

ON AN ELEMENTARY PROBLEM OF MODELLING DEAD TIME SYSTEMS

Toma L. DRAGOMIR Alexandru CODREAN Vlad CEREGAN

“Politehnica” University of Timișoara

300223 Timișoara, Bd. V. Pârvan N0. 2, Romania, tel: +40256403222, fax.: +40256403214, tldragomir@yahoo.com

Abstract: *The paper discusses the fundamental problem of accurate modelling of the simplest transfer element with dead time when simulation environments based on block diagrams are used. The core of the paper consists in presenting a way of modelling the output signal stored in the memory of this system for the time interval $[t_s - \tau, t_s)$ where t_s is the initial time of the simulation and τ the dead time. This set of values is called the initial segment and the implementation is done by an initial segment generator (isg). The paper explains by references to examples the manner in which the isg implementation can be done correctly using the possibilities offered by the Matlab / Simulink environment, draws attention to the errors that may appear in the simulation operations by omitting the isg and presents two case studies regarding control loops that contain elementary subsystems with dead time.*

Key words: *dead time system, simulation, initial segment, Matlab / Simulink.*

1. Introduction

The input-output delayed transfer of information arises in a large number of processes. Two practical aspects are of great importance for simulating such transfer operations: a) knowing the short-time prehistory of the input signal, before the moment in which the simulation starts, b) the usage of instruments capable of manipulating this prehistory.

While usually only constant and zero initial conditions are taken into account, these aspects are in fact omitted. Books, tutorials, articles are content with this level of approach (see for exemple [1] and [2]).

However, there exist some papers that mention the importance of the prehistory of dead time systems and introduce functions for modelling dead time systems. For example, in [3] we can find the Matlab function dde23 that is used for solving delayed differential equations. This function requires the specification of the prehistory of the dead time elements in one of the following forms: by defining a time function, by using a

constant vector, or by storing the result of a previous integration. The problem of using this function is covered in a systematic and explicit manner in [4] and [5]. In [6] new Matlab applications are developed to solve delayed differential equations (with one or more dead times). The authors propose extensions to Matlab's library with new functions for solving delayed differential equations. For example, to take into account the initial segments, they use the Matlab function fde45 based on an extension of the ode45 method. The initial segments are considered as arguments of this new function. The given examples consider the initial segments as constants, and thus not exceeding the possibilities of the Transport Delay block from Simulink.

For the regular users of the Matlab- Simulink environment, interested especially in Simulink, it is very important to dispose of simple and modular means of simulating. In the same time the user must be notified on the effects of not using the short-time prehistory of dead time systems. He needs to know what and how much can be lost, what and how much can be gained by considering the prehistory of a dead time system. The answer to these questions depends on the followed objectives, on the time interval in which the behaviour of the system is studied and on the system's structure. For example, if the system used in simulations is a stable control loop, and the simulation time interval on which the system's response is calculated is long, the omission of the system's prehistory can be treated as a non-persistent perturbation which, at the beginning introduces a simulation error that then is compensated.

In this context the paper covers in an elementary manner the problem of prehistory modelling of the simplest transfer system with dead time (that known as the transfer function $e^{-\tau s}$). To solve the problem we use as a basic modelling instrument an auxiliary signal generator called initial segment generator (section 2). Then, two case studies illustrate the way of using this

instrument (section 3).

2. Elementary system with dead time with two-sided and right-sided signals

An elementary system with dead time, also called transport delay- transfer element (td-TE) achieves by definition an input-output dependency of the form:

$$y(t) = u(t - \tau), \tau > 0, t \in \mathbb{R}. \quad (1)$$

In (1): u is the input, y is the output, τ - the dead time, also called transport delay, and t - the time representing the independent variable of the system. The td-TE transfers the input signal towards the output through propagation in τ seconds, and that means implicitly that the td-TE has the capacity to store for any t the values of u on a compact time interval $[t - \tau, t]$. Theoretically, this implies an infinity of stored values, namely an infinite number of states. For this reason td-TE is considered an infinite dimensional system.

Figure 1 shows a physical system, well known as an example of td-TE : the conveyor belt.

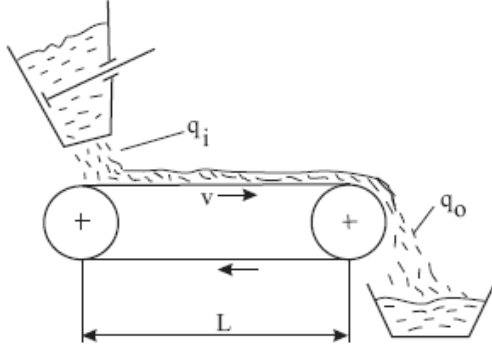


Fig. 1 The conveyor belt – an intuitive example of a physical system with dead time.

