AN ANALYSIS OF MIGRATION MODELS FOR LINEARIZED BIOGEOGRAPHY-BASED OPTIMIZATION APPLIED FOR PID TUNING PROBLEM

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Abstract: *Linearized Biogeography-Based Optimization (LBBO) is a new version of Biogeography-Based Optimization (BBO). BBO is an evolutionary optimization algorithm based on the mathematical model of organism distribution of Biological systems. BBO permits a recombination for the features of candidate solutions (habitats) by means of emigration and immigration. This paper presents a new migration model based on the sigmoid function (S curve) to be one of the nonlinear migration models. This paper also presents an analysis of three linear and three nonlinear different migration models, including the sigmoid model, in LBBO and tests their performance with the non-noisy 23 benchmark functions that have been accepted for 2005 Congress on Evolutionary Computation (CEC). Another test with seven transfer functions is carried out and the performance study explores that sigmoid migration model has the best performance between the different models that will be discussed. The proposed LBBO algorithm with the sigmoid migration function (LBBO-S) had been tested with 23 benchmarks and then compared with the 20 algorithms that have been accepted for 2005 CEC. The proposed algorithm achieved advanced rank between them and it gave better results and lower variance, which proved to have competitive performance with state-of-the-art evolutionary algorithms. An application of the proposed sigmoid model is applied here to tune Proportional Integral Derivative (PID) controller, which is widely used in industrial control systems. Enhancement the performance and ensuring the system stability of an industrial process by tuning PID controller parameters is an important issue. By using Matlab/Simulink and the objective function is chosen to be the squared error integral criteria, LBBO algorithm with the sigmoid migration model is applied to the seven transfer functions, and a comparison with Particle Swarm Optimization (PSO), BBO, and Modified Biogeography-Based Optimization (MBBO) is carried out. The results of the simulation proved that the proposed algorithm (LBBO-S) is an effective tuning method and has better performance compared with other algorithms*

Key words: *Biogeography-Based Optimization (BBO), Evolutionary Algorithm (EA), and Proportional Integral Derivative control.*

1. Introduction.

Biogeography-Based Optimization (BBO) is a relatively newer evolutionary algorithm. Since it was introduced by Dan Simon [1] for the first time, many papers were presented in that area to improve the BBO performance as: Equilibrium Species Counts and Migration Model Tradeoffs for BBO [2], BBO with Blended Migration for Constrained Optimization Problem [3], Linearized BBO with Re-initialization and Local Search [4], and a Modified BBO (MBBO) [5].

BBO has some problems, for example, although most of our real-world problems are non-separable, BBO deals with one independent variable at a time so it is suitable for separable optimization problems. Another version of BBO is introduced by Dan Simon [4] which is called Linearized Biogeography-Based Optimization (LBBO) applies some modifications to the standard BBO as gradient descent, boundary search, re-initialization, and restart. A new migration model in LBBO will be introduced in this paper which is called sigmoid migration model, and a comparison among different migration models will be carried out to examine their performance using the non-noisy 23 benchmarks that have been used in 2005 congress and then it is tested with a set of transfer functions of different orders.

As an application of the proposed sigmoid migration model we apply it to PID parameters tuning problems, which finding the PID control parameters' optimum value is a very difficult task. Most conventional PID tuning techniques require a considerable technical experience to apply those formulas, so PID controller parameters are rarely tuned optimally due to the conventional techniques difficulties. In the past, Ziegler-Nichols rules were used based on open and closed loop tests [6, 7]. Now, intelligent control replaces the conventional techniques, so it became a focus of research such as Artificial Neural Network (ANN) controller, fuzzy controller and evolutionary algorithms based controller [8-11].

Many optimization techniques based on the Evolutionary Algorithm (EA) principle have been used for solving a variety of engineering problems as Ant Colony Optimization (ACO), Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Bacterial Foraging scheme, and fish swarm Algorithm [12-18].

The rest of this paper will be organized as follows: Section 2 reviews Biogeography-Based Optimization. Section3 presents the Linearized Biogeography-Based Optimization. In Section 4 explores six different migration models and compares between them. Section 5 compares our proposed algorithm with the algorithms that have been accepted for CEC2005. Section 6 application and comparison of PSO, BBO, MBBO, and LBBO will be carried out and discussed. Finally, the conclusions are stated in section 7.

