Empirical Wavelet Transform-Based Power Quality Indices for Balanced and Unbalanced Three-Phase Systems

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Abstract: Three-phase Power-Quality Indices (PQIs) can be used to quantify and hence evaluate the quality of the Electric Power Systems (EPS) waveforms. In this paper the three-phase power components definitions contained in the IEEE Standard 1459-2000 for-balanced and unbalanced three-phase systems with non-sinusoidal situations are reformulated using the Empirical Wavelet Transform (EWT). EWT technique is applied on a balanced and unbalanced three-phase signals to estimate three-phase PQIs. This technique first estimates the frequency components and then adaptively tunes the wavelet and scaling function based on the boundaries to decompose the signal accurately and there by using EWT-based reformulated indices, three phase PQIs can be accurately estimated. It can be observed from the results that EWT-based PQIs for balanced and unbalanced three-phase supply are very close to the true values.

Key words*: Empirical Wavelet Transform, Power Quality Indices, Balanced three-Phase system, un balanced three-Phase system,*

1. Introduction

Power Quality (PQ) is defined as a combination of voltage quality and current quality [1].Voltage quality can be defined based on how much deviations exist between the actual voltage (or current) and the ideal voltage (or current) waveforms .The ideal voltage (or current) should have sinusoidal wave shape with fixed magnitude and fixed frequency as their nominal values. Voltage and/or current deviations are considered as power quality disturbances. The PQ disturbances can be categorized into stationary and non stationary signals. The stationary signal is the one in which mean and variance do not vary with time while in the case of a nonstationary signal, either of them or both may vary with time. Since most PQ disturbances are noisy and nonstationary in nature, and advanced signal processing technique is required to accurately decompose the nonstationary power signal and determine the location of frequency components in time as well.

The traditional Power Quality Indices definitions can be found in the IEEE standard 1459-2000[2,3] based on the frequency domain approach using the Fourier Transform:(FT).Frequency domain approaches suffers from high computational burden and loss of time information

Reference [4] presents an interesting review of mostly used signal-processing techniques for the estimation of PQ indices and algorithms for classifying power disturbances. The Discrete Wavelet Transform (DWT) has been frequently used for evaluating the quality of nonstationary power signals [4] .

Discrete Wavelet Transform (DWT) has been successfully used for solving PQ problems in the electric power system network [4]-[6].Wavelet packet Transform (WPT) is a generalization of the wavelet transform that can provide a time-frequency representation of any nonstationary waveforms without loosing any time or frequency related information. WPT was proven to be very effective to analyze many power quality disturbances and accurately measure many electrical quantities. The main advantage of using WPT over other wavelet transforms is that it can provide uniform frequency bands therefore identifying more frequency components especially at high frequency in [7].

The literature survey reveals that the analysis of a signal with the discrete wavelet transform (DWT) or the Wavelet Packet Transform (WPT) requires proper selection of mother wavelet, decomposition levels, and sampling frequency. The selection of these parameters along with a suitable choice of a mother wavelet differs for the signals containing different frequency components and this limits the application of DWT and WPT to analyze real-time nonstationary signals.

To overcome these drawbacks, various adaptive techniques have been proposed, such as the S-transform and recursive Newton-type algorithm, to assess the PQ indices for stationary and nonstationary signals. In the literature, parametric high resolution

methods, such as the prony, ESPRIT, and root-MUSIC methods [8] have been proposed to calculate the PQ indices. Recent contributions have extended the concepts to hybrid methods using DFT and parametric methods [9]-[12].

Recently a new approach, empirical wavelet transform (EWT), has been proposed to build a family of adaptive wavelets capable of extracting different components of a signal. This method has an advantage of adaptability according to the analyzed signal and can isolate the different modes of the signal. An attempt has been made in this paper to utilize the self-adaptiveness of the EWT in estimating the PQIs. The accurate frequency estimation and adaptive wavelets makes this technique well suited to analyze the stationary and highly distorted non-stationary signals.

The remainder of this paper is organized as follows: Section II presents a review of EWT technique required to analyze the signal. Then, the EWT-based PQ indices for three-phase system are listed in Section III. In order to investigate the effectiveness of this method, EWT-based three-phase PQ indices are calculated in section IV. Finally, the conclusion is given in Section V.

2. Empirical Wavelet Transform

EWT is a recently proposed method to adaptively detect the different modes of the signal and consequently construct the empirical wavelets to represent the signal by different modes detected. Empirical wavelets means constructing a set of wavelets adapted to the processed signal, i.e. in Fourier domain means constructing a set of band-pass filters [13, 15]. Adaptation here lies in detecting filter supports according to the information located in the processed signal. Modes can be viewed as the principal components (referred to as amplitude modulated and frequency modulated (AM-FM) components) of the signal which represents the signal completely [13, 15]

The adaptableness in this transform is provided by the segmentation of Fourier axis is done in away so as to separate different portions of the spectrum which correspond to modes that are centered around a specific frequency and of compact support. To find such boundaries we find the local maxima's in the Fourier spectrum [13].

Consider a real signal $x(n)$, which is sampled at a frequency of f_s. First apply the FFT to the discrete signal $x(n)$ to find the frequency spectrum $X(\omega)$ and

then obtain the set of maxima $\delta = \{\delta_n\}_{n=1,2,\ldots,N}$ in the Fourier spectrum by means of the magnitude and frequency distance thresholds and infer their corresponding frequency Ω_n. Here N is the number of frequency components assessed using FFT. Now with this set of frequencies $\Omega = \{\Omega_n\}_{n=1,2,\ldots,N}$ corresponding to maxima, the Fourier spectrum $[0, f_s/2]$ is segmented into N segments, where each segment is defined as $\Lambda_n = [\omega_{n-1}, \omega_n]$. Assuming $\omega_0 = 0$ and $\omega_N = f_s/2$, the boundaries ω_n are obtained as given in(1),representing the center of two successive maxima

$$\omega_n = \frac{\Omega_n + \Omega_{n+1}}{2} \ for, 1 \le n \le N-1. \tag{1}$$

Where Ω_n and Ω_{n+1} are the frequencies, and ω_n is their corresponding boundary. Then, the Fourier segments will be $[0, \omega_1], [\omega_1, \omega_2], \ldots, [\omega_{N-1}, f_s/2]$ .After attaining the set of bounds $\omega = \{\omega_n\}_{n=1,2,\ldots,N-1}$, we designate ω_n to be the limits between each segment $(\omega_0 = 0 \ and \ \omega_N = \pi)$ .Each segment is denoted $\Lambda_n = [\omega_{n-1}, \omega_n]$, then it is easy to see that $\bigcup_{n=1}^{N} \Lambda_n = [0, \pi]$ .centered about each ω_n, we define a transition phase T_n of width $2\tau_n$ .A bank of N wavelet filters, consist of one low-pass filter and N-1 band-pass filters are defined based on the well-detected boundaries[13]-[15].