The belt has a length L , a constant speed v and is loaded with a material that has an input flow $q_i(t)$. The material leaves the belt with an output flow $q_o(t)$. The input – output relation is

$$q_o(t) = q_i(t - \tau), t \in \mathbb{R}, \tau = \frac{L}{v}. \quad (2)$$

The relation (2) has the same form as (1) and the transport depends on the storage for every moment t of the values from the set

$$\{q_o(\tilde{t}) \mid \tilde{t} \in [t - \tau, t]\} \quad (3)$$

It's important to observe that both systems, (1) and (2), operate with two-sided signals, namely with signals defined on the set of real numbers \mathbb{R} .

When simulating the real behaviour of a system our interest shifts to what happens starting at a given

moment t_0 called initial moment. For simplicity we consider that $t_0=0$. A key remark is that now, instead of two-sided signals, we are dealing with right-sided signals. From the viewpoint of the first τ seconds, i.e. $t \in [0, \tau)$, there can exist a multitude of situations. For instance (fig. 1 is referred only from reason of intuition):

i) The moment $t = 0$ is characterized by the fact that in the last τ seconds the belt was on the move. Consequently on the time interval $t \in [0, \tau)$ the material which was loaded in the interval $[-\tau, 0)$, unloads:

$$q_o(t) = q_i(t - \tau), t \in [0, \tau). \quad (4)$$

ii) The moment $t=0$ is characterized by the fact that, previously, the conveyor was motionless, being started at $t = 0$ with the speed v . Consequently, on the time interval $t \in [0, \tau)$ an initial distribution of material is being unloaded $q_{ix}(x)$ for $x \in [0, L]$ (fig. 2)

$$q_o(t) = q_{ix}(L - vt), t \in [0, \tau) \quad (5)$$

We can observe that in both cases the starting hypothesis is that at the moment $t = 0$ the conveyor has an initial load which is a result of a particular short-time prehistory of τ seconds and which leads in the first τ seconds to a discharge expressible through the segment

$$\{(t, q_d(t)) \mid t \in [0, \tau)\} \quad (6)$$

in which

$$q_d(t) = \begin{cases} q_i(t - \tau) & [\text{case i)}] \\ q_{ix}(-v(t - \tau)) & [\text{case ii)}] \end{cases} \quad (7)$$

The above mentioned situation is interpreted in figure 2 using two belts.

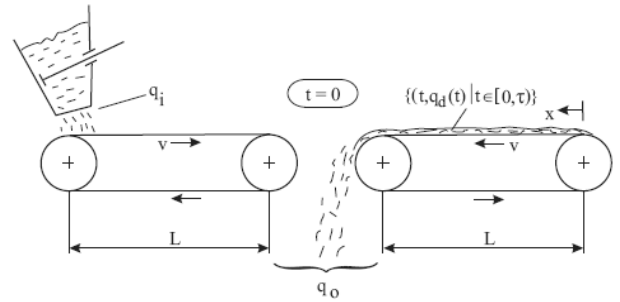


Fig. 2. The two conveyor belts system as a physical model for simulating a td-TE starting with a given moment $t=0$.

The belt from the right transports the initial load (the short-time prehistory) and empties it in the first τ seconds. After that its presence doesn't matter anymore. The belt to the left is initially empty. It is being loaded starting with the moment $t=0$ (right-sided signal) and it is being unloaded starting with the moment $t = \tau$.

Therefore, in simulations or every time we are interested in characterizing a td-TE starting with a given

moment (in this case $t = 0$), the models of td-TE with right-sided signals, and two-sided signals are

$$q_o(t) = \begin{cases} q_d(t), & t \in [0, \tau) \\ q_i(t - \tau), & t \geq \tau \end{cases} \quad (8)$$

respectively

$$q_o(t) = \begin{cases} q_d(t) \cdot [\sigma(t) - \sigma(t - \tau)] \\ q_i(t - \tau) \cdot \sigma(t - \tau) \end{cases}, \quad t \in \mathbb{R}. \quad (9)$$

Here $\sigma(t)$ is the unit step function.

Generalizing, in operating with transfer of right-sided signals for $t \geq 0$ the model (1) has to be rewritten in the form

$$y(t) = \begin{cases} f(t - \tau) \cdot [\sigma(t) - \sigma(t - \tau)] \\ u(t - \tau) \cdot \sigma(t - \tau) \end{cases}, \quad t \in \mathbb{R}_+, \quad (10)$$

where the function $f(t - \tau)$, $t \in [0, \tau)$, called the *initial segment generator* (isg) of the td-TE, carries the short-time prehistory of td-TE.

The advantage of isg is obvious:

- a) gives greater accuracy to modeling.

By introducing the isg defined by the set of points

$$\{(t, g(t)) \mid g(t) = f(t - \tau), t \in [0, \tau)\} \quad (11)$$

the model obtained represents with a higher fidelity the reality.

Concurrently, introducing the isg is disadvantageous because:

- b) it complicates the modeling process.

