Sliding Mode Control Strategy for a 6 DOF Quadrotor Helicopter

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***Abstract-* In this work, we gives the full dynamical model of a commercially available quadrotor helicopter and presents its behaviour control at low altitudes through sliding mode control. The control law is very well known for its robustness against disturbances and invariance during the sliding regime. The plant on the other hand, is nonlinear one with state variables are tightly coupled the control objective is the position tracking. Simulations results have shown that the algorithm successfully drives the system towards the desired trajectory.**

Keywords— Sliding mode Control, quadrotor, Dynamic modelling

1- INTRODUCTION

Autonomous Unmanned Air vehicles (UAV) are increasingly popular platforms, due to their use in military applications, trafc surveillance, environment exploration, structure inspection, mapping and aerial cinematography, in which risks to pilots are often high. Rotorcraft has an evident advantage over fixed-wing aircraft for various applications because of their vertical landing/take-off capability and payload. Among the rotorcraft, quadrotor helicopters can usually afford a larger payload than conventional helicopters due to four rotors. Moreover, small quadrotor helicopters possess a great maneuverability and are potentially simpler to manufacture. For these advantages, quadrotor helicopters have received much interest in UAV research [1,2].

The quadrotor we consider is an underactuated system with six outputs and four inputs, and the states are highly coupled, several recent works were completed for the design and control in pilotless aerial vehicles domain such that quadrotor [1,3,4]. Also, related models for controlling the vertical take-off and landing (VTOL) aircraft are studied by Hauser et al. [5]. A model for the dynamic and configuration stabilization of quasi-stationary flight conditions of a four rotors VTOL, based on Newton formalism, was studied by Hamel et al. [6] where the dynamic motor effects are incorporated and a bound of perturbing errors was obtained for the coupled system. Castillo et al. [7] performed autonomous take-off, hovering and landing control of a four rotors by synthesizing a controller using the Lagrangian model based on the Lyapunov analysis. In [8], authors take into account the gyroscopic effects and show that the classical model independent PD controller can stabilize asymptotically the attitude of the quadrotor aircraft. Moreover, they used a new Lyapunov function, which leads to an exponentially stabilizing controller based upon the PD^2 and the compensation of coriolis and gyroscopic torques. While in [9] the authors develop a PID controller in order to stabilize altitude. In [10], a PID controller and a LQ controller were proposed to stabilize the attitude. The PID controller showed the ability to control the attitude in the presence of minor perturbation and the LQ controller provided average results, due to model imperfections Madani et al. studied a full-state backstepping technique based on the Lyapunov stability theory and backstepping control [11], Yet another backstepping control method was proposed by P. Castillo et al. They used this controller with a saturation function and it performed well under perturbation [12]. In [13] a mixed robust feedback linearization with linear GH infinie controller is applied to a nonlinear quadrotor unmanned aerial vehicle, Robust adaptive-fuzzy control was applied in [14]. This controller showed a good performance against sinusoidal wind disturbance. In [15] presente the comparison between a based model method and a fuzzy inference system, In [16] the quadrotor has been controlled in 3 DOF using the combination of the backstepping technique and a nonlinear robust PI controller, in [17] The control strategy includes feedback linearization coupled with a PD controller for the translational subsystem and a backstepping-based PID
nonlinear controller for the rotational subsystem of the quadrotor, in References [18, 19] used a feedback linearization approach to stabilise the Four Rotors Helicopter.

The sliding mode control has been applied extensively to control quadrotors. The advantage of this approach is its insensitivity to the model errors, parametric uncertainties, ability to globally stabilize the system and other disturbances [20]. In [21] author use the Sliding mode control of a class of underactuated systems and he took the quadrotor as a sample application, In [22] the authors presents a continuous sliding mode control method based on feedback linearization applied to a Quadrotor UAV, In [23] presents a new controller based on backstepping and sliding mode techniques for miniature quadrotor helicopter, In [24] This paper presents two types of nonlinear controllers for an autonomous quadrotor helicopter. One type, a feedback linearization controller involves high-order derivative terms and turns out to be quite sensitive to sensor noise as well as modelling uncertainty. The second type involves a new approach to an adaptive sliding mode controller using input augmentation in order to account for the underactuated property of the helicopter.