The empirical wavelets are defined as bandpass filters on each Λ_n and based on Λ_n a wavelet tight frame be able to defined, to do so, we use the notion used in the building of both Little wood-Paley and Meyer's wavelets [13], a wavelet tight frame $B = \left\{ \{\phi_n(t)\}_{n=1}^{N-1}, \{\psi_n(t)\}_{n=1}^{N-1} \right\}$ is defined. And $\forall n > 0$, their Fourier transforms, i.e. the empirical scaling function $\overset{\wedge}{\phi}_n(\omega)$ and the empirical wavelets $\overset{\wedge}{\psi}_n(\omega)$ by expressions of (2) and (3), respectively.

$$\overset{\wedge}{\phi}_n(\omega) = \begin{cases} 1; if \ |\omega| \le \omega_n - \tau_n \\ \cos\left[\frac{\pi}{2}\beta\left(\frac{1}{2\tau_n}\left(|\omega| - \omega_n + \tau_n\right)\right)\right] if \ \omega_n - \tau_n \le |\omega| \le \omega_n + \tau_n \\ 0; otherwise \end{cases} \tag{2}$$

and

$$\hat{\psi}_n(\omega) = \begin{cases} 1; if\, \omega_n + \tau_n \leq |\omega| \leq \omega_{n+1} - \tau_{n+1} \\ \cos\left[\dfrac{\pi}{2}\beta\left(\dfrac{1}{2\tau_{n+1}}\left(|\omega| - \omega_{n+1} + \tau_{n+1}\right)\right)\right] if\, \omega_{n+1} - \tau_{n+1} \leq |\omega| \leq \omega_{n+1} + \tau_{n+1} \\ \sin\left[\dfrac{\pi}{2}\beta\left(\dfrac{1}{2\tau_n}\left(|\omega| - \omega_n + \tau_n\right)\right)\right] 0; if\, \omega_n - \tau_n \leq |\omega| \leq \omega_n + \tau_n \\ 0; otherwise \end{cases} \quad (3)$$

The function $\beta(x)$ is an arbitrary function such that

$$\beta(x) = \begin{cases} 0; for,\, x \leq 0 \\ \beta(x) + \beta(1-x) = 1\, \forall x \in [0,1] \\ 1; for,\, x \geq 1 \end{cases} \quad (4)$$

Many functions satisfy these properties, the most used in the literature [11] is

$$\beta(x) = x^4\left(35 - 84x + 70x^2 - 20x^3\right) \quad (5)$$

Regarding the choice of τ_n, several choices are possible. The simplest is to select τ_n proportional to $\omega_n : \tau_n = \gamma\omega_n$ where $0 < \gamma < 1$. To meet the requirement of tight frame [13], the parameter γ must fulfill the following equation:

$$\gamma < \min_n\left(\frac{\omega_{n+1} - \omega_n}{\omega_{n+1} + \omega_n}\right) \quad (6)$$

Therefore, $\forall n > 0$, (2) and (3) simplify to (7) and (8)

$$\hat{\phi}_n(\omega) = \begin{cases} 1; if\, \big|\omega\big| \leq (1-\gamma)\omega_n \\ \cos\left[\dfrac{\pi}{2}\beta\left(\dfrac{1}{2\gamma\omega_n}\left(|\omega| - (1-\gamma)\omega_n\right)\right)\right]; if\, (1-\gamma)\omega_n \leq |\omega| \leq (1+\gamma)\omega_n \\ 0; otherwise \end{cases} \quad (7)$$

and

$$\hat{\psi}_n(\omega) = \begin{cases} 1; if\, (1+\gamma)\omega_n \leq \big|\omega\big| \leq (1-\gamma)\omega_{n+1} \\ \cos\left[\dfrac{\pi}{2}\beta\left(\dfrac{1}{2\gamma\omega_{n+1}}\left(|\omega| - (1-\gamma)\omega_{n+1}\right)\right)\right]; if\, (1-\gamma)\omega_{n+1} \leq |\omega| \leq (1+\gamma)\omega_{n+1} \\ \sin\left[\dfrac{\pi}{2}\beta\left(\dfrac{1}{2\gamma\omega_n}\left(|\omega| - (1-\gamma)\omega_n\right)\right)\right]; if\, (1-\gamma)\omega_n \leq |\omega| \leq (1+\gamma)\omega_n \\ 0; otherwise \end{cases} \quad (8)$$

We can now define the Empirical Wavelet Transform (EWT), $W_f(n,t)$ in the similar way as for the classic wavelet transform. By performing the inner product operation between the applied signal f, and with

empirical wavelets we can obtain the detail coefficients as given in (9) and (10) respectively [13]-[15].

$$W_f(n,t) = \langle f, \psi_n \rangle = \int f(\tau)\psi_n(\tau - t)d\tau = IFFT\left(F(\omega) \times \psi_n(\omega)\right) \quad (9)$$

By performing the inner product operation between the applied signal f, and with empirical scaling functions we can obtain the approximation coefficients as given

$$W_f(0,t) = \langle f, \phi_1 \rangle = \int f(\tau)\phi_1(\tau - t)d\tau = IFFT\left(F(\omega) \times \phi_1(\omega)\right) \quad (10)$$

Where $\phi_1(\omega)$ and $\psi_n(\omega)$ are defined by (7) and (8), respectively. The reconstruction is obtained by

$$f(t) = W_f(0,t) * \phi_1(t) + \sum_{n=1}^{N} W_f(n,t) * \psi_n(t) \quad (11)$$

$$f(t) = IFFT\left(W_f(0,\omega) \times \phi_1(\omega) + \sum_{n=1}^{N} W_f(n,\omega) \times \psi_n(\omega)\right) \quad (12)$$

Resulting this formalism, the empirical mode f_k, as defined

$$f_0(t) = W_f(0,t) * \phi_1(t) \quad (13)$$

$$f_k(t) = W_f(k,t) * \psi_k(t) \quad (14)$$

Where $*$ represents the convolution

3. EWT Based Three Phase Power Quality Indices

This section presents the brief review of PQ indices recommended in [1] and [2] for three-phase system and the EWT based reformulated indices. Consider the three-phase 3- wire system having non sinusoidal periodic phase voltages and line currents containing the fundamental and harmonic components as

$$v_x(t) = \sum_{n=1}^{H_{max}} V_{xn}\sin(2\pi f_{xn}t - \theta_{xn}) \quad (15)$$

and

$$i_x(t) = \sum_{n=1}^{H_{max}} I_{xn}\sin(2\pi f_{xn}t - \varphi_{xn}) \quad (16)$$

Where x corresponds to R, S and T phases, t is the time and H_{max} is the maximum frequency component present in the signal $v_{xn}, f_{xn}, \theta_{xn}$ are the amplitude, frequency and phase angle of the nth component of x - phase voltage signal respectively, $I_{xn}, f_{xn}, \varphi_{xn}$ are the amplitude, frequency and phase angle of the nth component of x -phase current signal respectively. Now, for this set of three phase voltage and currents, EWT is applied and the corresponding mono component signal coefficients of line voltage and line currents are obtained. The approximate coefficients are

obtained by inner product of the input Signal with the scaling function $\Phi_{x,1,k}$ and the detail coefficients are obtained by inner product of the input signal with the empirical wavelets $\psi_{x,j,k}$ as explained in detail in section 2 of this paper.

$$W_{x,1,k} = <v_x(k), \Phi_{x,1,k}>; \ W'_{x,1,k} = <i_x(k), \Phi_{x,1,k}> \quad (17)$$

$$W_{x,j,k} = <v_x(k), \psi_{x,j,k}>, \ W'_{x,j,k} = <i_x(k), \psi_{x,j,k}> \quad (18)$$

Where j is the harmonic level and x corresponds to the phase.

