OPTIMIZED SLIDING MODE CONTROLLER FOR A CLASS OF TITO PROCESS WITH INTERACTION

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Abstract: *Control of Multi Input Multi Output (MIMO) process possessing interaction is a common problem in the field of process control. This paper is about the design of Sliding Mode Controller (SMC) for the control of Two Input - Two Output (TITO) process with elements of process transfer function matrix represented as First Order plus Dead Time (FOPDT) model. The conventional SMC designed delivered poor control performance, when applied for the control of TITO process, for the case where the ratio of dead time and time constant of the FOPDT model in the process transfer function matrix is greater than one. Interestingly, in the design of SMC, if the discontinuous part of SMC is multiplied by a weighting factor, the controller performance improved. In this work, Genetic Algorithm and Simulated Annealing optimization techniques are employed to find the best value of the weighting factors. The proposed SMC delivered a better closed loop performance, when applied for the control of TITO processes.*

Key words: *MIMO process, FOPDT process, TITO process, sliding mode controller, Nelder-Mead tuning method, Genetic Algorithm, Simulated Annealing Algorithm.*

1. Introduction

Most of the industrial and chemical processes consist of multiple loops, which are MIMO processes. Chemical processes having strong interaction between their variables, results in non-linear behaviour and is also complex in nature. The presence of parameter uncertainties, and/or non-minimum phase behaviour will further degrade the closed loop performance. The above problem can be eliminated by using centralized multivariable control or by decentralized control. Due to many practical advantages such as simple control structure, fewer tuning parameters, PID controller has been widely used for the control of multi-loop processes with uncertain interactions. The parameters of PID controller tuned using optimization algorithms, controller designed using pole placement technique, and on-line controller tuning are reported in [1], [2] and [3] respectively. The above references focus on the control of MIMO processes, with optimally tuned PID controllers.

Sliding mode control is based on Variable Structure Control (VSC) theory. A variable structure controller applied for the control of bioreactor, with two inputs and two controlled variables, exhibited very good tracking performance with minimal percentage overshoot and, good decoupling of the variables is presented in [4]. Three tanks test-bed process, which is a coupled MIMO process, controlled by using a First Order Sliding Mode Controller (FOSMC), with saturation function and, FOSMC with saturation combined with Integral controller is presented in [5]. Higher Order Sliding Mode Controller (HOSMC) is proposed in [6], for the control of MIMO process with unknown nonlinearities, which also guarantees robustness with respect to uncertainties. An adaptive SMC designed for the control of MIMO nonlinear process with uncertainty in the process dynamics and control distribution gain is presented in [7]. A novel sliding mode control methodology is proposed in [8], for the control of an uncertain nonlinear MIMO processes. The proposed dynamic integral sliding mode controller is delivered improved performance in the presence of uncertainties. Control of twin-rotor multi-input–multi-output system (TRMS) using adaptive second-order sliding mode (SOSM) controller is reported in [9]. The adaptive mechanism in SOSM controller is used to stabilize the TRMS, with significant cross couplings, and the system is able to reach the desired position and accurately track a specified trajectory. SMC control theory is applied to analyze the dynamic behavior of switching regulator and to establish the system stability conditions [10]. In recently, model-free sliding mode control system (MFSMCS) structure is used in the control MIMO system. It provides robustness against parameter variations and disturbances [11]. In general, control of highly uncertain nonlinear systems is a quite difficult task, for which SMC has proved to be an effective control technique [12].

Design of SMC controller comprises of continuous and discontinuous parts. The robust performance of SMC is based on its tuning parameters, which is present in the discontinuous part. The design of discontinuous part involves the determination of two parameters K_D and δ. Over the years, Nelder-Mead tuning method has been used to determine these values. This classical tuning method often resulted in unsatisfactory response, when applied for the control of TITO process, for the case where the ratio (ε) of dead time and time constant is greater than one. However the performance of conventional SMC improved, when the discontinuous part of SMC is multiplied by a weighting factor. In this paper, the introduced weighting factor is optimized and improves the closed loop performance of SMC. The existence region of single-phase dynamic voltage restorers is maximized with the help of optimized SMC. The dynamic response is improved by using the optimum sliding coefficient in SMC [13].

The main contribution of this work is to improve the closed loop response of the TITO process, by controlling it with an optimized SMC, where the optimized weighting factors in the discontinuous part is obtained by employing the evolutionary optimization algorithms such as Genetic Algorithm and Simulated Annealing technique. In the above optimization techniques, an objective or cost function is defined to evaluate the performance of the SMC.

The paper is organized as follows. Section 2 presents the description of TITO process model. A review of SMC and its parameter tuning is presented in section 3. Section four presents three TITO models that are considered in this work. The proposed SMC with the weighting factor is presented and simulation results are also discussed in section 5. Conclusions are drawn in Section 6.

2. TITO process

The TITO process has two input and two output variables with interacting control loops. To design a controller for such a process, proper pairing of manipulated and controlled variables is must. After identification of these variables, appropriate control configuration is to be determined. The block diagram of the open loop TITO process is given in Figure 1.

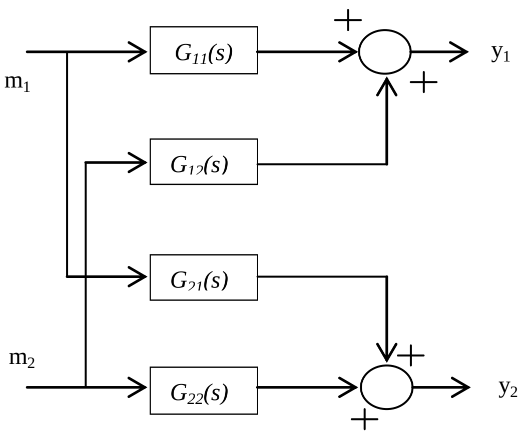

Fig. 1. Block diagram of open loop TITO process

The input and output relationships of the above process is given by,

$$\begin{aligned} y_1(s) &= G_{11}(s)m_1(s) + G_{12}(s)m_2(s) \\ y_2(s) &= G_{21}(s)m_1(s) + G_{22}(s)m_2(s) \end{aligned} \quad (1)$$

Where, $G_{11}(s)$, $G_{12}(s)$, $G_{21}(s)$, and $G_{22}(s)$ are the four transfer functions relating two outputs and two inputs. The change in manipulated variables m_1 and m_2 will affect both controlled outputs y_1 and y_2. Two control loops are formed by coupling m_1 with y_1, and m_2 with y_2 as shown in Figure 2. The above coupling should result in minimal interaction. Relative Gain Array (RGA) method is used for pairing the manipulated and controlled variables [14].

