STABILITY ANALYSIS AND CONTROL OF FRACTIONAL ORDER DEPTH MODEL OF AUV-A Case Study

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Abstract: Autonomous Underwater Vehicle (AUV) is a highly nonlinear and complex system. Modeling and Control of such systems is always a challenging task. The designed scheme presented is based on theory of fractional Calculus. Stability analysis is done using Riemann's Theory. Integer order PID is designed for Fractional order Depth system of AUV. Same PID controller is applied to integer order depth system and the results are compared.

Key words: Fractional order, stability, AUV, depth system, state space, PID, FO, FOPID etc.

I. INTRODUCTION AND LITERATURE REVIEW

Autonomous Underwater Vehicles (AUV) are extensively used for ocean survey, mapping and data sampling. The control of vehicle is difficult due to nonlinearities and hydrodynamic forces subject to uncertainties. Therefore many control strategies have been proposed in past years. PID, Sliding mode and H_∞, LQG/LTR are some of these control strategies.

The present study is focused on model based control design. Modeling of AUV involves statics and dynamics Statics concerns with equilibrium state of body and dynamics is concerned when body is under accelerated motion. Study of dynamics is divided into Kinetics and kinematics. Kinetics is an analysis of forces causing motion. Kinematics is geometrical aspect of motion.

Nowadays there is better understanding of the potential of fractional calculus in modeling and control of many engineering systems[1][2]. The significance of fractional order system is that it is a generalization of classical integer theory, which leads to more accurate model and robust control performance.

In present article the mathematical modeling of depth system of AUV is described in terms of fractional differential equation. Further it is used for stability analysis. Considering the variables in the system the theoretical model is explained for the order of 0.1 to 1.5. Stability criteria is based on Riemann surface. In addition to it the fractional order state space model is simulated for controllability, observabillity. The frequency response is also obtained for different values of α , fractional order of a system.

The paper has been organized in following way:
In section II basics of fractional calculus are discussed. Section III presents Modeling of AUV followed by its subsystems. In section IV the depth model is expressed in fractional order form. Stability analysis of fractional order depth system is presented using Riemann surface theory. In section V integer order PID controller is designed and simulated for fractional order and integer order depth system of AUV and simulation results are compared. Section VI concludes the work.

II. FRACTIONAL CALCULUS

Fractional calculus is generalized integration and differentiation. The integral-differential operator is : [3]

$$\begin{aligned} {}_aD^r{}_t &= d^r / dt^r : r > 0 \\ {}_aD^r{}_t &= 1 : r = 0 \\ {}_aD^r{}_t &= \int_a^t (d\tau)^{-r} : r < 0 \end{aligned} \tag{1}$$

a and t are limits of operations. The three definitions are used for general fractional differentegral are Grunwald Letnikov (GL), Riemann-Liouville (RL) and the Caputo.

The GL definition is given by

$${}_aD^\alpha{}_t f(t) = \lim_{h\to 0} h^{-r} \sum_{j=0}^{[t-a/h]} (-1)^j \binom{r}{j} f(t-jh) \tag{2}$$

RL Definition is given by

$${}_aD^\alpha{}_t f(t) = 1/\Gamma(n-r) d^n / dt^n \int_a^t f(\tau)/(t-\tau)^{r-n+1} d\tau \tag{3}$$

(n-1) < r < n and Γ () is Gamma function.

The Caputo definition is given by

$$ {}_aD^{\alpha}{}_t f(t) = 1/\Gamma(r-n)\int_a^t f^n(\tau)/(t-\tau)^{r-n+1}d\tau \quad (4)$$

(n-1) < r < n

The Caputo derivative is in the same form that of integer order differential equation with initial condition a=0.