3.1. Effective RMS calculations

The RMS value calculation of the non-sinusoidal voltage using EWT can be extended to three phase systems by considering any phase x of the three phases(R, S or T) as defined below

$$V_{x,rms} = \sqrt{V_{x,1}^2 + V_{x,H}^2} = \sqrt{V_{x,1}^2 + \sum_{j>1} V_{x,j}^2} \quad (19)$$

Where $V_{x,1} = \sqrt{\dfrac{1}{N}\sum_k W_{x,1,k}^2}$ and $V_{x,j} = \sqrt{\dfrac{1}{N}\sum_k W_{x,j,k}^2}$

The RMS value of the non-sinusoidal current is defined as

$$I_{x,rms} = \sqrt{I_{x,1}^2 + I_{x,H}^2} = \sqrt{I_{x,1}^2 + \sum_{j>1} I_{x,j}^2} \quad (20)$$

Where

$$I_{x,1} = \sqrt{\dfrac{1}{N}\sum_k W_{x,1,k}'^2} \text{ and } I_{x,j} = \sqrt{\dfrac{1}{N}\sum_k W_{x,j,k}'^2}$$

Here $V_{x,1}$ and $I_{x,1}$ are the RMS values of the fundamental frequency component of the phase-x voltage and current, respectively. $V_{x,j}$ and $I_{x,j}$ are the sets of RMS values of the phase-x voltage and current of each frequency components higher than one. Also $W_{x,1,k}$ and $W'_{x,1,k}$ are the EWT coefficients of phase-x voltage and current of first component at sample k while $W_{x,j,k}$ and $W'_{x,j,k}$ are the EWT coefficients of phase-x voltage and current of any frequency level j, higher than fundamental at sample k. N corresponds to the total number of samples of the signal.

The effective RMS values for three phase voltages and currents, recommended in [3], can be calculated as

$$V_e = \sqrt{V_{e1}^2 + V_{eH}^2} \ ; \ I_e = \sqrt{I_{e1}^2 + I_{eH}^2} \quad (21)$$

Where the fundamental effective RMS values for three phase voltage and current are

$$V_{e1} = \sqrt{\frac{V_{RS1}^2 + V_{ST1}^2 + V_{TR1}^2}{9}} \ ; I_{e1} = \sqrt{\frac{I_{R1}^2 + I_{S1}^2 + I_{T1}^2}{9}}$$

The Non-fundamental effective RMS values for three phase voltage and current are

$$V_{eH} = \sqrt{\frac{V_{RSH}^2 + V_{STH}^2 + V_{TRH}^2}{9}} \ ; I_{eH} = \sqrt{\frac{I_{RH}^2 + I_{SH}^2 + I_{TH}^2}{3}}$$

Where $V_{RS}, V_{ST} \& V_{TR}$ are the line-to-line voltage for phases labeled R, S, T. The subscript "1" refers to the fundamental component (50Hz) while subscript H refers to the nonfundamental harmonic components. $I_R, I_S \& I_T$ Are the line currents for phases R, S, T

3.2 Equivalent Total Harmonic Distortion (THD)

The EWT based equivalent total harmonic distortion of voltage and current are defined as

$$V_{ethd} = \frac{V_{eH}}{V_{e1}} \text{ and } I_{ethd} = \frac{I_{eH}}{I_{e1}} \quad (22)$$

3.3. Active Power

Fortescue [16] is credited with introducing the concept of symmetrical components which states that any unbalanced three-phase vectors can be decomposed into a set of balanced three-phase vectors. This set consists of positive-sequence components having the same phase sequences (R, S, T), negative -sequence components having the same phase sequences (R, T, S), and zero-sequence components having equal magnitudes with the same phase.

The concept of symmetrical components can be defined in the time-frequency domain by applying time shift instead of phase shift for the phase voltage and current. In order to get the sequence components in the wavelet domain, the time shift is applied to the voltage and currents of phases S and T. First applying time advance $(k+m)$ to the coefficients, where m represent the time corresponding to the angle 120^0 therefore we obtain the advanced versions of the current phases S and T to be i_S', i_T' and shifted versions of the voltages of the same phases to be v_S' and v_T' respectively. Second applying time delay $(k-m)$ to the coefficients,

therefore we obtain the delayed versions of the current phases S and T to be $i_S^{''}, i_T^{''}$ and shifted versions of the voltages of the same phases to be $v_S^{''}$ and $v_T^{''}$ respectively. Using the unshifted phase R and shifted phases S and T the sequence components in the wavelet domain can be obtained as follows:

$$i^0 = \frac{1}{3}\left[i_R + i_S + i_T \right], \qquad v^0 = \frac{1}{3}\left[v_R + v_S + v_T \right]$$

$$i^+ = \frac{1}{3}\left[i_R + i_S^{'} + i_T^{''} \right], \qquad v^+ = \frac{1}{3}\left[v_R + v_S^{'} + v_T^{''} \right]$$

$$i^- = \frac{1}{3}\left[i_R + i_S^{''} + i_T^{'} \right], \qquad v^- = \frac{1}{3}\left[v_R + v_S^{''} + v_T^{'} \right],$$

The fundamental positive sequence active power P_1 defined using the symmetrical components is

$$P_1^+ = 3\left(\frac{1}{T}\int_0^T v^+ i^+ dt \right) \tag{23}$$

Where v^+, i^+ are the instantaneous positive sequence voltage and current in the wavelet domain [8]

