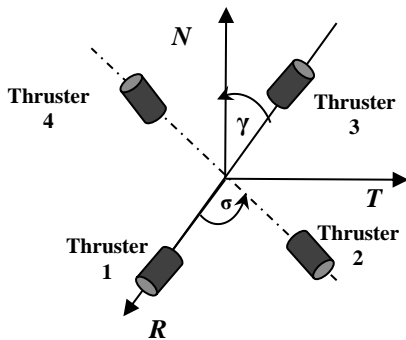


Fig. 4. Configuration force vector

The characteristics for this satellite can be found in Table 2.

TABLE II
CHARACTERISTICS OF SATELLITE

Characteristics of satellite	Value
Spacecraft masse (kg)	4500
Can angles of thruster (°)	50
Slaw angles of thruster (°)	15
Maximum force modulus (N)	0.17
Specific impulse (s).	3800s

The objective is to determine the set of manoeuvres to be executed in order to keep the satellite in a latitude and longitude box centered at the station longitude $l_0 = 10$ deg with $\lambda_{max} = 0.01$ deg and $\varphi_{max} = 0.01$ deg.

Figure 5, Figure 6 and Figure 7 illustrates the historical time of the optimal acceleration control components in RTN frame for one year.

Negative and positive values of optimal acceleration radial allow maintain the satellite in a latitude window equal to width 0.02deg, positive values of optimal tangential acceleration fix the satellite into a longitude width 0.02deg, and the variation of optimal acceleration normal centered the satellite in the box.

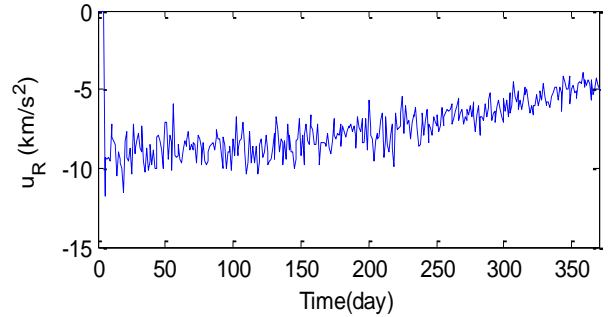


Fig. 5. Optimal acceleration radial control for one year

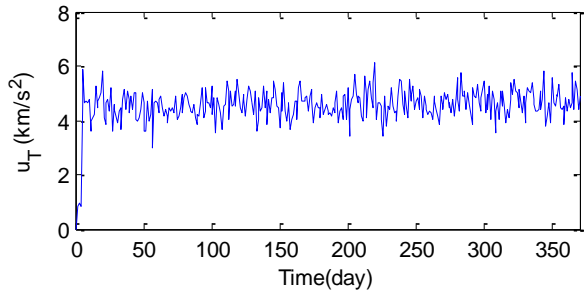


Fig. 6. Optimal acceleration tangential control for one year

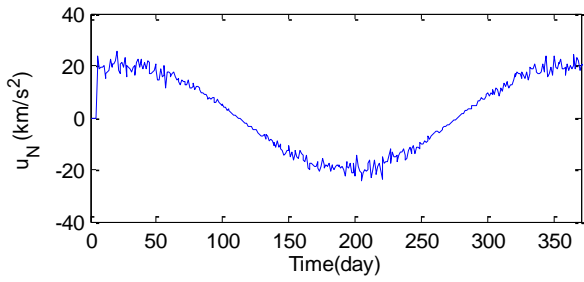


Fig. 7. Optimal acceleration normal control for one year

Figure 8. illustrates the evolution of the three components of propulsive force in *RTN* frame for one year with the maximum force modulus $F_{max}=0.17N$.

The variation in propulsive Force allows producing the optimal acceleration to control the satellite in latitude and longitude box. The thruster forces values are lower than maximum propulsive forces.