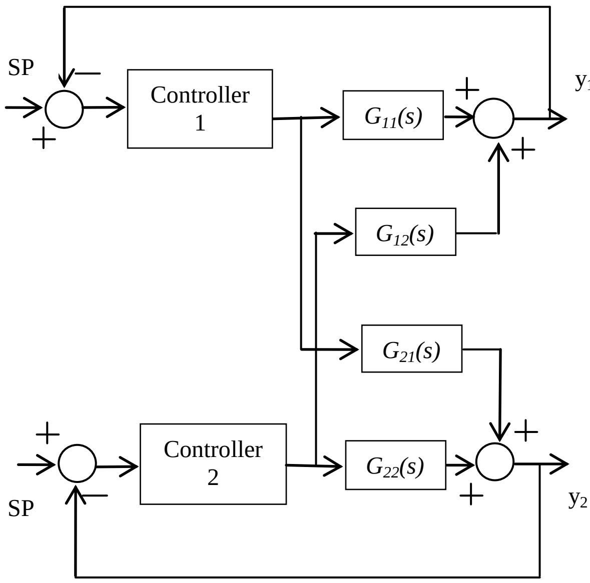

Fig. 2. Block diagram of closed loop TITO process

The main advantage of the RGA method is that it requires only the steady state process gain, which will provide the relation between two manipulated and controlled variables. Hence pairing of variables can be based on the values of relative gain. The elements of RGA matrix λ_{ij} or λ_{ji} between i^{th} or j^{th} manipulated variable and i^{th} or j^{th} controlled variable is calculated as follows,

$$\lambda_{ij} = \frac{\text{Open loop gain between } i^{th} \text{ output}(y_i) \text{ and } j^{th} \text{ manipulated variable } (m_j) \text{ with constant } m_i}{\text{Closed loop gain between } i^{th} \text{ output}(y_i) \text{ and } j^{th} \text{ manipulated variable } (m_j) \text{ with constant } y_j},$$

for $i=1,2$

For a TITO process, the RGA matrix is represented as,

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{12} \end{bmatrix} = \begin{bmatrix} \dfrac{K_{11}K_{22}}{K_{11}K_{22} - K_{12}K_{21}} & \dfrac{-K_{12}K_{21}}{K_{11}K_{22} - K_{12}K_{21}} \\ \dfrac{-K_{12}K_{21}}{K_{11}K_{22} - K_{12}K_{21}} & \dfrac{K_{11}K_{22}}{K_{11}K_{22} - K_{12}K_{21}} \end{bmatrix} \quad (2)$$

Where, K_{ij} is the open loop gain between i^{th} manipulated variable and j^{th} controlled variable.

The transfer function matrix of TITO process is represented as follows,

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} m_1(s) \\ m_2(s) \end{bmatrix} \quad (3)$$

Where, $G_{ij}(s)$ is a transfer function representing a First Order plus Dead Time (FOPDT) model and is given by,

$$G_{i\,j}(s) = \frac{K_{i\,j}\, e^{-Td_{i\,j}\, s}}{\tau_{i\,j} s + 1}, \text{ for } i = 1, 2 \quad (4)$$

With T_{dij} and $\tau_{i\,j}$ are the dead time and time constant of i^{th} manipulated variable and j^{th} controlled variable of the process respectively.

3. Review of sliding mode controller

Consider a FOPDT process represented by the continuous domain transfer function,

$$G(s) = \frac{x(s)}{u(s)} = \frac{K\, e^{-\tau_d\, s}}{\tau\, s + 1} \quad (5)$$

The sliding surface $s(t)$ using integro-differential equation, which acts on the tracking error is [15],

$$s(t) = \left(\frac{d}{dt} + \lambda \right)^n \int_0^t e(t)\, dt \quad (6)$$

Where $e(t)$ is the tracking error, λ is the tuning parameter and n is the order of the process. The parameter λ is obtained by using the relation,

$$\frac{\tau_d + \tau}{\tau_d\, \tau} = 2\lambda \quad (7)$$

In SMC control law, $u(t)$ consists of continuous and discontinuous parts, with the controller output given by,

$$u(t) = u_c(t) + u_d(t) \quad (8)$$

The continuous part of SMC is the function of controlled variable and error signal,

$$u_c(t) = f\left(x(t), e(t)\right) \quad (9)$$

The continuous part of SMC equation is,

$$u_C(t) = \frac{\tau_d\, \tau}{K} \left\{ \lambda^2 e(t) + \frac{x(t)}{\tau_d\, \tau} \right\} \quad (10)$$

The discontinuous part of SMC is given by [16],

$$u_d(t) = K_D \frac{s(t)}{|s(t)| + \delta} \quad (11)$$

Where, K_D is the tuning parameter which depends on reaching mode and δ is a tuning parameter of discontinuous part of SMC. These tuning parameters are determined using the Nelder-Mead tuning equations [17],

$$K_D = \frac{0.51}{|K|} \left(\frac{\tau}{\tau_d} \right)^{0.76} \quad (12)$$

$$\delta = 0.68 + 0.12 |K| K_D * 2 * \lambda \quad (13)$$

The complete controller equation is,

$$u(t) = \frac{\tau_d\, \tau}{K} \left\{ \lambda^2 e(t) + \frac{x(t)}{\tau_d\, \tau} \right\} + K_D \frac{s(t)}{|s(t)| + \delta} \quad (14)$$

4. TITO process models

The TITO process considered in this work, it consists of FOPDT models as the elements of the process transfer function matrix. This work is extended for the study of three categories of TITO processes, based on the ratio (ε) of dead time to time constant in the FOPDT models of the diagonal elements $G_{11}(s)$ and $G_{22}(s)$ of the process transfer function matrix. The first category of the processes selected has this ratio less than one, the second category has the ratio greater than one, and the third category has a ratio less than one in one element and greater than one in the other. Under each category two different TITO models are used to evaluate the performance of the SMC designed.