This aspect is associated with the block diagram form fig. 3 built after relations (10) and (11). The complications are related to specifying the initial segment (11) because models that incorporate the td-TE may correspond to a large variety of concrete problems. In case i) the initial segment is obtained by reconsidering a real short-time prehistory of the system. In case ii) we must assume an initial distribution of the output y . Basically; the isg corresponds to the conveyor placed in the right side of figure 2.

An unwanted aspect consists in the fact that considering the short-time prehistory, respectively isg, can lead for many researches to the necessity of reexamining and reconsidering of some of the previous results. Therefore, this aspect concerns in principle a takeover in a critical manner of the results from the literature in the case of many dead time systems.

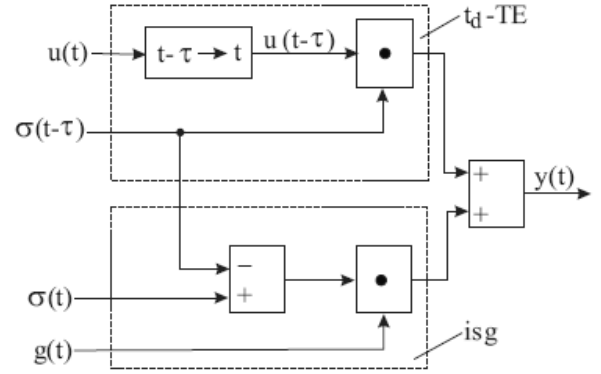


Fig. 3. Regarding the implementation of the isg.

Regarding the implementation of the isg we consider, as an example, the possibilities offered by the Matlab / Simulink environment. The implementation based on the principle from figure 3 can be done under the form suggested in figure 4. The concept of initial output is replaced with that of “initial segment generator” produced by the signal generator isg. Practically: i) we set to 0 the *Initial output* parameter of Transport Delay module and to value τ the *Time Delay* parameter and ii) we provide a supplementary modul to produce the short-time prehistory: the signal generator isg.

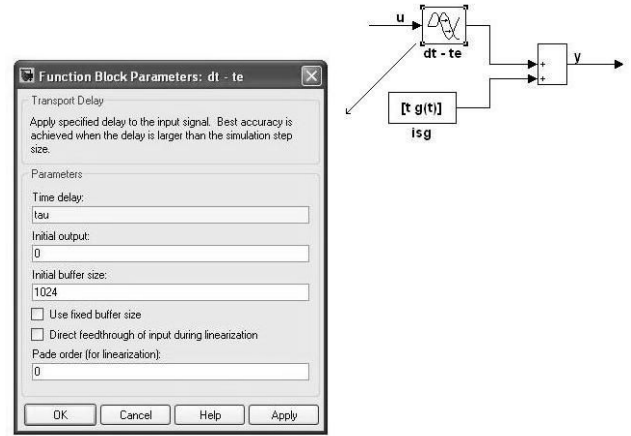


Fig. 4. The implementation of the isg block.

The isg block can be implemented under different forms, for example by memory block for the initial segment $\{(t, g(t)) \mid t \in (-\tau, 0]\}$ as in figure 5 or by a simulink structure like the one in figure 6.

In the first case, the initial segment from the input of the td-TE of the reference system is stored in Workspace, using a *To Workspace* block, and from here it is transferred in the *From Workspace* block of the system used for the simulation.

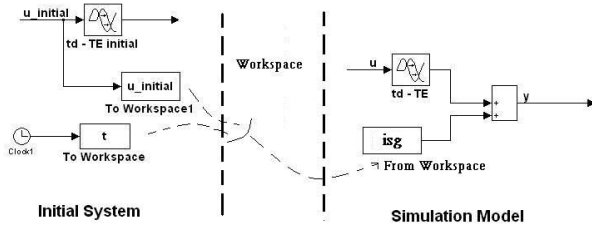


Fig. 5. The implementation of the isg using the To Workspace – Workspace – From Workspace chain.

In the second case, that based on the simulink structure, only a particular example is presented. In figure 6 the Simulink structure generates the transcendent function: $g(t) = A \cdot \sin(\omega t + \phi) + e^{-2t}$, $t \in [0, \tau)$. The values of parameters A , ω and ϕ are set by the Sine Wave block; the sigma(t) block is set to 1 at the moment $t = 0$, and the sigma(t-tau) block goes from 0 to -1 at $t = \tau$.

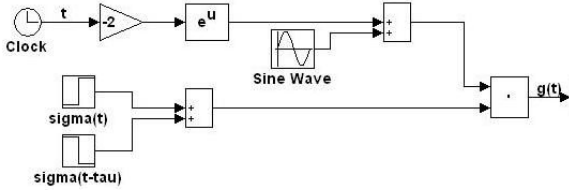


Fig. 6. Example of an isg implementation based on using the simulink structure.

Before presenting some examples regarding the usage of the isg it is necessary to refer to the option *Pade order (for linearization)* that appears in the Dialog Box of the Simulink Transport Delay Block. This option can be set in the following manner:

- if the implicit option remains: 0, we are found in the situation in which the td-TE acts according to the model (1);
- if the option selects a Pade aproximation of a certaind degree, the conveyor from figure 3 is replaced by a linear transfer block based on a Pade aproximation of the selected order.