Then, we present a control technique based on the development and the synthesis of a control algorithm based upon sliding mode approach ensuring the locally asymptotic stability and desired tracking trajectories expressed in term of the center of mass coordinates along *(X, Y, Z)* axis and yaw angle, while the desired roll and pitch angles are deduced . Finally all synthesized control laws are highlighted by simulations which gave results considered to be satisfactory.

2 - QUADROTORS DYNAMICS MODELING

A sketch of the quadrotor rotorcraft system studied in this study is shown in Fig. 1, where the Euler angles and the cartesian coordinate frame are shown. The equations of motion are given in (1) and the values of some variables seen are tabulated in Table. 1

Let $E(O, X, Y, Z)$ denote an inertial frame, and $B(O', x, y, z)$ denote a frame rigidly attached to the quadrotor

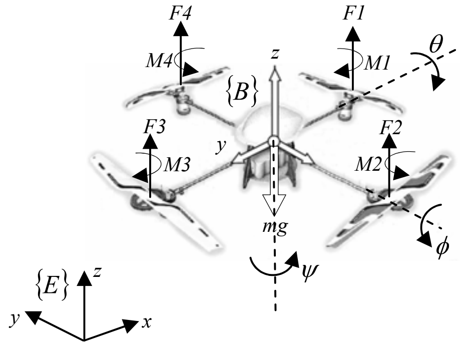

Fig. 1 General view of the quadrotor

We will make the following assumptions:

- The quadrotor structure is rigid and symmetrical.
- The center of mass and *o'* coincides.
- The propellers are rigid.
- Thrust and drag are proportional to the square of the
- propellers speed.

$$\begin{cases} m\,\ddot{x} = -u\sin\theta \\ m\,\ddot{y} = u\cos\theta\sin\phi \\ m\,\ddot{z} = u\cos\theta\cos\phi - m\,g \\ \ddot{\psi} = \tau_\psi \\ \ddot{\theta} = \tau_\theta \\ \ddot{\phi} = \tau_\phi \end{cases} \tag{1}$$

$$\tau = \begin{pmatrix} \tau_\psi \\ \tau_\theta \\ \tau_\phi \end{pmatrix} = J^{-1}\left(\tau - C(\eta,\dot{\eta})\dot{\eta}\right) \tag{2}$$

Here $\eta = (\psi\ \theta\ \phi)^T$, $J(\eta) = T_\eta\ I\ T_\eta$ and

$$T_\eta = \begin{pmatrix} -\sin\theta & 0 & 1 \\ \cos\theta\sin\phi & \cos\phi & 0 \\ \cos\theta\cos\phi & -\sin\phi & 0 \end{pmatrix} \tag{3}$$

$$I = \begin{pmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & Izz \end{pmatrix} = \begin{pmatrix} Ixx & 0 & 0 \\ 0 & Iyy & 0 \\ 0 & 0 & 2Ixx \end{pmatrix} \tag{4}$$

The Coriolis and centripetal vector denoted by $C(\eta,\dot{\eta})$ is defined as below and computed as given by (8).

$$C(\eta,\dot{\eta}) = \dot{J} - \frac{1}{2}\frac{\partial}{\partial\eta}\left(\dot{\eta}^T J\right) \tag{5}$$

$$J = I_{xx}\begin{pmatrix} 1+(\cos\theta)^2(\cos\phi)^2 & -\cos\theta\sin\phi\cos\phi & -\sin\theta \\ -\cos\theta\sin\phi\cos\phi & 2-(\cos\phi)^2 & 0 \\ -\sin\theta & 0 & 1 \end{pmatrix} \tag{6}$$