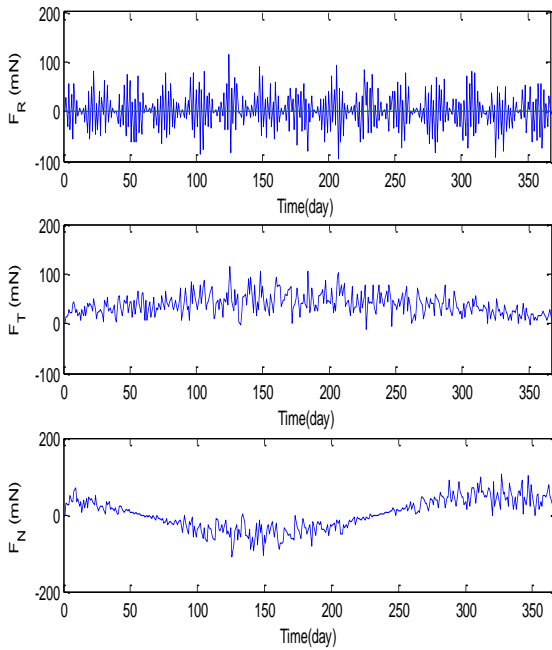


Fig. 8. Propulsive force for one year in RTN frame

Figure 9. Shows the controlled and uncontrolled manoeuvres time histories of the latitude for the station keeping box in width 0.02deg. In *SKBox* no-controlled, the variation in latitude value is not fixing in the box. For this problem the *SKBox* controlled is used for fixing the variation in latitude into the box.

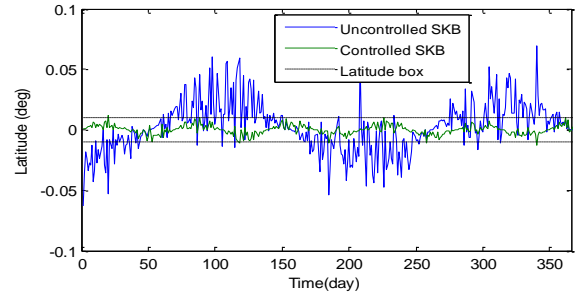


Fig. 9. Time histories of the latitude for one year

Figure 10. Illustrates the controlled and uncontrolled manoeuvres time histories of the true longitude for the station keeping box in width 0.02deg. In *SKBox* no-controlled, the variation in longitude value is not fixing in the box. For this problem the *SKBox* controlled is used for fixing the variation in longitude into the box.

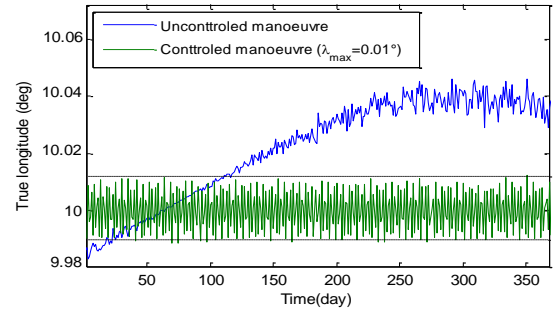


Fig. 10. Time histories of true longitude for one year

Conclusion

In this paper a new method for station keeping box of geostationary satellite equipped with electric propulsion has been developed, we considered a novel approach based on direct method for solution of continuous optimal control. Using this method, satellite position can be directly controlled based on the optimal acceleration for thruster, simulation results have demonstrated that the satellite can be tightly controlled in the station keeping box.

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Authors

Louardi Beroual, he was born in 1981. He received his Engineering Diploma in electronic engineering in 2004, a master degree in communication from the University of Constantine (Algeria) in 2007, Currently, Ph.D student studying at the University of Batna. His current research interests are: Station keeping (SK) maneuvers for a geostationary Satellite.



Djamel Benafia, he was born in Constantine (Algeria) in 1967. He received his Engineering Diploma in electronic engineering in 1990, a Master in communication in 1994 and the Ph.D in microwaves in 1999 from the Electronic Institute, University of Constantine. In 1994 he joined the Electronic Department, University of Banta (Algeria). In 2000 he received the Associate Professor degree. In 2005 he received the Professor degree.