A. *State Space Representation of Fractional Order System*

The general fractional order n term system can be shown in following form [4]:

$$ {}_aD^{\alpha_n}y(t) + {}_{a_{n-1}}D^{\alpha_{n-1}}y(t) + ...{}_{a0}D^{\alpha_0}y(t) = u(t) \quad (5)$$

By Laplace Transform of equation (5), the fractional transfer function G(s) is

$$ G(s) = \frac{1}{a_nS^{\alpha_n} + ...+ a_1S^{\alpha_1} + a_0S^{\alpha_0}} \quad (6)$$

The fractional order system in state space can be expressed as: [5]

$$ {}_aD_t^{\alpha} = Ax(t) + Bu(t) \quad (7)$$
$$ y(t) = Cx(t) $$

$x(t), u(t), y(t)$ are the state, input and output vectors and $\alpha = [\alpha_{1,}\alpha_2....\alpha_n]^T$ are the fractional orders. n-term fractional differential equation is in eq. (8)

$$\begin{pmatrix} {}_0D^{\alpha_1}x_1(t) \\ {}_0D^{\alpha 2}x_2(t) \\ {}_0D^{\alpha 2}x_3(t) \\ . \\ {}_0D^{\alpha_n}x_n(t) \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ . & . & . & . \\ . & . & & \\ -a_0/a_n & -a_1/a_n & . & a_{n-1}/a_n \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ . \\ x_n(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ . \\ 1/a_n \end{pmatrix} u(t)$$

$$ y(t) = \begin{pmatrix} 1 & 0 ....0 & 0 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \\ 0 \\ . \\ x_n(t) \end{pmatrix} \quad (8)$$

The controllability and observabillity concepts can be applied like conventional state space system. From equation (8): [6]

$$ C = Controllability = \begin{pmatrix} B & AB & A^2B...A^{n-1}B \end{pmatrix} \quad (9)$$

$$ O = Observabillity = \begin{pmatrix} C \\ CA \\ CA^2 \\ . \\ CA^{n-1} \end{pmatrix} \quad (10)$$

If both the matrices have rank n the system is controllable and observable. [7]

B. *Stability Considerations in Fractional Order (FO) Systems using Riemann Surface*

The stability of fractional order system is different than integer order system. Fractional order systems may be stable systems with their roots on the right half of ω plane. The principle sheet of Riemann surface is given as $-\pi < \arg(s) < \pi$, where $\omega = s^q$,the corresponding ω can be given as $-q\pi < \arg(s) < q\pi$. The ω plane for right half plane of Riemann sheet is given as $-q\pi/2 < \arg(s) < q\pi/2$. Roots of equation in ω plane are observed and Riemann surface stability of the system can be analyzed.

C. *Frequency Response of FO system*

The evaluation of transfer function of fractional order system along imaginary axis s=jω gives frequency response. From Bode Plot the frequency response can be obtained. By factorizing the function [8]

$$ G_s = \frac{P(S^{\alpha})}{Q(S^{\alpha})} = \frac{\prod_{k=0}^{m} s^{\alpha} + z_k}{\prod_{k=0}^{n} (s^{\alpha} + \lambda_k)}$$

$$ z_k, P(z_k) = 0, \lambda_k, Q(\lambda_k) = 0, z_k \neq \lambda_k $$

The magnitude of curve has a slope equal to $\pm\alpha 20db/dec$ and for phase plot the slope is $\pm\alpha\pi/2$.

III. MODELING OF AUV

The complete mathematical model of AUV is divided into three subsystems:

i) Surge system
ii) Speed system
iii) Depth system.

Here depth system of AUV is considered. From the statics and dynamics of AUV linear and angular motion equations are obtained. Based on Newton's law two basic equations are used for modeling of AUV. Refer equations (11) & (12).