In all the examples that follow it is considered that the Pade order (for linearization) is set as 0.

3. Case studies regarding the usage of the isg

3.1. Simulation of a remote control system

In figure 7 a remote control system affected by 3 dead times is represented: the plant P is controlled from a distance by the controller RG . The forward propagation takes τ_D seconds ($u_D(t) = u(t - \tau_D)$), the backward propagation (feedback) takes τ_{FB} seconds ($y_{FB}(t) = y(t - \tau_{FB})$), and the plant P has a dead time of τ_P seconds.

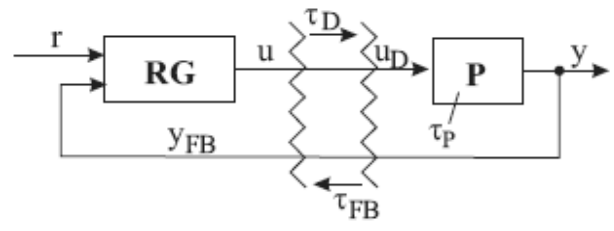


Fig. 7. Telecontrol system.

We suppose that the controlled plant is linear and has the transfer function

$$H_P(s) = \frac{1}{s} \cdot e^{-\tau_P s}, \tau_P = 1 \text{ second} \quad (12)$$

and that the controller is a simple P – controller (a proportional one): $u(t) = K \cdot e(t) = K \cdot [r(t) - y_{FB}(t)]$. Due to the presence of 3 dead times it is necessary to use three isg blocks as shown in figure 8: the isg-tauD, isg-tauP, and isg-tauFB blocks corresponds, respectively, to the τ_D , τ_P ($y(t) = x(t - \tau_P)$, where x is a state variable of the process) and τ_{FB} dead times.

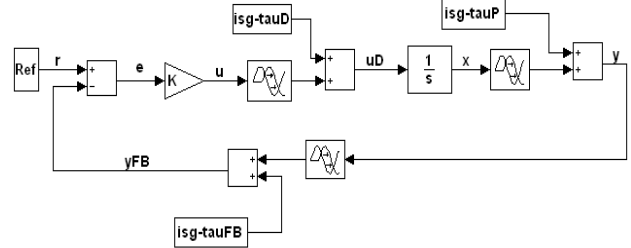


Fig. 8. A remote control system with three td-TEm.

The open loop transfer function of the control system in figure 8 is $\tilde{H}(s) = K \cdot \frac{1}{s} \cdot e^{-\tau \cdot s}$, $\tau = \tau_D + \tau_P + \tau_{FB}$ and it

explains while for some problems, as that of system's stability, only the equivalent dead time τ is important. [7, 8]. Figure 9 show the system's response during the so called *reference regime* defined (arbitrarily) by: the values from the legend L1, ii) setting the three isg with null segments (as if the blocks were missing), iii) resetting the Initial Output of the dialog blocks of the td-TEs to 0.

To demonstrate the importance of isg we chose for reproducing by simulation the reference regime at initial moment $t_s = 10$ seconds. From the reference regime we find that at this moment the integrator's output (from the process P) is $x = x_s = 0.751$, and the process's output is $y = y_s = 0.67$.

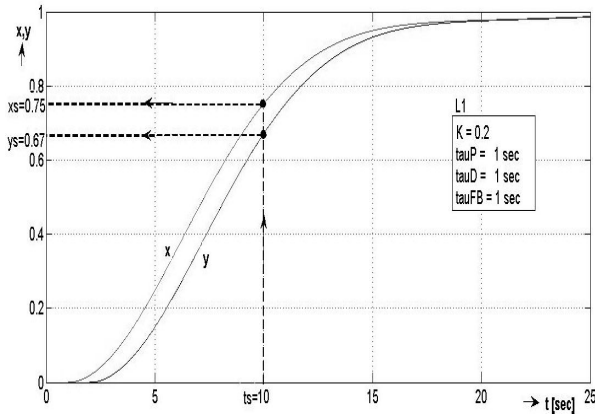


Fig. 9. The dynamic characteristics $x(t)$ și $y(t)$ of the reference regime of the system from fig. 8 ($K=0.2$).

The simulation scenario imagined in the next part of this section investigates the reference regime's reproduction, without and with isg, starting with the moment $t = t_s$.

In figure 10 the result of a first attempt of reproducing the reference regime is illustrated in which the isg blocks are not used. The simulation is defined by:

i) the values from the legend L1, ii) setting the three isg blocks with null initial segments (as if the blocks were missing), iii) setting Initial Output of the dialog blocks to the three td-Tes, to the values from the legend L2, iv) setting the initial state to the value from L2.