2. Biogeography-Based Optimization.

BBO is based on the science of biogeography. Biogeography is a science that deals with the migration of plants and animals between their habitats (islands). Every possible solution in BBO is presented by a habitat. Each habitat has a Habitat Suitability Index (HSI) which it is considered a measure of the solution fitness. A good solution which has high HSI has a good performance in the optimization process, while a habitat with low HSI has a bad performance. Any solution yk has a number of features called a suitability index variable (SIV) such as rainfall, topography, diversity of vegetation, temperature, etc. The problem dimension determines the number of SIV in each solution yk [19].

The offspring generation in BBO is obtained through two main operations. Firstly, the recombination between the solutions in BBO is done by immigration and emigration of the solution features between the different habitats. As an island tends to have high species' count, the species tend to leave the island to share their good features with other habitats. So islands with a good HSI will have high emigration rate μk and low immigration rate λk. A small species count will be on a habitat with low HSI, hence, low emigration rates μk and high immigration rates λk [20]. The second operation is the mutation process which is done with a manner like mutation in Genetic Algorithm.

2.1. Migration

As shown in algorithm 1, the immigration rate λk is used probabilistically to decide whether the solution will immigrate or not.

Algorithm 1. r is a random number ~ U(0,1), λk∈(0,1), and yk,s is the kth candidate solution and sth solution features, s ∈ (1,n) where n is the problem dimension

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If $r < \lambda_k$
  Immigrate to $y_{k,s}$ (*i.e.* $y_{k,s} \longleftarrow y_{j,s}$)
else
  Do not immigrate to $y_{k,s}$
End if

2.2. Mutation

Mutation is done for each variable like mutation in GA as described in algorithm 2.

Algorithm 2. r is a random number ~ U(0,1), probability of mutation $p_m \in (0,1)$, L_s and U_s is the minimum and maximum values of the variables

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If $r < p_m$
  $y_{k,s} \longleftarrow U(L_s, U_s)$
End if

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3. Linearized Biogeography-Based Optimization (LBBO) [4].

Due to limitations of BBO as it changes with one variable at a time in each solution, and it has a weakness of its local search ability, so a gradient descent is being used in LBBO. Several modifications to the original BBO as Migration equation, boundary search, re-initialization, and restart will be discussed.

3.1. LBBO Migration

As in original BBO, the immigration rate λk is used probabilistically to decide whether a solution zk to immigrate or not. The solution z_k is linearly combined with the k emigrating solutions such that z_k moves towards each emigrating solution y_j with an amount that is proportional to its emigration rate μ_j as stated in equation 1:

$$z_k \longleftarrow z_k + \mu_j (y_j - z_k) \quad (1)$$

3.2. Boundary Search

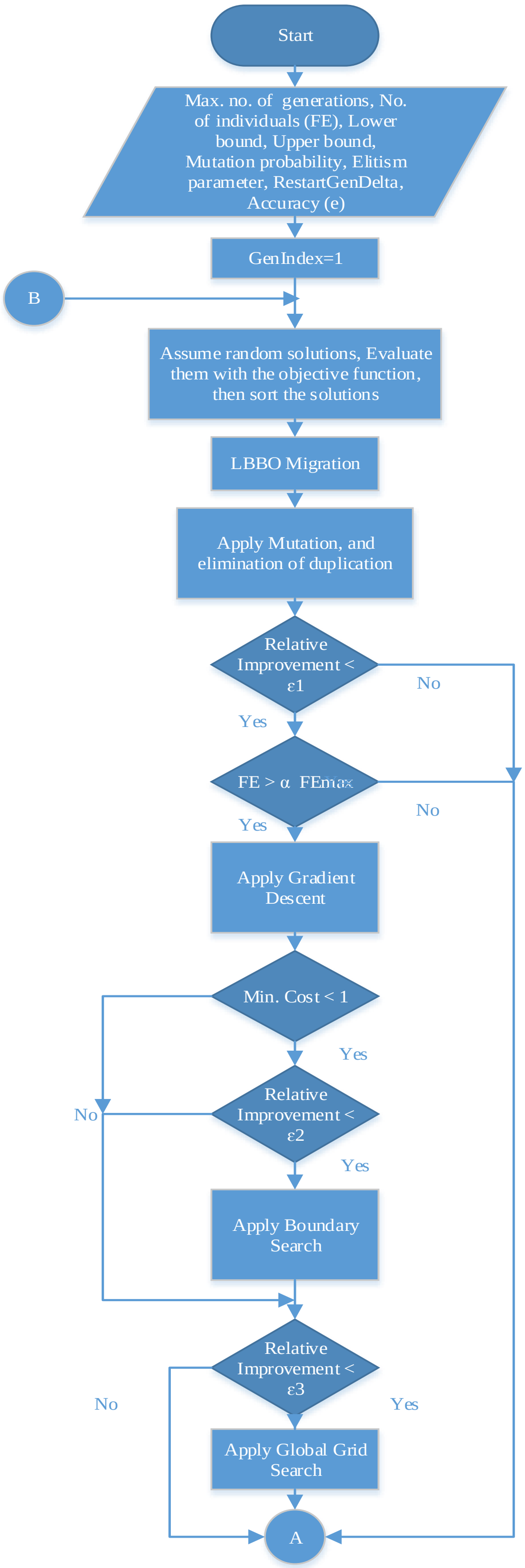
A boundary search is applied as there are many optimization problems having their solution on the boundary of the search space. If the best individual in the population is within a certain threshold of the search space boundary, then it is moved to the search space boundary.