4.1 TITO models for first category with ε < 1 in both $G_{11}(s)$ and $G_{22}(s)$

The process transfer function matrix of the models selected under this category is,

$$G_{P1}(s) = \begin{bmatrix} \dfrac{2e^{-0.8s}}{s+1} & \dfrac{0.8e^{-0.5s}}{2s+1} \\ \dfrac{1.2e^{-1s}}{0.5s+1} & \dfrac{1.5e^{-0.6s}}{s+1} \end{bmatrix};$$

$$G_{P2}(s) = \begin{bmatrix} \dfrac{2e^{-0.4s}}{s+1} & \dfrac{0.8e^{-0.5s}}{2s+1} \\ \dfrac{1.2e^{-1s}}{0.5s+1} & \dfrac{1.5e^{-0.2s}}{s+1} \end{bmatrix}$$

The ratio of dead time to time constant values of $G_{11}(s)$ is 0.8 and 0.4 in $G_{P1}(s)$ and $G_{P2}(s)$ respectively, and the ratio of $G_{22}(s)$ is 0.6 and 0.2 in $G_{P1}(s)$ and $G_{P2}(s)$ respectively. The RGA matrix of the above process is determined by using Eq. (2) and is given below.

$$\Lambda=\begin{bmatrix} 1.47 & -0.47 \\ -0.47 & 1.47 \end{bmatrix} \quad (15)$$

Since RGA matrix elements are based only on gains, it is same for all categories of models considered. From Eq. (15), it is seen that, the diagonal elements of the RGA matrix are positive. Hence the first controlled variable is to be controlled by adjusting the first manipulated variable, and the second manipulated variable is chosen to control the second controlled variable.

4.2 TITO models for second category with ε > 1 in both G_{11}(s) and G_{22}(s)

The process transfer function matrix of the models selected under this category is,

$$G_{P3}(s)=\begin{bmatrix} \frac{2e^{-1.4s}}{s+1} & \frac{0.8e^{-0.5s}}{2s+1} \\ \frac{1.2e^{-1s}}{0.5s+1} & \frac{1.5e^{-1.2s}}{s+1} \end{bmatrix};$$

$$G_{P4}(s)=\begin{bmatrix} \frac{2e^{-1.8s}}{s+1} & \frac{0.8e^{-0.5s}}{2s+1} \\ \frac{1.2e^{-1s}}{0.5s+1} & \frac{1.5e^{-1.6s}}{s+1} \end{bmatrix}$$

The ratio of dead time to time constant values of $G_{11}(s)$ is 1.4 and 1.8 in $G_{P3}(s)$ and $G_{P4}(s)$ respectively, and the ratio of $G_{22}(s)$ is 1.2 and 1.6 in $G_{P3}(s)$ and $G_{P4}(s)$ respectively. Since the FOPDT gains in $G_{P3}(s)$ and $G_{P4}(s)$ are the same as given in category one, RGA matrix is also the same as given in Eq. (15).

4.3 TITO models for third category with ε > 1 in both G_{11}(s) or G_{22}(s)

For further analysis, the third category is also considered, where the ratio of dead time to time constant in the element $G_{11}(s)$ is less than one and $G_{22}(s)$ is greater than one in $G_{P5}(s)$. In $G_{P6}(s)$, the ratio is greater than one in $G_{11}(s)$ and less than one in $G_{22}(s)$. The process transfer function matrix of the models selected under this category is,

$$G_{P5}(s)=\begin{bmatrix} \frac{2e^{-0.8s}}{s+1} & \frac{0.8e^{-0.5s}}{2s+1} \\ \frac{1.2e^{-1s}}{0.5s+1} & \frac{1.5e^{-1.2s}}{s+1} \end{bmatrix};$$

$$G_{P6}(s)=\begin{bmatrix} \frac{2e^{-1.4s}}{s+1} & \frac{0.8e^{-0.5s}}{2s+1} \\ \frac{1.2e^{-1s}}{0.5s+1} & \frac{1.5e^{-0.6s}}{s+1} \end{bmatrix}$$

The ratio of dead time to time constant values of $G_{11}(s)$ is 0.8 and 1.4 in $G_{P5}(s)$ and $G_{P6}(s)$ respectively, and the ratio of $G_{22}(s)$ is 1.2 and 0.6 in $G_{P5}(s)$ and $G_{P6}(s)$ respectively. The controller parameters of SMC for the above categories are obtained by using Eq. (7) and the Nelder-Mead tuning algorithm given in Eq. (12) and Eq. (13). The controller parameters of the processes in category one, two and three are listed in Table 1, Table 2 and Table 3 respectively.

Table 1
Controller Parameters (Category One)

SMC Settings	$G_{P1}(s)$		$G_{P2}(s)$	
	Loop 1	Loop 2	Loop 1	Loop 2
λ	1.125	1.333	1.75	3
K_D	0.302	0.5013	0.5116	1.155
δ	0.843	0.9206	1.109	1.9274

Table 2
Controller Parameters (Category Two)

SMC Settings	$G_{P1}(s)$		$G_{P2}(s)$	
	Loop 1	Loop 2	Loop 1	Loop 2
λ	0.857	0.916	0.777	0.812
K_D	0.197	0.296	0.163	0.237
δ	0.761	0.777	0.741	0.749

Table 3
Controller Parameters (Category Three)

SMC Settings	$G_{P5}(s)$		$G_{P6}(s)$	
	Loop 1	Loop 2	Loop 1	Loop 2
λ	1.125	0.9167	0.8571	1.333
K_D	0.302	0.296	0.1974	0.5012
δ	0.843	0.777	0.7612	0.9205

4.4 Control TITO process with conventional SMC

The controller parameters of three categories of TITO models are given in Table 1, Table 2 and Table 3 respectively. These parameters are used for obtaining the closed loop responses of TITO processes considered, for unit step change is applied. The closed loop responses of processes $G_{P1}(s)$ and $G_{P2}(s)$ obtained using simulation are shown in Figures 3(a) and 3(b). It can be seen that, the closed loop responses are less oscillatory and has overshoot, with zero steady state errors.