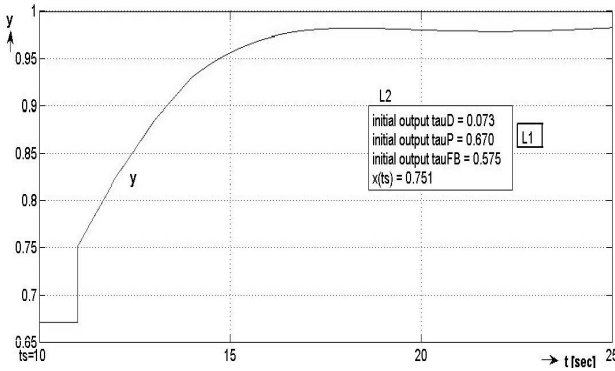


Fig. 10. The simulation of the control system's behaviour from fig. 8 with the Initial output parameters of the transport delays blocks set to constant values and without using the isg ($K=0.2$).

It can be observed that the output signal $y(t)$ from fig. 10 differs from the reference variation from fig. 8. So the manner in which the simulation was conceived is incorrect.

In figure 11 the result of a second attempt to reproduce the reference regime is shown; now the isg blocks are used. The simulation is defined by:

i) the values from the legend L1, ii) setting the three isg: isg-tauD, isg-tauP and isg-tauFB, with the segments $\{(t, u(t)|t \in [9,10])\}$, $\{(t, x(t)|t \in [9,10])\}$, respectively $\{(t, y(t)|t \in [9,10])\}$ determined during the reference regime, iii) setting Initial Output from the dialog blocks of the three td-TEs to 0, iv) setting the initial state to the value from L3.

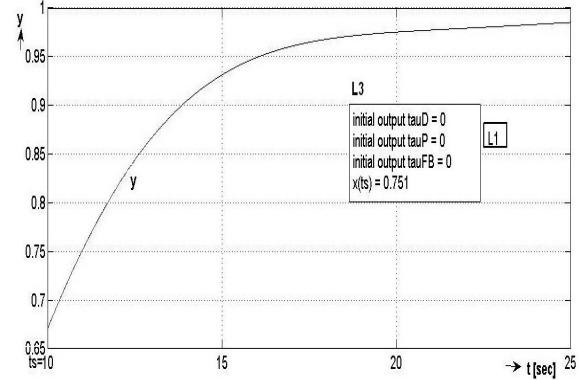
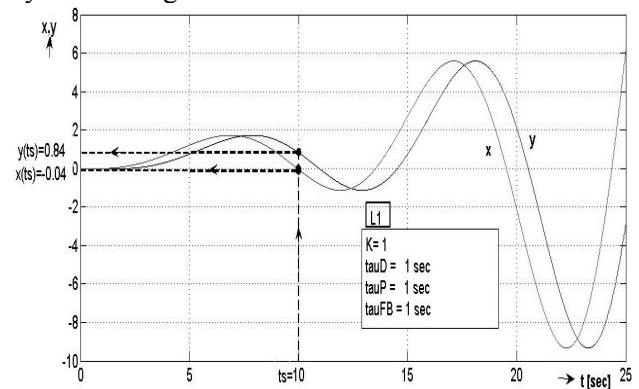


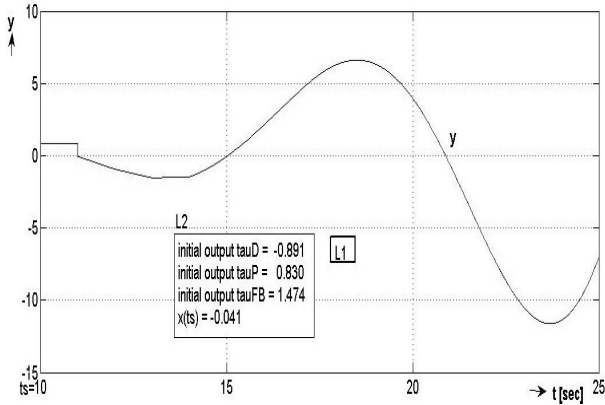
Fig. 11. The simulation of the control system's behaviour from fig. 8 using the isg ($K=0.2$).

By comparing figure 11 with figure 10 it's obvious that implementing the isg is more than necessary in order to reproduce in a correct manner the reference regime starting from $ts=10s$. More, this result is general. It holds for every ts .

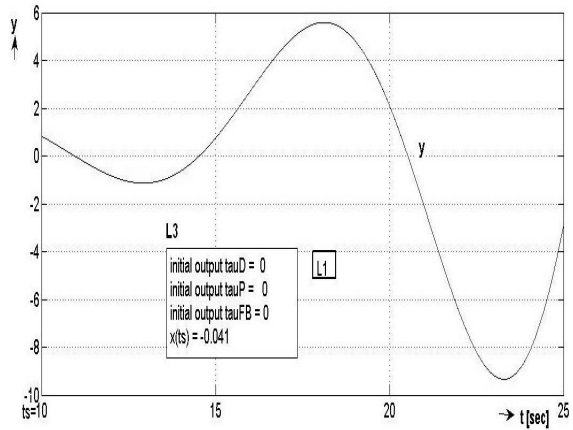
Also, it can be noted that besides giving greater accuracy to the modeling process, the proposed solution of implementing the isg (Section 2) is quite straight forward.