The harmonic active power of a phase x is defined as

$$P_{x,H} = \sum_{j>1} P_{x,j}$$

Where $P_{x,j} = \frac{1}{N}\sum_k W_{x,j,k} X W_{x,j,k}^{'}$

The total harmonic active power of three phases is

$$P_H = P_{R,H} + P_{S,H} + P_{T,H} \tag{24}$$

The total active power of three phases is defined as

$$P = P_1^+ + P_H \tag{25}$$

3.4. Apparent Power

The fundamental positive sequence apparent power based on the symmetrical components approach is defined as

$$S_1^+ = 3V_1^+ I_1^+ \tag{26}$$

The fundamental effective apparent power is defined as

$$S_{e1} = 3V_{e1}I_{e1} \tag{27}$$

The fundamental unbalanced power is

$$S_{u1} = \sqrt{S_{e1}^2 - S_1^{+2}} \tag{28}$$

The current distortion power and voltage distortion power can be defined as

$$D_{eI} = 3V_{e1}I_{eH} \text{ and } D_{ev} = 3V_{eH}I_{e1} \tag{29}$$

The harmonic apparent power is given by

$$S_{eH} = 3V_{eH}I_{eH} \tag{30}$$

The non-fundamental effective apparent power is defined as

$$S_{eN} = \sqrt{D_{eI}^2 + D_{ev}^2 + S_{eH}^2} \tag{31}$$

The total effective apparent power is

$$S_e = \sqrt{S_{e1}^2 + S_{eN}^2} \tag{32}$$

The non-active power N is defined as

$$N = \sqrt{S_e^2 - P_e^2} \tag{33}$$

3.5. Reactive Power

The fundamental positive sequence reactive power is defined as

$$Q_1^+ = \sqrt{\left(S_1^+\right)^2 - \left(P_1^+\right)^2} \tag{34}$$

3.6. Power Factor

The fundamental positive sequence power factor is defined as

$$PF_1^+ = \frac{P_1^+}{S_1^+} \tag{35}$$

The total three phase power factor is defined as

$$PF = \frac{P}{S_e} \tag{36}$$

3.7. Harmonic Pollution

The Harmonic pollution is defined as the ratio of the non fundamental effective apparent power to the fundamental effective apparent power

$$HP = \frac{S_{eN}}{S_{e1}} \tag{37}$$

3.8. Load unbalance

The load unbalance is the ratio of fundamental apparent power to the fundamental positive sequence power that can measure the system unbalance. It is given as

$$LU = \frac{S_{u1}}{S_1^+} \tag{38}$$

4. NUMERICAL EXAMPLES

This section contains Four numerical examples that are solved using the definitions contained in the IEEE Standard and the definitions based on the EWT.First,

the true values are computed as per the IEEE standard definitions and then the EWT is applied to decompose the signal into mono frequency components. By applying EWT-based reformulated indices, on to these mono components we can accurately estimate the three phase PQIs.The percentage difference has been calculated for all the indices using the mathematical expressions shown below by

$$\% \, difference = \frac{\left| Indices_{true} - Indices_{calculated} \right|}{Indices_{true}} * 100 \qquad (39)$$

4.1. Balanced Three-Phase System

The first numerical example considers balanced three-phase load supplied from nonsinusoidal balanced three-phase source as shown in Fig.1.The time domain equations for the three-phase source are [10]

$$v_R\left(t\right) = 100 \sin\left(\omega_1 t\right) + 20 \sin(\omega_3 t) + 5 \sin(\omega_5 t) \qquad (40)$$

$$v_S\left(t\right) = 100 \sin\left(\omega_1 t - 120^0\right) + 20 \sin(\omega_3 t - 120^0) + 5 \sin(\omega_5 t - 120^0) \qquad (41)$$

$$v_T\left(t\right) = 100 \sin\left(\omega_1 t + 120^0\right) + 20 \sin(\omega_3 t + 120^0) + 5 \sin(\omega_5 t + 120^0) \qquad (42)$$

Here ω_1, ω_3 and ω_5 are the angular frequency at the fundamental $\left(f_1 = 60Hz\right)$, third harmonic $\left(f_3 = 180Hz\right)$ and fifth harmonic $\left(f_5 = 300Hz\right)$, respectively.
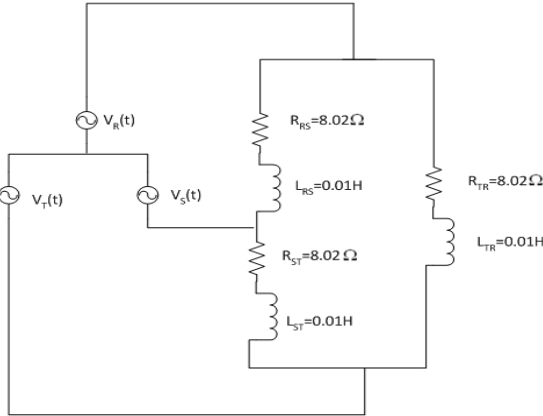

Fig.1.Balanced three-phase load supplied from nonsinusoidal balanced three phase source.

Balanced three-phase line voltages $v_R\left(t\right), v_S\left(t\right), v_T\left(t\right)$, is applied to balanced three-phase load as shown as in Fig.1.From which we can calculate line currents $i_R\left(t\right), i_S\left(t\right), i_T\left(t\right)$, and the calculated line currents are shown in equation (43) to (45).

$$i_R\left(t\right) = 33.85 \sin\left(\omega_1 t - 25.176^0\right) + 4.3256 \sin(\omega_3 t - 54.6586^0) + 0.7312 \sin(\omega_5 t - 66.951^0) \qquad (43)$$

$$i_S\left(t\right) = 33.85 \sin\left(\omega_1 t - 145.176^0\right) + 4.3256 \sin(\omega_3 t - 174.658^0) + 0.7312 \sin(\omega_5 t - 186.951^0) \qquad (44)$$

$$i_T\left(t\right) = 33.85 \sin\left(\omega_1 t + 94.824^0\right) + 4.3256 \sin(\omega_3 t + 65.342^0) + 0.7312 \sin(\omega_5 t + 53.049^0) \qquad (45)$$

For this set of three-phase line voltages and line currents after being sampled with a sampling frequency $\left(f_s = 7680Hz\right)$, EWT is applied and the corresponding mono-component signal coefficients of line voltage and line currents are obtained. It can be clearly observed from the filtered signals shown in Fig.2. to Fig.7.that EWT technique is able to accurately estimate the frequency components, which are present in the signal and provide its mono-components, It can also be noticed that no two frequency components are combined in a single mode. In all these Figures mode 2 corresponds to fundamental component and mode3 and mode4 are corresponding to harmonics of the signals. By using EWT-based reformulated indices, on to these fundamental components and harmonics we can accurately estimate the three phase PQIs. Table1 compares the PQIs obtained from the IEEE standard definitions with those obtained from the EWT.As the Results in Table 1 show, the percentage differences are too small in the case of effective RMS and equivalent total harmonic distortion. We also observe that, the percentage differences for active power, apparent power, distortion power, and reactive power are considerably small; however for power factor, pollution factor and load unbalance indicates zero percentage differences which proves the accuracy of the EWT-based definitions.
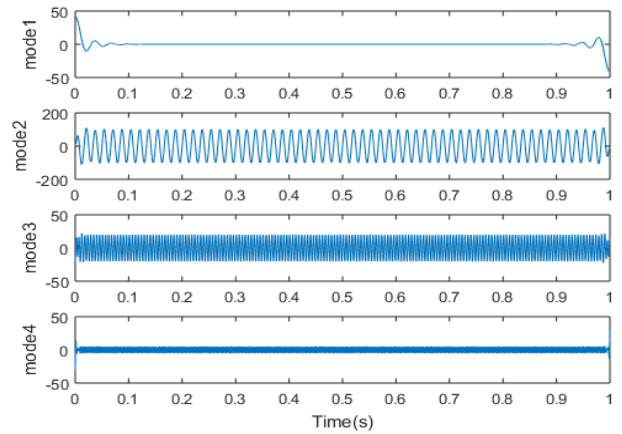