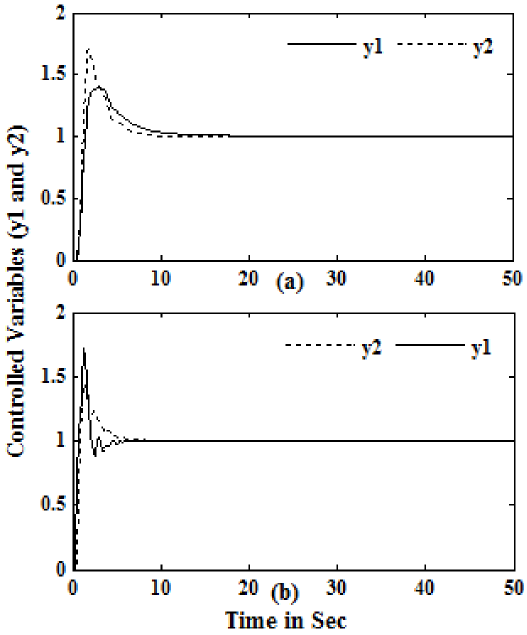

Fig. 3. Servo responses of TITO processes (a) G_{P1} (s) and (b) G_{P2} (s) with SMC

The closed responses of processes $G_{P3}(s)$ and $G_{P4}(s)$ obtained using simulation are shown in Figures 4(a) and 4(b). It is seen that, the closed loop responses are sluggish, with non-zero steady state error.

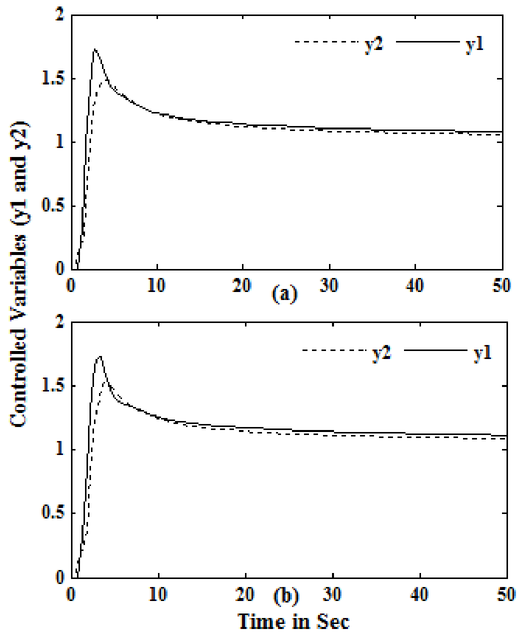

Fig. 4. Servo responses of TITO process (a) G_{P3}(s) and (b) G_{P4}(s) with SMC.

The closed responses of processes $G_{P5}(s)$ and $G_{P6}(s)$ obtained using simulation are shown in Figures 5(a) and 5(b). It is seen that, the closed loop responses are sluggish, with the steady state error approaching non-zero very slowly.

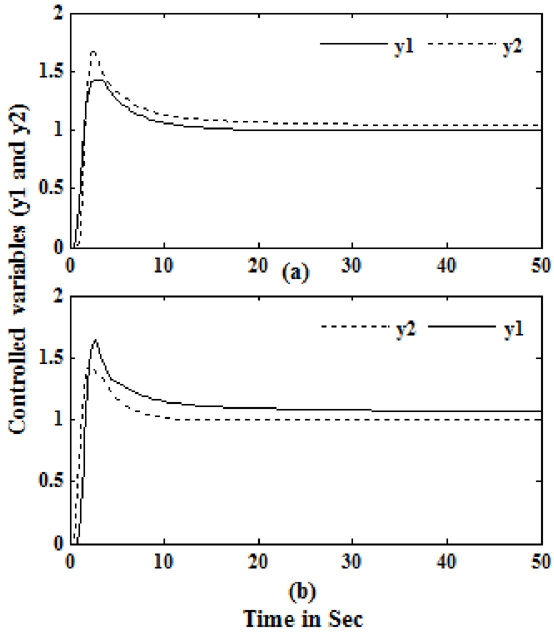

Fig. 5. Servo responses of TITO processes (a) G_{P5}(s) and (b) G_{P6}(s) with SMC.

Figure 4(a) and Figure 4(b) show that the controlled variables y_1 and y_2 has not reached the desired set-point values. This is because $G_{11}(s)$ and $G_{22}(s)$ have ratio ε greater than one in $G_{P3}(s)$ and $G_{P4}(s)$. The servo response of Figure 5(a) shows that, the second controlled variable y_2 has not reached the set point value. This is again because $G_{22}(s)$ has ratio ε greater than one in $G_{P5}(s)$. Figure 5(b) shows that, the first controlled variable y_1 has not reached the desired set point value. This is because in $G_{11}(s)$ the ratio ε is greater than one in $G_{P6}(s)$. If, one of the loop transfer function, either $G_{11}(s)$ or $G_{22}(s)$ has the ratio ε greater than one, that loop's controlled variable is not settling at the set-point value. This shows that the performance of conventional SMC is not effective, if the ratio ε is greater than one in any one of the loop transfer functions $G_{11}(s)$ and $G_{22}(s)$ in process transfer function matrix. The theoretical deliberations and consistent results are confirmed through computer simulations using MATLAB.