In figures 12 a, b, c the study described through the figures 9, 10 and 11 is resumed, for the case in which the controller RG from fig. 8 has the gain $K = 1$, the system beeing unstable.





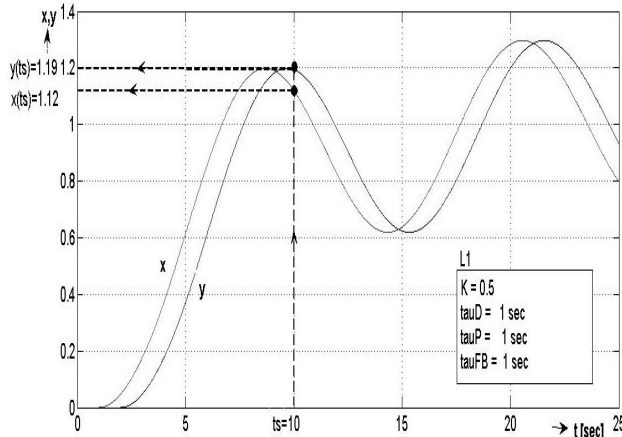
- b -



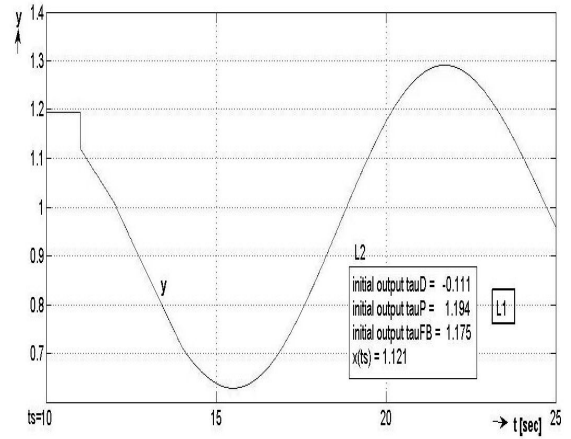
- c -

Fig. 12. The reference regime (a), the regime simulated without using the isg (b) and the regime simulated using the isg (c) for the control system from fig. 8 when $K = 1$ (unstable system).

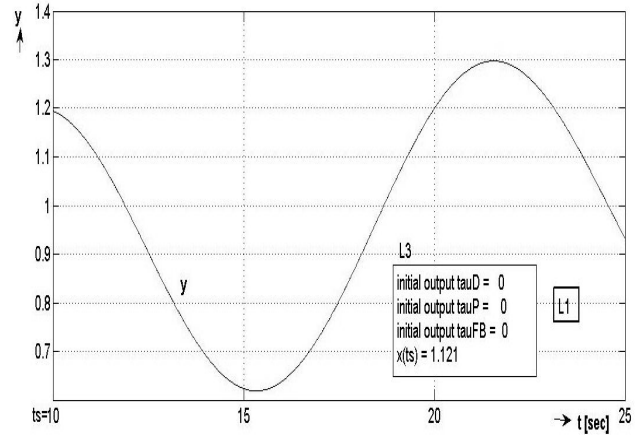
In figures 13 a, b, c the same study is resumed for the case in which the controller RG from fig. 8 has the gain $K = 0.5$, the system being at the limit of stability.



- a -



- b -



- c -

Fig. 13 The reference regime (a), the regime simulated without using the isg (b) and the regime simulated using the isg (c) for the control system from fig. 8 when $K = 0.5$ (the system at the limit of stability).

As expected, the conclusions drawn from the case where $K=0.2$ regarding the effect of incorrect modeling of the td-TE are preserved.

3.2. Simulation of a nonlinear control system with limit cycles

The second example refers to the nonlinear control system in fig 14. The plant is linear and has the transfer function

$$H_P(s) = \frac{1}{s(s+1)} \cdot e^{-\tau_P s}, \quad \tau_P = 1 \text{ second}.$$

The controller is of bipositional type (bang-bang controller)

$$u = \begin{cases} -1, & \text{for } e < 0 \\ 1, & \text{for } e > 0 \end{cases}$$

without hysteresis. This system is also widely studied in [9]. The control is characterized by periodic regimes that appear in form of stable limit cycles.