$$\dot{J} = I_{xx}\begin{pmatrix} \dot{\theta}\,s_{2\theta}\,c_\phi^2 + \dot{\phi}\,s_{2\phi}\,c_\theta^2 & \dot{\theta}\,s_\theta\,s_\phi c_\phi + \dot{\phi}\,c_{2\phi}\,c_\theta & \dot{\theta}\,c_\theta \\ \dot{\theta}\,s_\theta\,s_\phi c_\phi + \dot{\phi}\,c_{2\phi}\,c_\theta & \dot{\phi}\,s_{2\phi} & 0 \\ \dot{\theta}\,c_\theta & 0 & 0 \end{pmatrix} \tag{7}$$

$$\begin{aligned} & C_{1,1} = C_{1,2} = C_{1,3} = 0 \\ & C_{2,1} = I_{xx}\left(\dot{\psi}\,c_\phi^2 s_{2\theta} + \dot{\theta}\,s\phi\,c\phi\,s_\theta - \dot{\phi}\,c\theta\right) \\ & C_{2,2} = I_{xx}\dot{\psi}\,s_\phi c_\phi s\theta \\ & C_{2,3} = -I_{xx}\dot{\psi}\,c\theta \\ & C_{3,1} = -I_{xx}\left(\dot{\psi}\,c_\theta^2 s_{2\phi} + \dot{\phi}\,c_\theta\,c_{2\phi}\right) \\ & C_{3,2} = -I_{xx}\left(\dot{\psi}\,c_\theta c_{2\phi} + \dot{\phi}\,c_{2\phi}\right) \\ & C_{3,3} = 0 \end{aligned} \tag{8}$$

where $I_{xx} = I_{yy} = ml^2, I_{zz} = 2ml^2$. Model inputs and the aerodynamic forces (f_i) created by each propeller are related to each other as described below and *l* is the distance from the motors to the centre of gravity and τ_{M_i} is the couple produced by each motor M_i .

$$\tau_\psi = \sum_{i=1}^{4}\tau_{M_i} \tag{9}$$

$$\tau_\theta = (F_3 - F_1)\,l \tag{10}$$

$$\tau_\phi = (F_2 - F_4)\,l \tag{11}$$

$$u = \sum_{i=1}^{4}F_i \tag{12}$$

m_i	Motor weight	0.08 *kg*
m_b	Battery weight	0.20 *kg*
m	Total weight of the quadrotor	0.52 *kg*
l	Distance from motors to the centre of gravity	0.205 *kg*
g	Gravitational acceleration	9.81 m/s^2

Table. 1 Physical parameters of the quadrotor

3- ROTOR DYNAMICS

The rotor is a unit constituted by D.C-motor actuating a propeller via a reducer. The D.C-motor is governed by the following dynamic equations:

$$\begin{cases} V = ri + L\dfrac{di}{dt} + k_e\omega \\ k_m = J_r + C_s + k_r\omega^2 \end{cases} \tag{13}$$

The different parameters of the motor are defined such:

V : Motor input
k_e, k_m : Electrical and mechanical torque constant respectively
k_r : Load constant torque
r : Motor internal resistance
J_r : Rotor inertia
C_s : Solid friction

Then the model chosen for the rotor is as follows

$$\dot{\omega}_i = bV_i - \beta_0 - \beta_1\omega_i - \beta_2\omega_i^2 \qquad i \in [1,4] \tag{14}$$

With:

$$\beta_0 = \frac{C_s}{J_r}, \beta_1 = \frac{k_e k_m}{rJ_r}, \beta_2 = \frac{k_r}{J_r} \text{ and } b = \frac{k_m}{rJ_r}$$

4- CONTROL STRATEGY

To achieve a robust path following for the quadrotor helicopter, two techniques, capable of controlling the helicopter in presence of sustained external disturbances, parametric uncertainties and unmodelled dynamics, are combined. The proposed control strategy is based on the decentralized structure of the quadrotor helicopter system, which is composed of the dynamic Equation (1). The overall scheme of the control strategy is depicted in Fig. 2.

The translational motion control is performed in two stages. In the first one, the helicopter height, z, is controlled and the total thrust, u, is the manipulated signal. In the second stage, the reference of pitch and roll angles (θ_r and ϕ_r, respectively) are generated through the two virtual inputs u_x and u_y, computed to follow the desired xy movement. Finally the rotation controller is used to stabilize the quadrotor under near quasi-stationary conditions with control inputs τ_ψ, τ_θ, τ_ϕ.