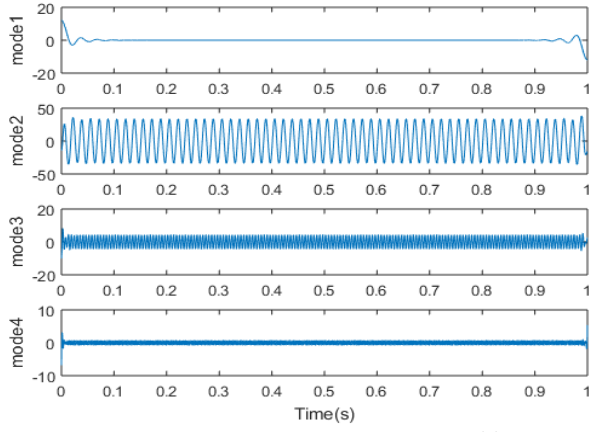Fig.2 Modes extracted by the EWT for $v_R\left(t\right)$

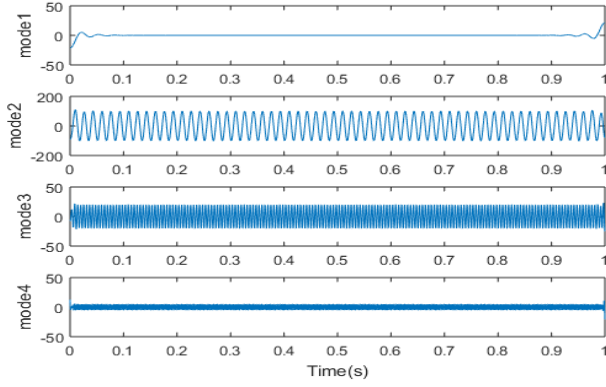Fig.3.Modes extracted by the EWT for $i_R(t)$

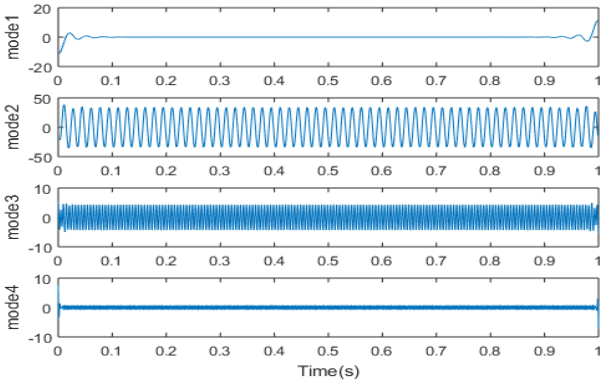Fig.4.Modes extracted by the EWT for $v_S(t)$

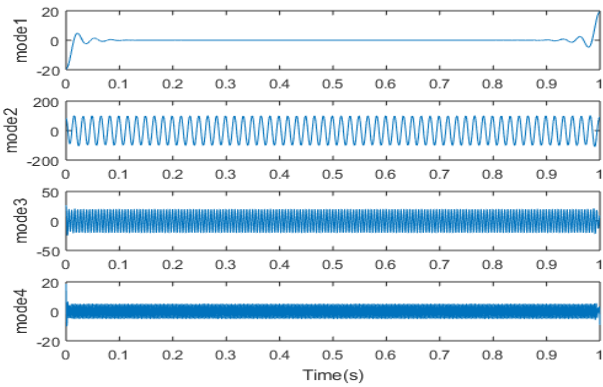Fig.5 Modes extracted by the EWT for $i_S(t)$
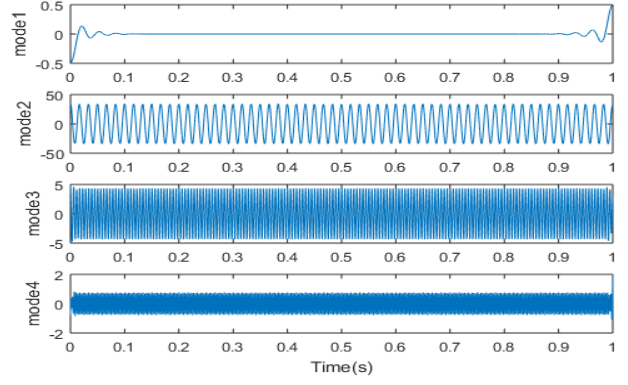

Fig.6.Modes extracted by the EWT $v_T(t)$

Fig.7.Modes extracted by the EWT for $i_T(t)$

Table 1 PQIs for Balanced Three-Phase Signals

Indices	IEEE standard Definitions	EWT based Indices	%diffe rence
Effective Voltage RMS	$V_{e1} = 70.700$	$V_{e1} = 70.6245$	0.106
	$V_{eH} = 14.560$	$V_{eH} = 14.5726$	0.086
	$V_{erms} = 72.180$	$V_{erms} = 72.1123$	0.093
Effective Current RMS	$I_{e1} = 23.900$	$I_{e1} = 23.900$	0.000
	$I_{eH} = 3.100$	$I_{eH} = 3.111$	0.354
	$I_{erms} = 24.100$	$I_{erms} = 24.108$	0.033
THD	$V_{ethd} = 0.206$	$V_{ethd} = 0.206$	0.000
	$I_{ethd} = 0.130$	$I_{ethd} = 0.130$	0.000
Distortion Index	$DIN_{ev} = 0.201$	$DIN_{ev} = 0.202$	0.497
	$DIN_{eI} = 0.1283$	$DIN_{eI} = 0.129$	0.545
Active Power	$P_1^+ = 4586$	$P_1^+ = 4591.35$	0.116
	$P_H = 88.00$	$P_H = 85.507$	2.832
	$P = 4674$	$P = 4674.85$	0.018
Apparent Power	$S_1^+ = 5069.19$	$S_1^+ = 5065.169$	0.079
	$S_{e1} = 5069.19$	$S_{e1} = 5065.169$	0.079
	$S_{U1} = 0.0000$	$S_{U1} = 0.0000$	0.000
Total Apparent Power	$S_e = 5218.6$	$S_e = 5215.48$	0.059
Reactive Power	$Q_1^+ = 2159.6$	$Q_1^+ = 2159.86$	0.012
Distortion Power	$D_{eI} = 657.51$	$D_{eI} = 659.145$	0.248
	$D_{eV} = 1043.95$	$D_{eV} = 1045.145$	0.114
	$S_{eH} = 135.40$	$S_{eH} = 136.07$	0.494
	$S_{eN} = 1241.16$	$S_{eN} = 1243.101$	0.156
	$N = 2321.1$	$N = 2318.385$	0.116
Power Factor	$PF_1^+ = 0.904$	$PF_1^+ = 0.904$	0.000
	$PF = 0.896$	$PF = 0.896$	0.000
Harmonic Pollution	$HP = 0.245$	$HP = 0.245$	0.000
Load Unbalance	$LU = 0.000$	$LU = 0.000$	0.000