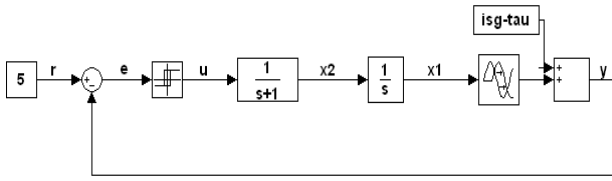
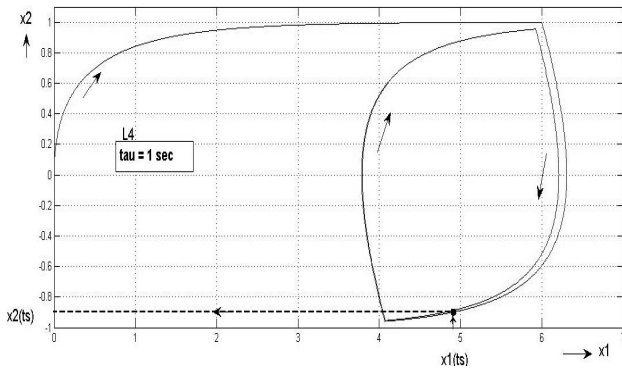
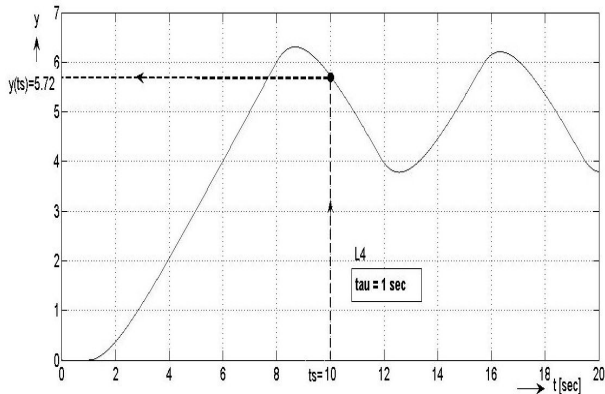


Fig. 14. Control system with bipositional controller.

The system's behaviour in the reference regime by applying an input signal $r = 5$ is shown in fig. 15. The initial conditions are: $x_1 = 0$, $x_2 = 0$, and for the td-TE the parameter Initial Output is set to 0. The isg block is set to a null segment. Fig. 15 a. shows the periodic evolution in form of limit cycles in the state plane. Fig. 15 b. shows the variation of y in the same time interval. The aim is to reproduce the reference regime starting with the initial moment $t_s = 10$ seconds.



- a -

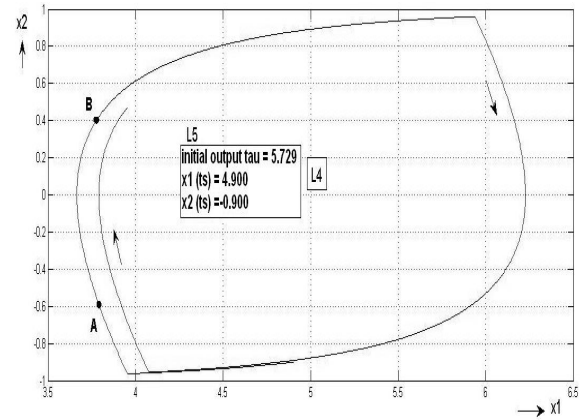


- b -

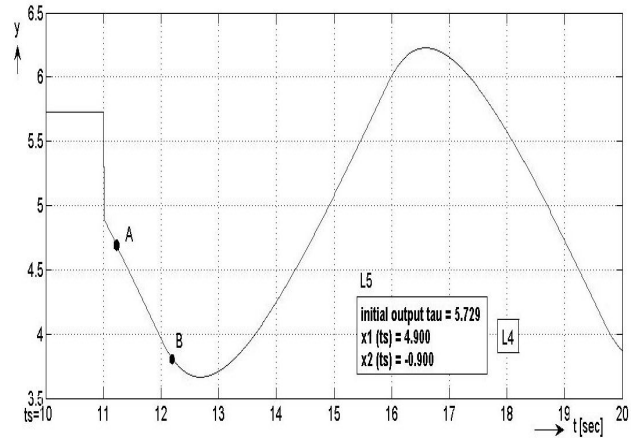
Fig. 15. The nonlinear control system from fig. 14 passes from the initial zero steady state state into a limit cycle corresponding to the constant input signal $r = 5$.

Fig. 16 illustrates the simulated system's response without taking into account the isg.

The difference observed by comparing fig. 16 with fig. 15 consists in the portion represented by the \widehat{AB} arc. The differences are most obvious in Fig. 16 a.



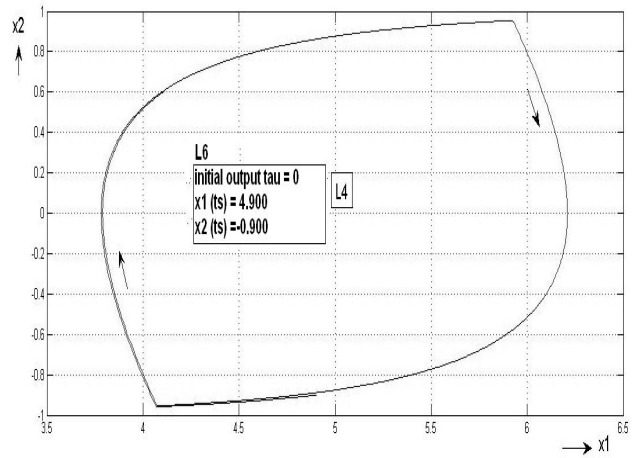
- a -



- b -

Fig. 16. The evolution of the system from fig. 14 starting with the state's value at the moment $t_s = 10$ (simulation without using the isg).