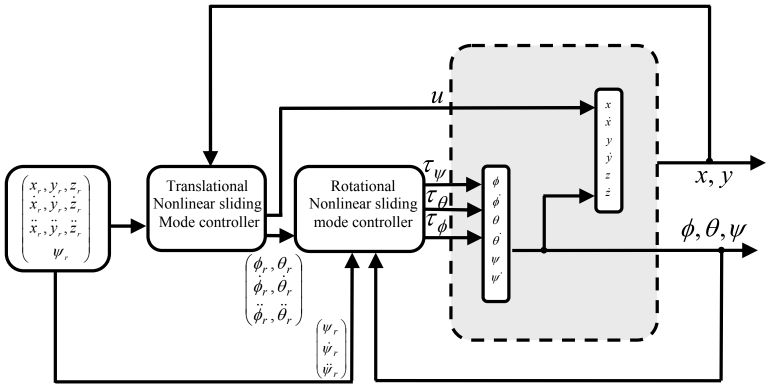

Fig. 2. Quadrotor helicopter control structure.

5. SLIDING MODE CONTROL DESIGN

A Sliding Mode Control is a Variable Structure Control (VSC). Basically, VSC includes several different continuous functions that map plant state to a control surface. The switching among these functions is determined by plant state which is represented by a switching function [25].

Considering the system to be controlled described by state-space equation:

$$x^{(n)} = f(x,t) + g(x,t)u \tag{15}$$

Where $x(t) = \left(x, x^{(1)}, \dots\dots\dots, x^{(n-1)}\right)$ is the vector of state variable $f(x,t)$ and $g(x,t)$ are both nonlinear functions present the system, u is the control part.

The design of the sliding mode control needed two steps. The choice of the sliding surface, and the design of the control law.

step1: the Choice of the Sliding Surface

Slotine in [26] propose the general form, when its consist of defined the scalar function for de sliding surface in the phase plan. The objective is the convergence of state variable x at its desired value .The general formulation of the sliding surface is given by the following equation [31]:

$$s(x) = \sum_{i=1}^{i=n} \lambda_i\ e_i = e_n + \sum_{i=1}^{n-1} \lambda_i\ e_i \tag{16}$$

When $\lambda_n = 1$, and $\lambda_i\,(i = 1 \dots n)$ present the plan coefficients.

Generally the sliding surface is given by the following linear function:

$$S(x) = e + \lambda\,\dot{e} \tag{17}$$

Where λ is constant positive value, and $e = x - x_d$.

When the function of commutation it's calculated the problem of tracking needed the conception of the law control with the stat vector $e(t)$ rested on the sliding surface then $s(x,t)=0$ for only $t \geq 0$.

A suitable control *u* has to be found so as to retain the error on the sliding surface $s(e,t)=0$. To achieve this purpose, a positive *Lyapunov* function *V* is defined as:

$$V(s,x,t)=\frac{1}{2}s^2(x,t) \quad (18)$$

The sufficient condition for the stability of the system is given by:

$$\dot{V}(s,x,t)=\dot{V}(s)=s.\dot{s}<-\eta.|s| \quad (19)$$

Where η is the positive value $(\eta>0)$.

Step2: the Choice of the Sliding Surface

The sliding mode control comports two terms which are equivalent control term and switching control term:

$$u=u_{eq}+u_s \quad (20)$$

u_{eq} is the equivalent part of the sliding mode control, i.e. the necessary known part of the control system when $\dot{s}=0$.

u_s Described the discontinues control is given by:

$$u_s=-k\ sign\ (s) \quad (21)$$

6- SLIDING MODE CONTROL OF THE QUADROTR

The model (1) developed in the first part of this paper can be rewritten in the state-space form:

$\dot{X}=f(x)+g(X,U)+\delta$ and $X=[x_1,...,x_{12}]^T$ is the state vector of the system such as:

$$X=[x,\dot{x},y,\dot{y},z,\dot{z},\psi,\dot{\psi},\theta,\dot{\theta},\phi,\dot{\phi}] \quad (22)$$