4.2. Unbalanced Three-Phase System

The second numerical example [10] considers an unbalanced three-phase load supplied from the same nonsinusoidal balanced three-phase source used in the previous example shown in Fig.8.
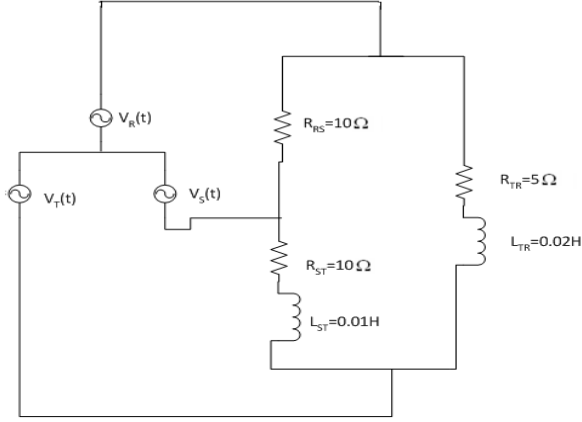

Fig.8.Unbalanced three-phase load supplied from nonsinusoidal balanced three-phase source.

$$v_R\left(t\right) = 100\sin\left(\omega_1 t\right) + 20\sin(\omega_3 t) + 5\sin(\omega_5 t) \qquad (46)$$

$$v_S\left(t\right) = 100\sin\left(\omega_1 t - 120^0\right) + 20\sin(\omega_3 t - 120^0) + 5\sin(\omega_5 t - 120^0) \qquad (47)$$

$$v_T\left(t\right) = 100\sin\left(\omega_1 t + 120^0\right) + 20\sin(\omega_3 t + 120^0) + 5\sin(\omega_5 t + 120^0) \qquad (48)$$

Here, ω_1, ω_3 and ω_5 are the angular frequency at the fundamental $\left(f_1 = 60Hz\right)$, third harmonic $\left(f_3 = 180Hz\right)$ and fifth harmonic $\left(f_5 = 300Hz\right)$, respectively. Balanced three-phase line voltages $v_R\left(t\right), v_S\left(t\right), v_T\left(t\right)$, is applied to unbalanced three-phase load as shown as in Fig.8.From which we can calculate line currents $i_R\left(t\right), i_S\left(t\right), i_T\left(t\right)$, and the calculated line currents are shown in equation (49) to (51).

$$i_R\left(t\right) = 19.26\sin\left(\omega_1 t - 32.8^0\right) + 2.5674\sin(\omega_3 t + 6.9^0) + 0.6991\text{in}(\omega_5 t + 18.6^0) \qquad (49)$$

$$i_S\left(t\right) = 31.57\sin\left(\omega_1 t + 229^0\right) + 5.73\sin(\omega_3 t + 214^0) + 1.2723\sin(\omega_5 t + 209.3^0) \qquad (50)$$

$$i_T\left(t\right) = 34.57\sin\left(\omega_1 t + 82.5^0\right) + 3.6575\sin(\omega_3 t + 53.6^0) + 0.5981\sin(\omega_5 t + 42^0) \qquad (51)$$

For this set of three-phase line voltages and line currents after being sampled with a sampling frequency $\left(f_s = 7680Hz\right)$, EWT is applied and the corresponding monocomponent signal coefficients of line voltage and line currents are obtained. It can be clearly observed from the filtered signals shown in

Fig.9.to Fig.16.that EWT technique is able to accurately estimate the frequency components, which are present in the signal and provide its monocomponents, it can also be noticed that no two frequency components are combined in a single mode. In all these Figures mode 2 corresponds to fundamental component and mode3 and mode4 are corresponding to harmonics of the signals. By using EWT-based reformulated indices, on to these fundamental components and harmonics we can accurately estimate the three phase PQIs.Table 2 compares the PQIs obtained from the IEEE standard definitions with those obtained from the EWT.As the Results in Table 2 show, the percentage differences are too small in the case of effective RMS and equivalent total harmonic distortion. We also observe that from the results shown in Table 2, the percentage differences for active power, apparent power, distortion power, and reactive power are considerably small; the fundamental unbalanced power S_{U1} has a nonzero value, the percentage difference for power factor is zero.
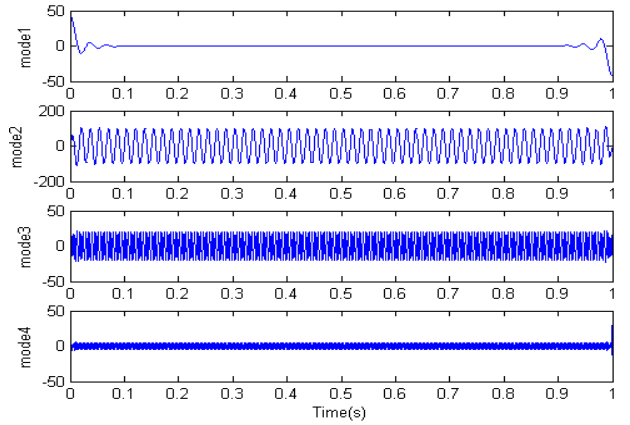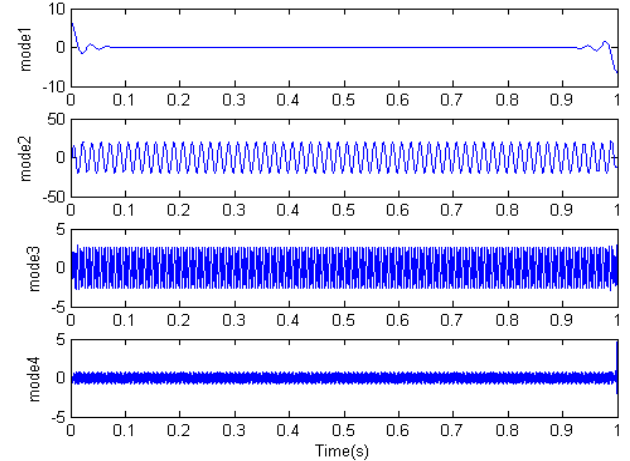

Fig.9. Modes extracted by the EWT for $v_R\left(t\right)$

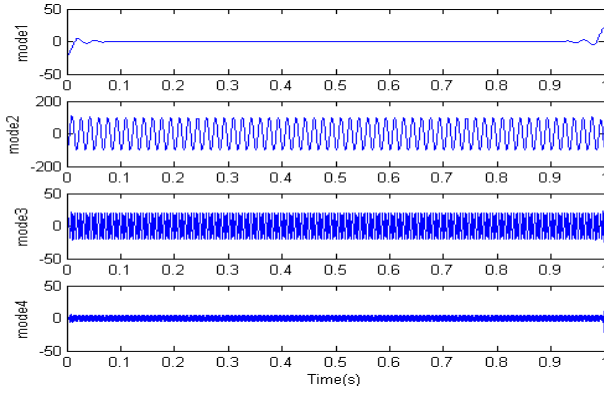Fig.10. Modes extracted by the EWT for $i_R\left(t\right)$