5. Proposed optimized sliding mode controller

It is seen that, the closed loop response of the processes in category two and three has poor steady state response. For improving the steady state response, the following modification is proposed in the discontinuous part of the SMC, which has c_i, K_D and δ as the tuning parameters. The discontinuous part of SMC is represented as,

$$u_d(t) = f\left(c_i, K_D, \delta, s(t)\right) \tag{16}$$

The parameter K_D is responsible for reaching mode, and hence increasing the value of K_D will result in improvement in settling of the controlled variable.

Hence increasing K_D value by a weighting factor c_i, will improve the responses. The proposed modification results in the discontinuous part of SMC given by,

$$u_{id}(t) = c_i * K_D \frac{s(t)}{|s(t)| + \delta} \text{, for } i=1,2 \qquad (17)$$

Where c_1 and c_2 are the weighting factors applied in the discontinuous part of the conventional SMC, for first and second loop respectively. Hence the proposed controller is,

$$u_i(t) = \frac{\tau_d \tau}{K} \left\{ \lambda^2 e(t) + \frac{x(t)}{\tau_d \tau} \right\} + c_i * K_D \frac{s(t)}{|s(t)| + \delta} \text{, for } i=1,2 \qquad (18)$$

Primarily evaluating the performance of the above proposed change, the values of c_1 and c_2 are manually selected as 2, and the closed loop responses of category two and three of the TITO process are obtained for the applied unit step change in the input. The responses obtained are given in Figure 6 and Figure 7.

From Figure 6 and Figure 7, it is observed that, the proposed method has improved the steady state performance of the closed loop response, with no significant improvement in the transient response. To further evaluate the effectiveness of the proposed change, the values of c_1 and c_2 are increased as 4 and 6, expecting a further improvement in the response. But, this resulted in an oscillatory response as given in Figure 8 and Figure 9. The responses y_1 and y_2 of $G_{p3}(s)$ are shown in Figures 8(a) and 8(b), and the responses of $G_{p4}(s)$ are shown in Figures 8(c) and 8(d).

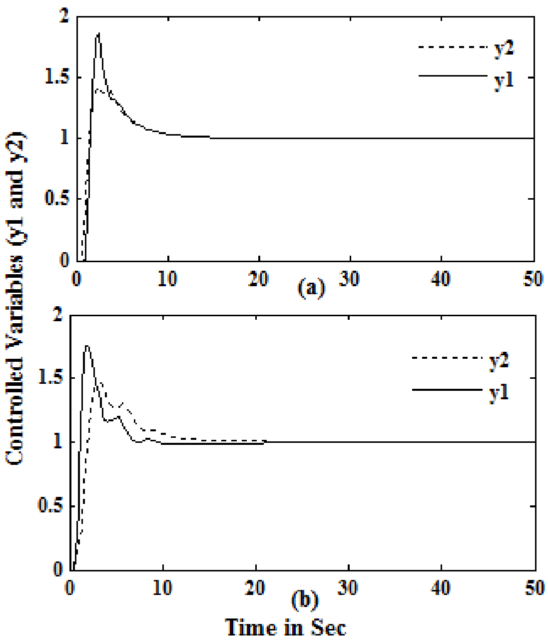

Fig. 6. Servo responses of TITO processes (a) $G_{P3}(s)$ and (b) $G_{P4}(s)$ with proposed SMC ($c_{1,2}$ =2).

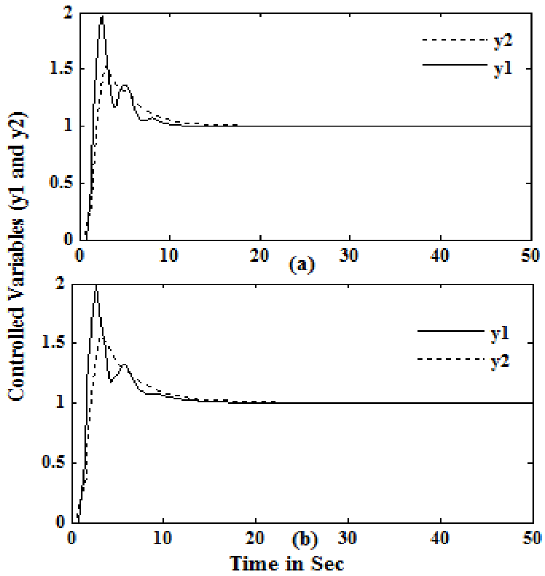

Fig. 7. Servo responses of TITO processes (a) $G_{P5}(s)$ and (b) $G_{P6}(s)$ with SMC ($c_{1,2}$ =2).

From Figures 8(a), 8(b), 8(c), and 8(d), the response obtained with the weighting factors c_1 and c_2 chosen as 4. It is seen that, the controlled variables y_1 and y_2 are oscillating around the set point values. Further increasing c_1 and c_2 to 6, results in increase in the oscillations of y_1 and y_2, which can be seen as continuous lines in Figure 8 (a), (b), (c), and (d). This indicates choosing any higher values of c_1 and c_2 is not going to improve the closed loop response of TITO process controlled using SMC.

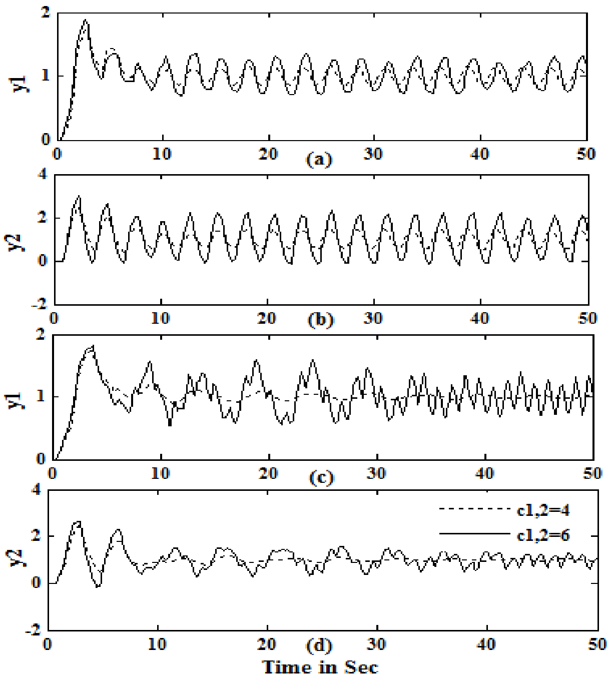

Fig. 8. Servo responses of controlled variables y_1(a, c) and y_2(b, d) of TITO process G_{P3} (s) and G_{P4} (s) with SMC ($c_{1,2}$ = 4, 6).