From (1) and (22) we obtain the following state representation:

$$\begin{cases} \dot{x}_1=x_2 \\ \dot{x}_2=u_x \\ \dot{x}_3=x_4 \\ \dot{x}_4=u_y \\ \dot{x}_5=x_6 \\ \dot{x}_6=\frac{1}{m}\cos x_9\cos x_{11}-g \\ \dot{x}_7=x_8 \\ \dot{x}_8=\tau_\psi \\ \dot{x}_9=x_{10} \\ \dot{x}_{10}=\tau_\theta \\ \dot{x}_{11}=x_{12} \\ \dot{x}_{12}=\tau_\phi \end{cases} \quad (23)$$

$$\begin{cases} u_x=-\frac{1}{m}\sin x_9 \\ u_y=\frac{1}{m}\cos x_9\sin x_{11} \end{cases} \quad (24)$$

To synthesize a stabilizing control law by sliding mode, the necessary sliding condition $(S\dot{S}<0)$ must be verified; so the synthesized stabilizing control laws are as follows:

$$\begin{cases} u_x=-k_1 sign(S_x)+\ddot{x}_r+\lambda_1 e_2 \\ u_y=-k_2 sign(S_y)+\ddot{y}_r+\lambda_2 e_4 \\ u=\frac{m}{\cos x_9\cos x_{11}}\{-k_3 sign(S_z)+\ddot{z}_r+g+\lambda_3 e_6\} \\ \tau_\psi=-k_4 sign(S_\psi)+\ddot{\psi}_r+\lambda_4 e_8 \\ \tau_\theta=-k_2 sign(S_\theta)+\ddot{\theta}_r+\lambda_5 e_{10} \\ \tau_\phi=-k_1 sign(S_\phi)+\ddot{\phi}_r+\lambda_6 e_{12} \end{cases} \quad (25)$$

Such as $(k_i,\lambda_i)\in R^2$

Proof

The tracking errors are defined by:

$$\begin{cases} e_i=x_{id}-x_i \\ e_{i+1}=\dot{e}_i \end{cases} \qquad i\in[1,11] \quad (26)$$

The sliding surfaces are chosen as follows:

$$\begin{cases} S_x=e_2+\lambda_1 e_1 \\ S_y=e_4+\lambda_2 e_3 \\ S_z=e_6+\lambda_3 e_5 \\ S_\psi=e_8+\lambda_4 e_7 \\ S_\theta=e_{10}+\lambda_5 e_9 \\ S_\phi=e_{12}+\lambda_6 e_{11} \end{cases} \quad (27)$$

The lyapunov function is defined by:

$$V(S_\phi) = \frac{1}{2} S_\phi^2$$

if $(\dot{V}(S_\phi) < 0)$ then $(S\dot{S} < 0)$, we can say that the necessary condition has verified and the stability of Lyapunov is guaranteed

$$S_x = e_2 + \lambda_1 e_1 \tag{28}$$

The chosen law for the attractive surface is the time derivative of (36) satisfying $(S\dot{S} < 0)$:

$$\begin{aligned} \dot{S}_x &= -k_1 \; sign\,(S_\phi) \\ &= \ddot{x}_{1d} - \dot{x}_2 + \lambda_1 \dot{e}_1 \\ &= \ddot{x}_r - u_x + \lambda_1 (\dot{\phi}_r - x_2) \end{aligned} \tag{29}$$

Than:

$$u_x = -k_1 sign(S_x) + \ddot{x}_r + \lambda_1 e_2 \tag{30}$$

$$u_x = u_{xeq} + \Delta u_x \tag{31}$$

According to (30) we obtain:

$$\begin{cases} \Delta u_x = -k_1 \; sign(S_x) \\ u_{xeq} = \ddot{x}_r + \lambda_1 e_2 \end{cases} \tag{32}$$