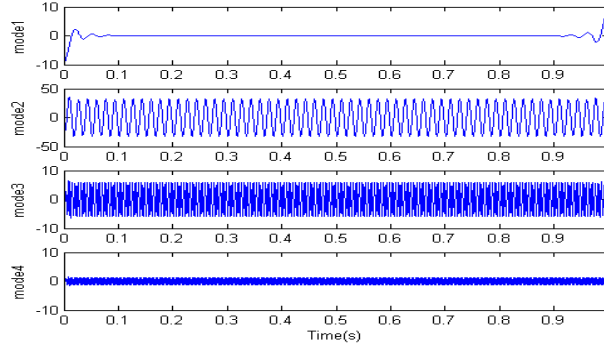Fig.11. Modes extracted by the EWT for $v_S(t)$

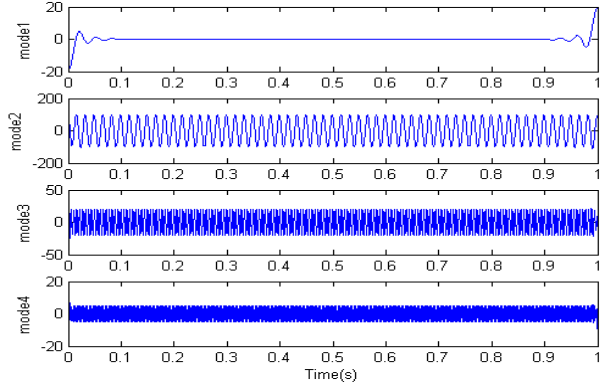Fig.12. Modes extracted by the EWT for $i_S(t)$
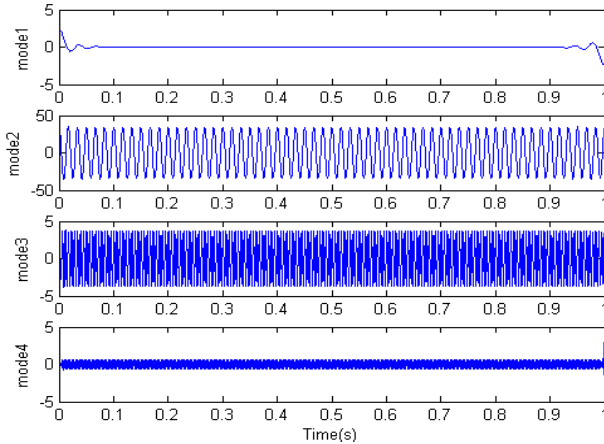

Fig.13. Modes extracted by the EWT for $v_T(t)$

Fig.14. Modes extracted by the EWT for $i_T(t)$

Table 2 PQIs for Unbalanced Three-Phase Signals

Indices	IEEE standard Definitions	EWT based Indices	%diffe rence
Effective Voltage RMS	$V_{e1} = 70.700$	$V_{e1} = 70.624$	0.106
	$V_{eH} = 14.560$	$V_{eH} = 14.572$	0.086
	$V_{erms} = 72.180$	$V_{erms} = 72.112$	0.093
Effective Current RMS	$I_{e1} = 20.670$	$I_{e1} = 20.652$	0.087
	$I_{eH} = 3.028$	$I_{eH} = 3.032$	0.132
	$I_{erms} = 20.890$	$I_{erms} = 20.873$	0.081
THD	$V_{ethd} = 0.206$	$V_{ethd} = 0.206$	0.00
	$I_{ethd} = 0.146$	$I_{ethd} = 0.146$	0.00
Distortion Index	$DIN_{ev} = 0.2035$	$DIN_{ev} = 0.202$	0.737
	$DIN_{eI} = 0.1457$	$DIN_{eI} = 0.145$	0.480
Active Power	$P_1^+ = 3713$	$P_1^+ = 3717.34$	0.116
	$P_H = 113$	$P_H = 108.31$	4.15
	$P = 3826$	$P = 3825.65$	0.009
Apparent Power	$S_1^+ = 4157$	$S_1^+ = 4162$	0.120
	$S_{e1} = 4384.1$	$S_{e1} = 4385.69$	0.036
	$S_{U1} = 1393$	$S_{U1} = 1387.74$	0.377
Total Apparent Power	$S_e = 4523.5$	$S_e = 4521.79$	0.037
Reactive Power	$Q_1^+ = 1869.3$	$Q_1^+ = 1873.95$	0.248
Distortion Power	$D_{eI} = 642.238$	$D_{eI} = 642.548$	0.048
	$D_{eV} = 902.865$	$D_{eV} = 902.879$	0.001
	$S_{eH} = 132.263$	$S_{eH} = 132.583$	0.241
	$S_{eN} = 1115.854$	$S_{eN} = 1116.08$	0.020
	$N = 2413$	$N = 2418$	0.207
Power Factor	$PF_1^+ = 0.893$	$PF_1^+ = 0.893$	0.000
	$PF = 0.846$	$PF = 0.846$	0.000
Harmonic Pollution	$HP = 0.254$	$HP = 0.255$	0.393
Load Unbalance	$LU = 0.335$	$LU = 0.333$	1.782

4.3. Balanced Distorted Signals:

This example considers balanced three-phase voltage and balanced current signals containing four frequency components as shown in Table 3 and 4, respectively.

Table 3 Parameters of the Three-Phase Voltage signal

Time(S)	Phase	Amplitude, Frequency and Phase angle			
0-1	R	100,50, 0^0	30,150, 0^0	8,250, 0^0	5,350, 0^0
	Y	100,50, 120^0	30,150, 120^0	8,250, 120^0	5,350, 120^0
	B	100,50, 240^0	30,150, 240^0	8,250, 240^0	5,350, 240^0

Table 4 Parameters of the Three-Phase Current signal

Time (S)	Pha se	Amplitude, Frequency and Phase angle			
0-1	R	28.62,50, 17.44^0	6.54,150 43.3^0	1.28,250 57.5^0	0.619,350 65.62^0
	Y	28.62,50, 137.44^0	6.54,150 163.3^0	1.28,250 177.5^0	0.619,350 185.6^0
	B	28.62,50, 102.56^0	6.54,150 76.69^0	1.28,250 62.48^0	0.619,350 54.38^0