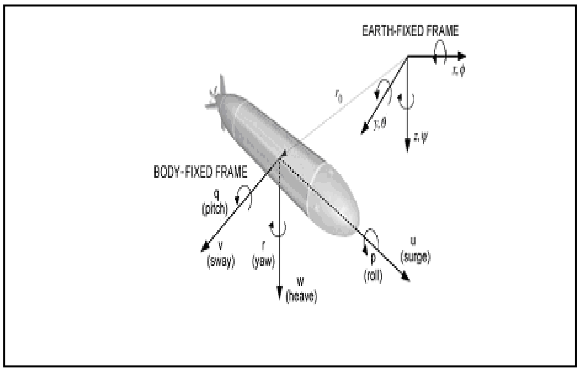

Fig.1 AUV Variables

$$\dot{\eta} = J(\eta)v \tag{11}$$

$$M\dot{v} + C(v)v + D(v)v + g(\eta) = \tau \tag{12}$$

$J(\eta)$ – Transformation Matrix

M – Inertial Matrix

$C(v)$ – Coriolis Matrix

$D(v)$ – Damping Matrix

η-Position and Orientation Vector

g(η)-Gravitational Matrix

τ – Control Input Matrix

From equations (11) and (12) linear zed depth model in state space system is as follows

$$\begin{pmatrix} m - x_{\dot{u}} & -(mx_g + z_g) & 0 & 0 \\ -(mx_g + M_{\dot{w}}) & I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{w} \\ \dot{q} \\ \dot{z} \\ \dot{\theta} \end{pmatrix} - \begin{pmatrix} Z_w & mU + Z_g & 0 & 0 \\ M_w & -mx_g U + M_{\dot{q}} & 0 & M_\theta \\ 1 & 0 & 0 & -U \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} w \\ q \\ z \\ \theta \end{pmatrix} = \begin{pmatrix} Z_{\delta_S} \\ M_{\delta_S} \\ 0 \\ 0 \end{pmatrix} \delta_S \tag{13}$$

Considering the state vector as x and control vector as u (14) is:

$$x = \begin{bmatrix} w & q & z & \theta \end{bmatrix}^T, u = \begin{bmatrix} \delta_S \end{bmatrix}^T \tag{14}$$

The state space system is typically expressed as $\dot{x} = Ax + Bu$ .With a forward velocity of vehicle as U = 1.54Knots. The REMUS-100 AUV model parameter values are as in Table 1.

Table 1: Inertia and Added Mass and Drag Coefficients

Parameter	Value
Ixx	1.77e-001 $kg.m^2$
I_{yy}	3.45e+000 $kg.m^2$
Izz	3.45e+000 $kg.m^2$
$Y_{\dot{v}}$	-35.5 kg
$Y_{\dot{r}}$	1.93 kg
$N_{\dot{v}}$	1.93 kgm
$N_{\dot{r}}$	-4.88 $kg-m^2/rad$
$M_{\dot{w}}$	-1.93e+000 kgm
$M_{\dot{q}}$	-4.88e+000 kgm^2/rad
Z_w	-6.66e+001 kg/s
M_w	-3.07e+001 $kg-m/s$

IV. STABILITY ANALYSIS OF FRACTIONAL ORDER DEPTH SYSTEM OF AUV

Based on REMUS AUV [9], the Mathematical model of depth system of AUV is expressed in state space form as under:

$$6.406 / s^3 + 0.82s^2 + 0.69s^1 \tag{15}$$

The transfer function of the depth system of AUV with α = 0.5 can be shown as :

T.F. = $1/0.156s^{1.5} + 0.128s + 0.108s^{0.5}$

The Fractional order differential equation is:

$$0.156\,{}_0D^{1.5}y(t) + 0.128\,{}_0D^1y(t) + 0.108\,{}_0D^{0.5}y(t) = u(t) \tag{16}$$

A. Controllability and Observability

Obtaining state space system as per equation (8) from equation (16) and like conventional integer system referring (9) and (10) the controllability and observabillity are obtained as 3. The system is controllable and observable when the rank of controllability and observabillity matrix is full. Hence the depth system is controllable and observable.