The same steps are followed to extract $u_y, u, \tau_\psi, \tau_\theta$ and τ_ϕ

The desired roll and pitch angles in terms of errors between actual and desired speeds are, thus, separately given by:

$$\phi_r = arctg\left(\frac{u_y}{-k_3 sign(S_z) + \ddot{z}_r + g + \lambda_3 e_6} \right) \tag{33}$$

$$\theta_r = arctg\left[\left(\frac{-k_3 sign(S_z) + \ddot{z}_r + g + \lambda_3 e_6}{u_x} \right)^{-1} \cos\phi \right] \tag{34}$$

7. SIMULATION RESULTS

Fig. 3 shows the tracking of desired trajectory by the real one and the evolution of the quadrotor in space and its stabilization.
Fig. 4 highlights the tracking of the desired trajectories along yaw angle (ψ) and (X,Y,Z) axis respectively. the tracking in yaw presents a rather weak permanent error when the desired trajectory is dynamic.
Fig. 5 represents the errors made on the desired trajectory tracking.
Fig. 6 presents the Pitch and roll angles response of a quadrotor helicopter. Finally fig 7 presents the robustness test to measurement noise added to the roll, pitch, and yaw angles (inertial measurement unit sensor noise)

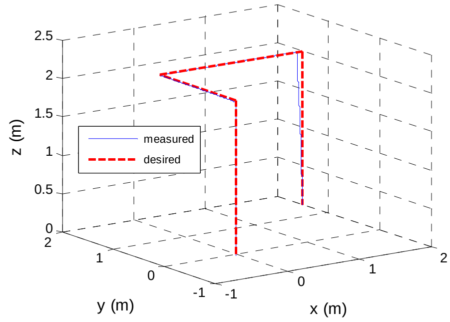

Fig. 3 Global trajectory of the quadrotor in 3D

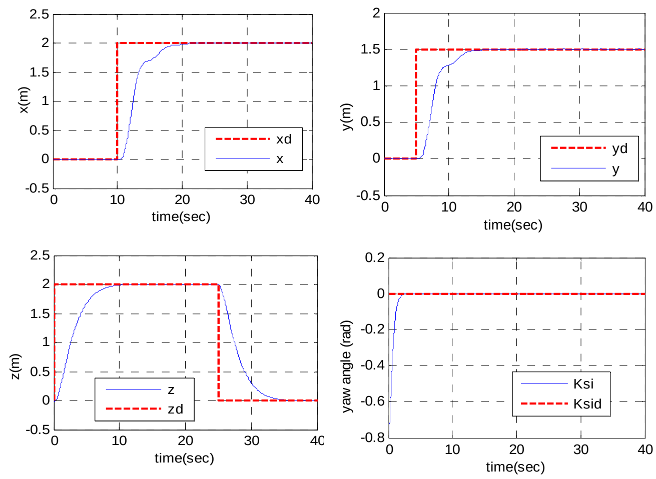

Fig. 4 Tracking simulation results of the desired trajectories along yaw angle (ψ) and (*X*, *Y*, *Z*) axis