Table 5 PQIs for Balanced Three-Phase Signals

Indices	IEEE standard Definitions	EWT based Indices	%differ ence
Effective Voltage RMS	$V_{e1} = 70.981$	$V_{e1} = 70.607$	0.426
	$V_{eH} = 22.378$	$V_{eH} = 22.16$	0.974
	$V_{erms} = 74.12$	$V_{erms} = 74$	0.161
Effective Current RMS	$I_{e1} = 20.302$	$I_{e1} = 20.207$	0.467
	$I_{eH} = 4.734$	$I_{eH} = 4.732$	0.042
	$I_{erms} = 20.87$	$I_{erms} = 20.75$	0.574
THD	$V_{ethd} = 0.318$	$V_{ethd} = 0.313$	1.572
	$I_{ethd} = 0.2345$	$I_{ethd} = 0.234$	0.213
Distortion Index	$DIN_{ev} = 0.30$	$DIN_{ev} = 0.299$	0.333
	$DIN_{eI} = 0.229$	$DIN_{eI} = 0.228$	0.436
Active Power	$P_1^+ = 4096.78$	$P_1^+ = 4097.34$	0.013
	$P_H = 223.31$	$P_H = 221.82$	0.667
	$P = 4320.09$	$P = 4319.17$	0.021
Apparent Power	$S_1^+ = 4297.78$	$S_1^+ = 4299.14$	0.031
	$S_{e1} = 4296.99$	$S_{e1} = 4298.41$	0.033
	$S_{U1} = 0.000$	$S_{U1} = 0.000$	0.000
Total Apparent Power	$S_e = 4609.43$	$S_e = 4607.65$	0.038
Reactive Power	$Q_1^+ = 1301.3$	$Q_1^+ = 1301.70$	0.030
Distortion Power	$D_{eI} = 1002.46$	$D_{eI} = 1002.38$	0.007
	$D_{eV} = 1345.43$	$D_{eV} = 1343.42$	0.149
	$S_{eH} = 314.92$	$S_{eH} = 314.60$	0.101
	$S_{eN} = 1707.8$	$S_{eN} = 1705.43$	0.138
	$N = 1608.7$	$N = 1604.75$	0.245
Power Factor	$PF_1^+ = 0.9538$	$PF_1^+ = 0.953$	0.083
	$PF = 0.938$	$PF = 0.937$	0.106
Harmonic Pollution	$HP = 0.3987$	$HP = 0.398$	0.175
Load Unbalance	$LU = 0.000$	$LU = 0.000$	0.000

The unbalanced three-phase currents are obtained by applying balanced three-phase voltages to unbalanced

three-phase load. Table 5 shows the PQIs for a balanced three-phase system calculated from the IEEE standard definitions and that estimated using the EWT. It can be seen that the deviation in the EWT-based PQIs is small.

4.4. Un balanced Distorted Signals:

This example considers balanced three-phase voltage and unbalanced current signals containing four frequency components as shown in Table 6 and 7, respectively[15].The unbalanced three-phase currents are obtained by applying balanced three-phase voltages to unbalanced three-phase load. Table 8shows the PQI s for an unbalanced three-phase system calculated from the IEEE standard definitions and that estimated using the EWT.It can be seen that the deviation in the EWT-based PQIs is small.

Table 6 Parameters of the Three-Phase Voltage signal

Time (S)	Pha se	Amplitude, Frequency and Phase angle			
0-1	R	100,50, 0^0	30,150, 0^0	8,250, 0^0	5,350, 0^0
	Y	100,50, 120^0	30,150, 120^0	8,250, 120^0	5,350, 120^0
	B	100,50, 240^0	30,150, 240^0	8,250, 240^0	5,350, 240^0

Table 7 Parameters of the Three-Phase Current signal

Time (S)	Ph ase	Amplitude, Frequency and Phase angle			
0-1	R	24.52,50 24.3^0	5.42,150 44.6^0	1.113,250 55.2^0	0.56,350 62.2^0
	Y	21.85,50 161.6^0	4.15,150 178.1^0	0.865,250, 190.5^0	0.43,350 199.9^0
	B	17.07,50 264.6^0	3.95,150, 274.2^0	0.785,250, 285.9^0	0.38,350 293.1^0

5. Conclusions

In this paper, the EWT is successfully applied to formulating the power components definitions contained in the IEEE standard 1459-2000 for balanced and unbalanced three-phase systems with non sinusoidal situations. This technique first estimates the frequency components and then adaptively tunes the wavelet and scaling function based on the boundaries to decompose the signal accurately. Four numerical examples considering balanced and unbalanced three-phase systems with non sinusoidal situations are solved using the IEEE Standard definitions and the EWT-

based definitions. It can be observed from the results that the EWT-based PQ indices for the balanced and unbalanced three-phase supply are very close to the true values. Based on the results, the EWT proved to be adaptive and also accurate in estimation of PQIs,hence this technique is very useful for its application in real time power quality monitoring and can extract relevant characterstics, which can be used as inputs to classify the power Quality disturbances.

Table 8 PQIs for Unbalanced Three-Phase Signals

Indices	IEEE standard Definitions	EWT based Indices	%difference
Effective Voltage RMS	$V_{e1} = 70.7107$	$V_{e1} = 70.605$	0.149
	$V_{eH} = 22.2374$	$V_{eH} = 22.208$	0.132
	$V_{erms} = 74.1249$	$V_{erms} = 74.022$	0.138
Effective Current RMS	$I_{e1} = 15.1107$	$I_{e1} = 15.099$	0.077
	$I_{eH} = 3.3087$	$I_{eH} = 3.308$	0.021
	$I_{erms} = 15.4687$	$I_{erms} = 15.458$	0.069
THD	$V_{ethd} = 0.3145$	$V_{ethd} = 0.3144$	0.031
	$I_{ethd} = 0.2190$	$I_{ethd} = 0.218$	0.456
Distortion Index	$DIN_{ev} = 0.300$	$DIN_{ev} = 0.30$	0.00
	$DIN_{eI} = 0.2139$	$DIN_{eI} = 0.213$	0.420
Active Power	$P_1^+ = 2713.15$	$P_1^+ = 2715.7$	0.093
	$P_H = 146.584$	$P_H = 146.97$	0.263
	$P = 2859.734$	$P = 2862.68$	0.103
Apparent Power	$S_1^+ = 3145.51$	$S_1^+ = 3147.9$	0.075
	$S_{e1} = 3205.464$	$S_{e1} = 3205.98$	0.016
	$S_{U1} = 617.113$	$S_{U1} = 608.8$	1.347
Total Apparent Power	$S_e = 3439.847$	$S_e = 3439.91$	0.001
Reactive Power	$Q_1^+ = 1591.55$	$Q_1^+ = 1591.93$	0.023
Distortion Power	$D_{eI} = 701.8815$	$D_{eI} = 701.52$	0.051
	$D_{eV} = 1008.068$	$D_{eV} = 1005.98$	0.207
	$S_{eH} = 220.7307$	$S_{eH} = 220.01$	0.326
	$S_{eN} = 1248.02$	$S_{eN} = 1247.80$	0.017
	$N = 1911.667$	$N = 1908.96$	0.141
Power Factor	$PF_1^+ = 0.8657$	$PF_1^+ = 0.8625$	0.369
	$PF = 0.8314$	$PF = 0.834$	0.312
Harmonic Pollution	$HP = 0.3893$	$HP = 0.389$	0.077
Load Unbalanc	$LU = 0.1962$	$LU = 0.1947$	0.764

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