B. Stability of Depth System of AUV using Riemann Surface

The stability analysis of fractional order system can be done using Riemann surface. The analytical solution of fractional order differential equation based on equation (16) for depth control system of AUV is obtained with u(t) = 0. It is seen that the solution is stable

when $\lim_{t \to \infty} y(t) = 0$. The characteristic equation of depth system is:

$$P(s): 0.156s^{1.5} + 0.128s^{1} + 0.108s^{0.5} \Rightarrow 0.156s^{15/10} + 0.128s^{10/10} + 0.108s^{5/10}$$

When m=10, $\omega = s^{1/10}$ the polynomial is as follows:

$$P(\omega): 0.156\omega^{15} + 0.128\omega^{10} + 0.108\omega^{5} = 0$$

The appropriate arguments are:

$$\omega = \begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.834j \pm 0.483 \\ 0.7173j \pm 0.644 \\ 0.202j \pm 0.943 \\ 0.391j \pm 0.881 \\ 0.959j \pm 0.0992 \\ -0.202j \pm 0.943 \\ -0.959j \pm 0.0992 \\ -0.834j \pm 0.483 \end{array}$$

The significance of Riemann sheet is given as :

$-\pi / m < \phi < \pi / m$ where, $\phi = \arg(m)$

From shown arguments below, it is seen that, not a single argument is satisfying the condition mentioned above. It means that the system is not stable. [10].

$$\begin{aligned}
|\arg(\omega_{1,2})| &= 0 \\
|\arg(\omega_{3,4})| &= 0 \\
|\arg(\omega_{5,6})| &= 0 \\
|\arg(\omega_{7,8})| &= 0 \\
|\arg(\omega_{9,10})| &= 0 \\
|\arg(\omega_{11,12})| &= 2.0960 \\
|\arg(\omega_{13,14})| &= 0.8394 \\
|\arg(\omega_{15,16})| &= 2.9306 \\
|\arg(\omega_{17,18})| &= 0.4173 \\
|\arg(\omega_{19,20})| &= 1.6739 \\
|\arg(\omega_{21,22})| &= 2.9306 \\
|\arg(\omega_{23,24})| &= 1.6739 \\
|\arg(\omega_{25,26})| &= 2.0960 \\
|\arg(\omega_{27,28})| &= 0.4173 \\
|\arg(\omega_{29,30})| &= 0.8394
\end{aligned}$$

The roots of equation in complex ω plane are shown in pole zero map (Fig.1) From the roots shown in the plot, $-\pi / m < \phi < \pi / m$ condition is not satisfied. Hence the system is not in stable zone.

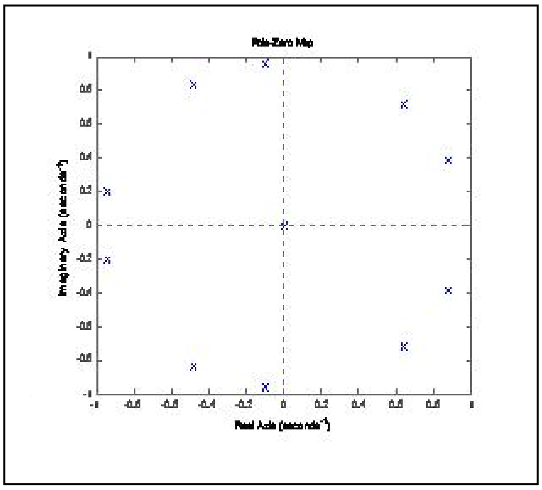

Fig.1 Roots of Equation in Complex ω Plane

C. Frequency Response of Fractional Order System

In this section Fractional order Depth system of AUV is analyzed. The step response and bode plot of fractional order depth system of AUV are obtained. The stability analysis is based on responses.

MATLAB/SIMULINK is used as simulation tool. FOMCON toolbox [11] is used for simulation.