Figures 9(a) and 9(b) shows that the responses of y_1 and y_2 of $G_{p5}(s)$, when the weighting factors c_1 and c_2 are selected as 4 and 6. Figures 9(c) and 9(d) shows the responses $G_{p6}(s)$ with same weighting factors.

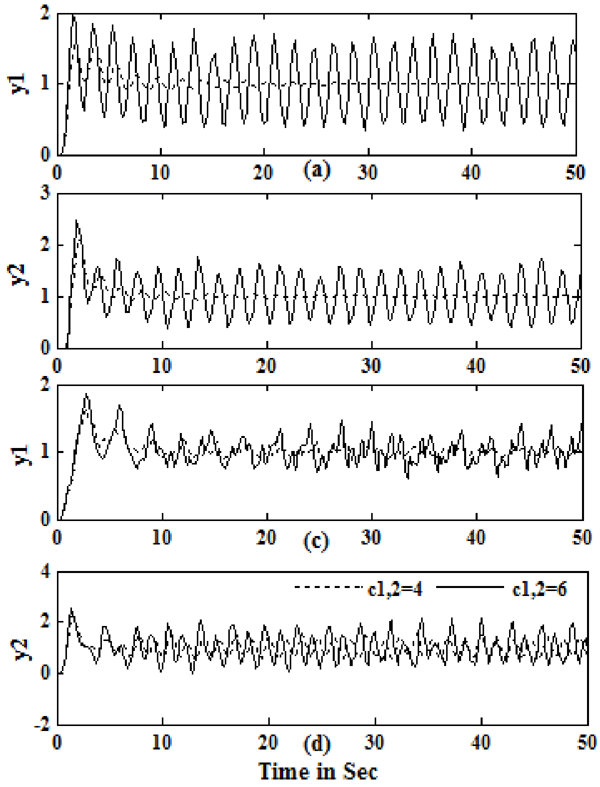

Fig. 9. Servo responses of controlled variables y_1(a, c) and y_2 (b, d) of TITO processesG_{P5}(s) and G_{P6}(s) with SMC ($c_{1,2}$ = 4, 6).

From Figures 9(a), 9(b), 9(c), and 9(d), where the closed loop responses of controlled variables y_1 and y_2 of TITO process (G_{P5} and G_{P6}) are plotted, with the weighting factors c_1 and c_2 are chosen as 4 and 6. As earlier, the responses of y_1 and y_2 are found to be oscillating around the set point values.

Owing to the above results, these weighting factor c_i is to be optimized and obtain the enhanced closed loop response. Genetic Algorithm and Simulated Annealing optimization algorithms are used for obtaining the optimum values of weighting factors c_1 and c_2. These optimum values are obtained with minimization of objective function being chosen as ITAE.

5.1 Genetic Algorithm

Among the many evolutionary algorithms, Genetic algorithm is one of the widely used methods. It is a very appropriate tool for parameters optimization tasks and has delivered good results. In many controller designs, controller parameter tuning is combined with an evolutionary optimization technique. Genetic Algorithm technique, when applied for controller tuning, used for the control of a non-linear MIMO system, resulted in better control performance [18].

5.2 Simulated Annealing

Simulated Annealing is a technique applied to solve combinatorial optimization problems, by minimizing the functions of two or more variables [19]. For controlling processes with time-varying dead time, the PID controller employed for controlling, is tuned using Simulated Annealing optimization technique, which delivered minimized ITAE performance index and overshoot, when compared with other conventional controller tuning methods [20]. Simulated Annealing is one of the best methods to tune a multivariable controller. The centralized cross-coupled PID controller used for the control of a complex system with strong couplings and high nonlinearity, is tuned using Simulated Annealing, which provided better closed loop performance when compared with other evolutionary algorithm [21].

5.3 The Procedure

The following steps are used to obtain the optimized weighting parameters proposed SMC. The optimized weighting factors (c_1 and c_2) are obtained by using ITAE performance index. Theses obtained parameters are used to obtain the closed loop response of TITO systems.

1. Construct a simulation model of TITO system with the controller algorithms in Simulink.
2. Develop a MATLAB m-file with an objective function that computes the ITAE index.
3. Use Optimization Toolbox in MATLAB to minimize the objective function.

Using the Simpson's 1/3 rule for the calculation of ITAE performance index. The developed Simulink model is executed for the each evaluation of the objective function. The first MATLAB/Simulink is to create a TITO simulation model with weighting factor of proposed SMC is presented in Eq. (18).

The first MATLAB file is the gamain.m and prays a functions *ga* and *simulannealbnd* from MATLAB Optimization Toolbox. This file contains time to stop the simulation, sample time and the initial estimations for weighting factors of the proposed SMC controller. The file *gafobs.m* has the MATLAB source code with a call to execute the simulation in Simulink and the implementation of the Simpson's 1/3 rule.

5.4 SMC with optimization of weighting factors (c_1 and c_2)

For tuning using Genetic Algorithm, the value of optimal parameters are obtained by setting the ranges of c_1 and c_2 between 1 to 4, for the maximum optimization time of 100 sec, sampling time of 0.05 sec and for a maximum iteration of 200. In Simulated Annealing, the initial value of an algorithm is considered as [1,1], and the optimum parameters constraints with minimum bound as 1 and the maximum as 4.