Fig. 17 shows the simulated response of the system with the isg taken into account. By comparing with fig. 15, it can be observed that there are no more differences.



- a -

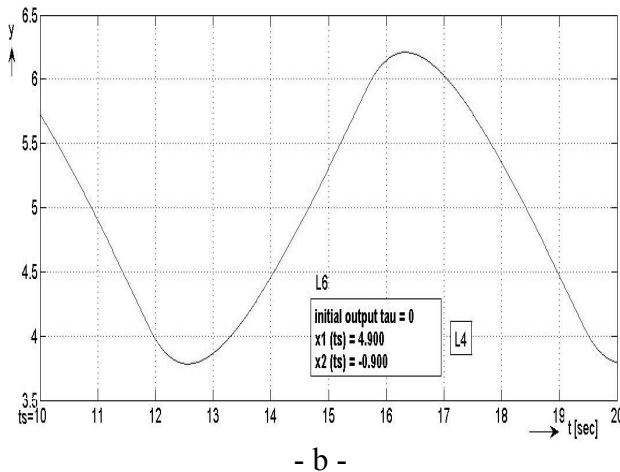


Fig. 17. The system's evolution from fig. 14 starting with the state's value at the moment $t_s = 10$ with the usage of the isg.

In nonlinear systems, taking into account during simulations the system's short-time prehistory by implementing an isg is even more crucial. For example, if the control system is characterized by an unstable limit cycle, the usage of an isg can make the difference between a stable and an unstable trajectory.

4. Conclusions

The accurate modelling of systems with dead time is a complex problem, which in the conditions of using block diagrams, requires a very carefully modelling of even the simplest dead time systems: the pure delay elements. In essence, it is necessary to take in account the short-time prehistory of this element, i.e. the set of values of the output signal stored in the memory of this system for the time interval $[t_s - \tau, t_s)$ where t_s is the initial time of the simulation and τ the dead time. This set of values is called the initial segment and the implementation is done by an initial segment generator (isg). In this context the paper:

- defines the concept of isg and gives a method of implementation using the possibilities offered by the Matlab/Simulink environment;
- draws attention to the errors that can appear in simulations by not using the initial segment;
- presents two case studies related to control

loops which get to function in a permanent stationary regime, respectively periodic regime (stable limit cycles).

As the studied system has an open loop structure or a closed loop structure the effects of considering or omitting the short-time prehistory can be different. The paper refers only to simulation problems.

References

1. Normey-Rico J.E., Camacho E.F.: Control of Dead-time Processes, Springer, London, 2007.
2. Boukas E-K., Liu Z-K.: *Deterministic and Stochastic Time-Delay Systems*, Birkhäuser, Boston, 2002.
3. ***, *Matlab 7.5.0 (R2007.b) (Help part)*
4. Shampine L.F., Thompson S., Kierzenka J.: „L.F. Shampine, S. Thompson, *Solving DDEs in MATLAB*, Appl. Numer. Math., 37 (2001), pp. 441-458
5. L.F. Shampine, I. Gladwell, and S. Thompson, *Solving ODEs with MATLAB*, Cambridge University Press, 2003.
6. Kwon W.H., Kim A., Han S.H., Pimenov V., Lozhnikov A., Onegova O.: *Time-Delay System Toolbox and its Application*. In: Proceedings of the 13th KACC, October 1998, p.147-151.
7. Föllinger O.: *Regelungstechnik, Einführung in die Methoden und ihre Anwendung*, 8. Auflage. Hüthig Verlag, Heidelberg, 1994.
8. Weinmann A.: *Test- und Prüfungsaufgaben Regelungstechnik*, Springer Verlag, Wien-New York, 1997.
9. Föllinger O.: *Nichtlineare Regelungen, Bd. 1: Grundbegriffe, Anwendung der Zustandsebene, direkte Methode.*, 8. Auflage. Oldenbourg, München, 1998.

Symbols and acronyms

- τ - dead time
 $\sigma(t)$ - unit step signal
 $e(t)$ - error signal
 $f(t)$ - various functions of time
 $g(t)$ - function associated to the isg
 $H(s)$ - transfer function
 $\tilde{H}(s)$ - transfer function of the system in open circuit
isg - initial segment generator
 K - gain
 L, v - length and speed of the conveyor belt
 q_0 - output flow
 q_i - input flow
 $r(t)$ - reference signal
 s - operational variable used in the case of the Laplace transform
 t, \tilde{t} - time
td-TE - time delay-Transfer Element
 u - input signal
 x - distance or state variable
 y - output signal