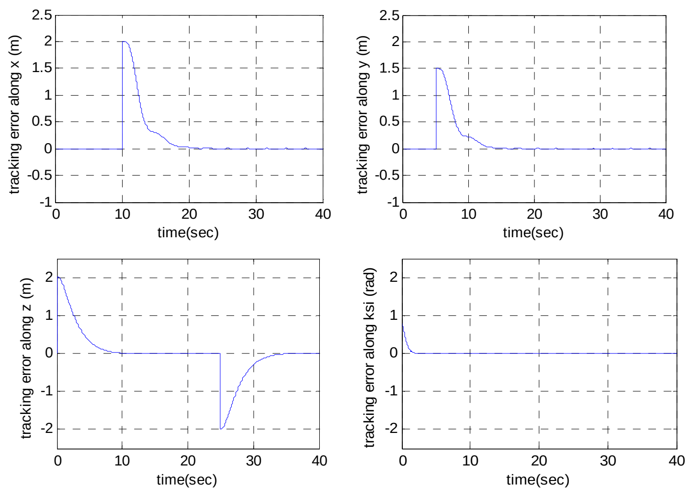

Fig. 5 Tracking errors according yaw (ψ) angle and (*X*, *Y*, *Z*) axis

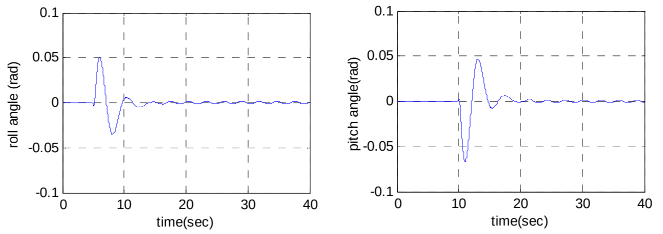

Fig. 6 Pitch and roll angles Response of a quadrotor helicopter

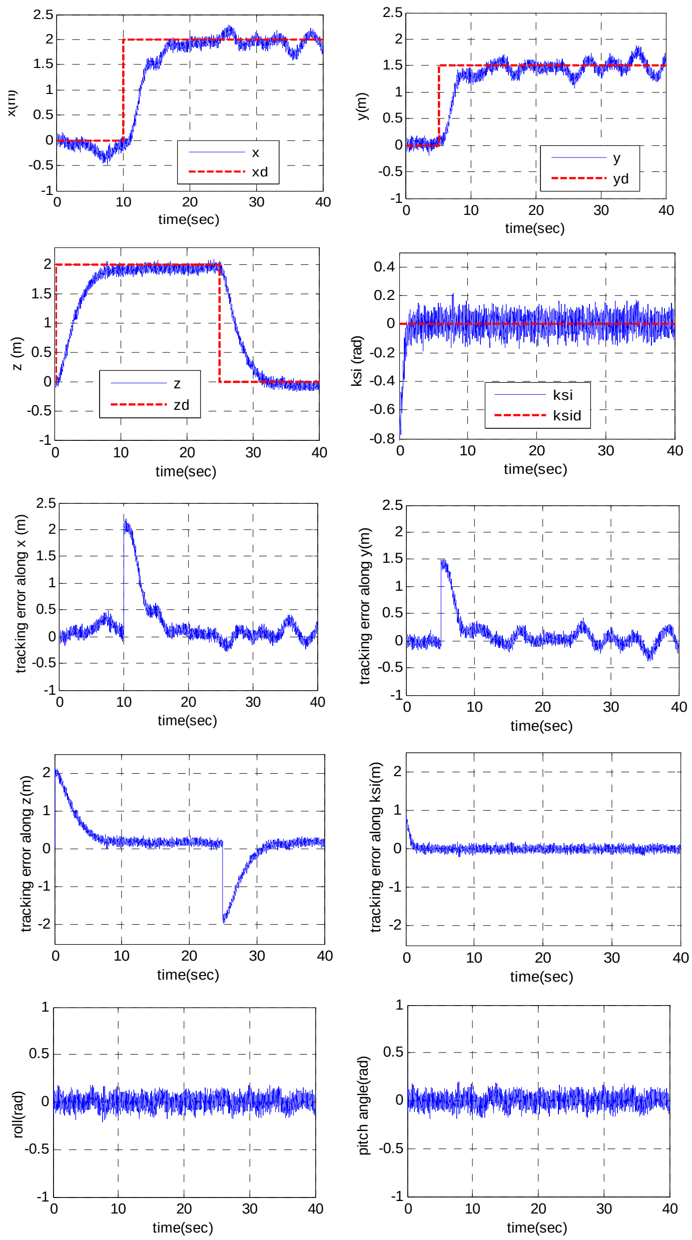

Fig .7. Robustness test to measurement noise

8. Conclusion

In this paper, we presented stabilizing control laws synthesis by sliding mode. Firstly, we start by the development of the dynamic model of the quadrotor taking into account the different physics phenomena which can influence the evolution of our system in the space, this says these control laws allowed the tracking of the various desired trajectories expressed in term of the center of mass coordinates of the system in spite of the complexity of the proposed model. As prospects we hope to develop other control techniques in order to improve the performances and to implement them on a real system.

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