The optimum values of c_1 and c_2 are given in Table 4 using Genetic Algorithm and Simulated Annealing respectively. The closed loop performances of y_1 and y_2 for $G_{P3}(s)$ and $G_{P4}(s)$ are shown in Figure 10 and Figure 11 respectively.

Table 4
The optimum parameters c_1 and c_2

Technique	**G_{P3}(s)**		**G_{P4}(s)**	
	c_1	c_2	c_1	c_2
Genetic Algorithm (GA)	2.978	2.507	3.272	2.828
Simulated Annealing (SA)	2.958	2.558	3.317	2.865

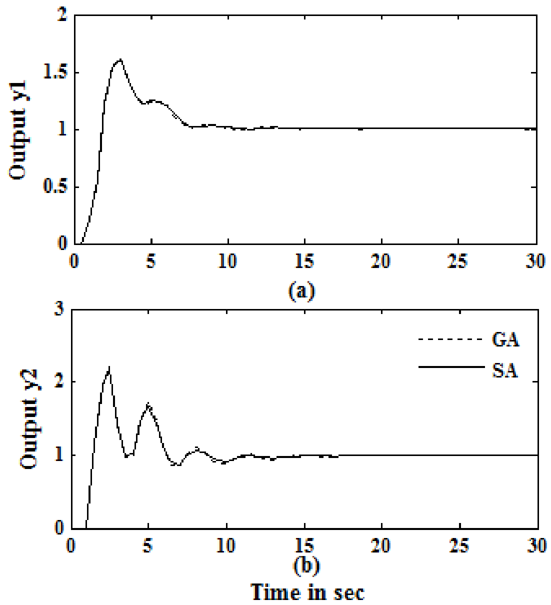

Fig. 10. Closed loop response y_1 and y_2 of G_{P3} with optimized SMC.

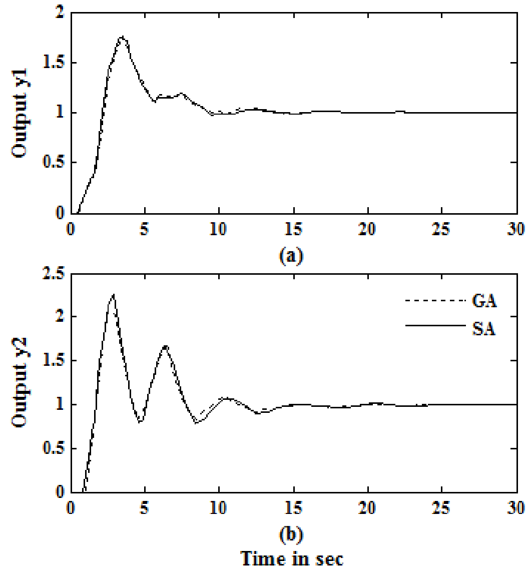

Fig. 11. Closed loop response y_1 and y_2 of G_{P4} with optimized SMC.

The optimum values of c_1 and c_2 are given in Table 5 using Genetic and Simulated Annealing Algorithms. It is seen that, the optimum parameters c_1 and c_2 obtained using the three methods are very closely to each other. The closed loop responses y_1 and y_2 for $G_{P3}(s)$ and $G_{P4}(s)$ are shown in Figure 12 and Figure 13 respectively.

Table 5
The optimum parameters c_1 and c_2

Technique	G_{P5}(s)		G_{P6}(s)	
	c_1	c_2	c_1	c_2
Genetic Algorithm (GA)	1.576	1.911	2.752	1.775
Simulated Annealing (SA)	1.178	1.921	2.683	1.836

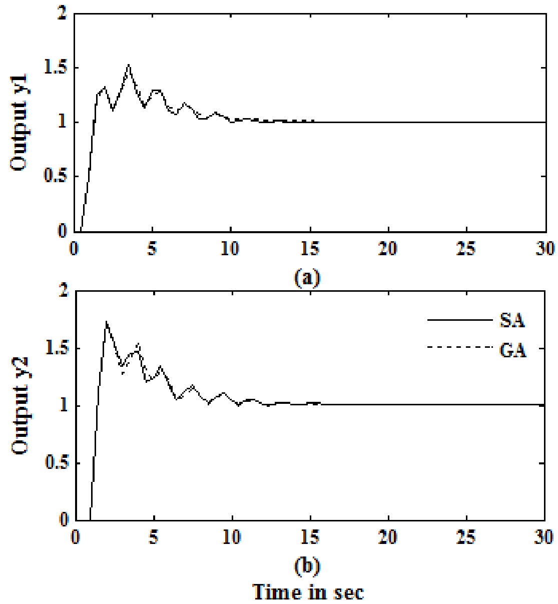

Fig. 12. Closed loop response y_1 and y_2 of G_{P5} with optimized SMC.

From the closed loop responses y_1 and y_2 of $G_{P3}(s)$ and $G_{P4}(s)$, and y_1 of $G_{P5}(s)$ and y_2 of $G_{P6}(s)$ has reached the desired values. Thus an improvement of the closed loop response of TITO processes $G_{P3}(s)$, $G_{P4}(s)$, $G_{P5}(s)$ and $G_{P5}(s)$ is obtained by using the proposed SMC.

From Table 4 and Table 5, it is found that, the optimum values of weighting factors c_1 and c_2 are obtained using three optimization methods are very close to each other. Computer simulation results show that the proposed SMC strategy compensates the unsatisfactory response of TITO system, for the case of ratio (ε) of dead time and time constant is greater than one, but also exhibits improved closed loop performance.