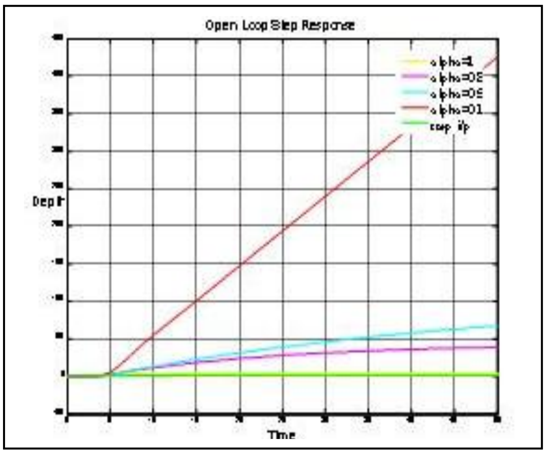

Fig.2 Open Loop Step Response for Different Values of α

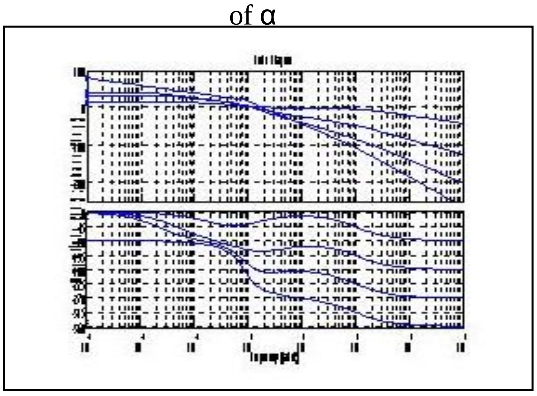

Fig.3 Frequency Response for different values of α

V. IOPID CONTROLLER FOR FO MODEL

In previous section it is seen that the Fractional order depth system of AUV is controllable, observable but is un stable. In present section conventional PID controller is designed for fractional order depth system of AUV. The unit step responses can show good comparison , hence unit step input is applied. The comparison of unit step response for closed loop integer order depth model and fractional order depth model with same integer order controller is shown in Fig. 4. [12]

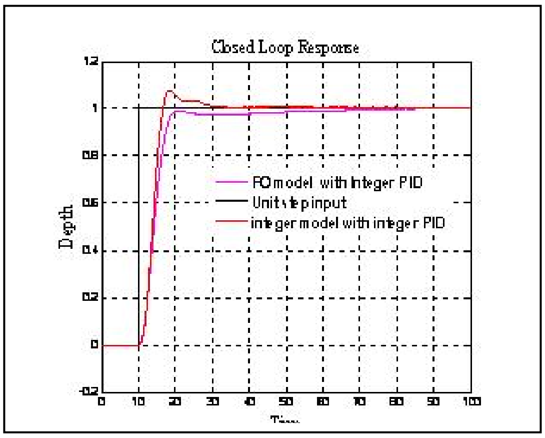

Fig.4.Comparison of Unit step Response for FO and Integer Model with Integer PID

P=0.031638 I=7.4428E-005 D=-0.034423	Integer order Model of depth system of AUV	Fractional order model of AUV
Rise Time	**3.9Sec**	**5.55Sec**
Settling time	**18.3Sec**	**30.3Sec**
Over Shoot	**7.76%**	**0%**
Peak Overshoot	**1.08**	**1**

Table 2: Comparison of dynamic parameters of FO & Integer Model

From the comparison of dynamic properties of fractional order system and integer order system with same integer PID controller designed for integer order system it is seen that overshoots are reduced in fractional order system but it stabilizes slower than integer order system. The settling time and rise time values are mentioned in Table 2.

VI. CONCLUSION

From the theory of fractional calculus it is seen that applying fractional derivatives modeling and control of dynamic systems is more realistic than integer order system. The different stability features can be studied for fractional order model before deciding suitable control strategy. They are like state space representation, controllability, observabillity, bode plots, pole-zero plots, step response etc.

The performance analysis of fractional order and integer order system for Integer PID controller shows that the integer controller is not adequate. The alternative and better approach is fractional order PID controller [13] for fractional order system. Which can lead the improvement in dynamic properties of fractional order system.

VIII. ACKNOWLEDGMENTS

Authors wish to thanks to Naval Research Board (NRB) who have supported this project under their grant in scheme Projects. Project no. ***NRB-236/SC/11-12***

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