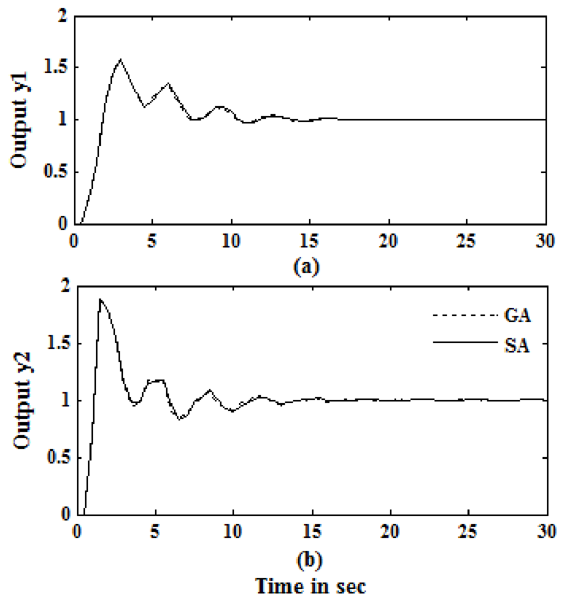

Fig. 13. Closed loop response y_1 and y_2 of G_{P6} with optimized SMC.

5.5 Application of optimized SMC

To further demonstrate the effectiveness of the proposed SMC, it is applied for the control of pilot plant binary distillation column [22], which is a TITO process, whose transfer function model is given below. From the model $G_{PR}(s)$, ratio ε of $G_{11}(s)$ and $G_{22}(s)$ are 1 and 2.47 respectively.

$$G_{PR}(s)=\begin{bmatrix}\frac{-0.16e^{-0.01s}}{0.01s+1} & \frac{0.6e^{-1.19s}}{21.0s+1}\\ \frac{-0.04e^{-0.01s}}{0.02s+1} & \frac{0.49e^{-0.47s}}{0.19s+1}\end{bmatrix}$$

Using Eq. (2) for the TITO process considered, RGA matrix obtained has positive diagonal elements. The process variables are controlled by manipulating the input variables with diagonal controller matrix. Hence the first controlled variable is to be controlled by adjusting the first manipulated variable, and the second manipulated variable is chosen to control the second controlled variable. The TITO processes are controlled by using the controller transfer function given by,

$$G_C(t)=\begin{bmatrix}u_1(t) & 0\\ 0 & u_2(t)\end{bmatrix} \qquad (19)$$

The controller expressions $u_1(t)$ and $u_2(t)$ are obtained using Eq. (18). SMC controller expressions $u_1(t)$ and $u_2(t)$ are derived using Eq. (14). The tuning parameters K_D and δ are obtained using Nelder-Mead tuning method. The output y_1 and y_2 are shown in Figure 14. It can be observed that the output y_1 has reached the desired set-point and output y_2 does not reach the desired set point.

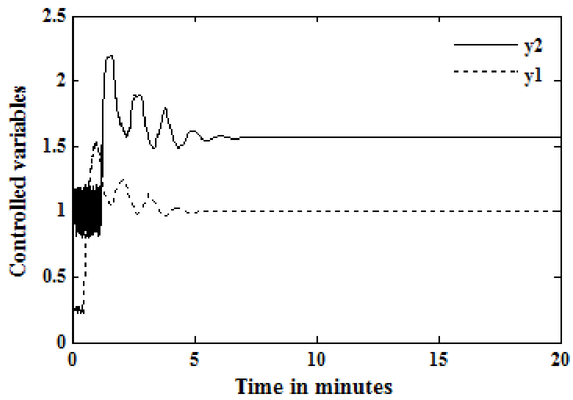

Fig. 14. Responses of pilot plant binary distillation column with conventional SMC

It can be seen that, the closed loop response of the binary distillation column obtained with the conventional SMC is poor. The proposed controller designed with the optimized values of the weighting factors c_1 and c_2, given in Table. 6. Figure 15 shows the responses y_1 and y_2 of binary distillation column, with optimized values of weighting factors in discontinuous part of SMC.

Table 6
The optimum parameters c_1 and c_2

Techniques	$G_{PR}(s)$	
	c_1	c_2
Genetic Algorithm (GA)	2.978	2.507
Simulated Annealing (SA)	2.958	2.558

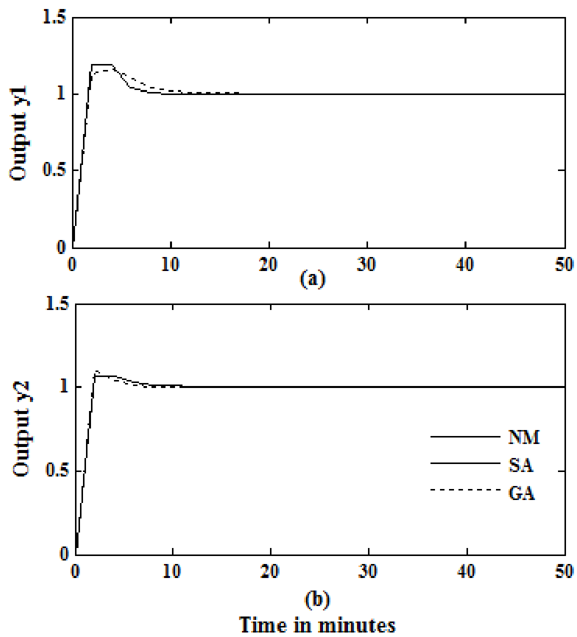

Fig. 15. Responses of pilot plant binary distillation column with optimized SMC

From the above closed loop responses of the binary distillation column, obtained with the proposed SMC, for the applied unit step change in input, it can be noted that, improved closed loop transient and steady state behaviour is obtained, when compared to that of the conventional SMC.

6. Conclusion

In this paper, design of conventional SMC and optimized SMC for the control of Two-Input and Two-output (TITO) processes are presented. Three classes of TITO process models are considered in this work, which differ with the ratio of dead time and time constant of the FOPDT elements of the process transfer function matrix. When these processes are controlled by conventional SMC, unsatisfactory closed loop performances are obtained. Optimized SMC obtained by the inclusion of weighting factor to the discontinuous part of SMC, using Simulated Annealing and Genetic Algorithms, when applied for the control of above three classes of TITO processes, delivered improved closed loop performance. Thus the optimized SMC is superior to the conventional SMC in terms of the closed loop performance.

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