3.3. Re-initialization

The Re-initialization process is performed every Nr (Nr is set typically equal to 1000) function evaluations. A new random individuals N are generated, keeping the best two individuals. This gives us a temporary population size of 2N+2. The best N individuals are then selected out of these 2N+2 individuals for the next generation.

3.4. Restart

If there is no improving in the population, a randomly-generated population will be started, and the entire population is replaced by the new randomly-generated population. The LBBO flow chart is shown in Figure 1. As with standard BBO, elitism is typically used where the best two solutions are kept from a generation to the next.



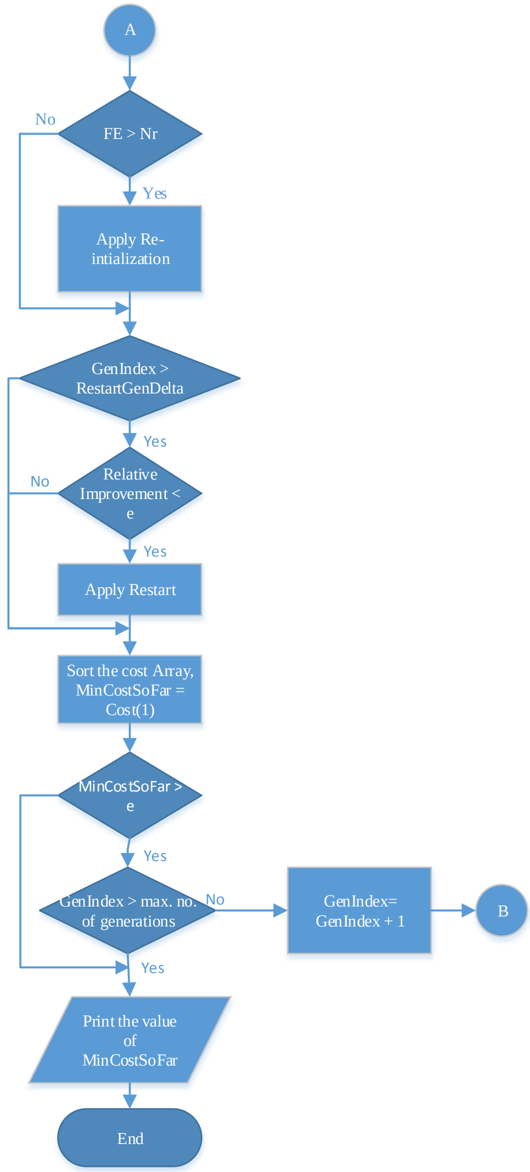


Fig. 1. Linearized Biogeography-Based Optimization Flow Chart.

4. Migration Models.

There are various mathematical migration equations according to the different mathematical models of the biogeography theory [21]. We use here 6 migration models to test the effect of migration curve variation on the optimization performance. Three functions of them are linear equations as shown in Figure 2, while the other three functions are nonlinear. Ko is the number of species at which the equilibrium will occur, E is the maximum possible emigration rate, and I is the maximum possible immigration rate. E and I do not have a remarkable influence on optimization performance [22], so we add both of E and I to be equal one.

4.1. Linear models

The three linear models stated below do not exist actually in biogeography, but they are used due to their simplicity compared with the nonlinear equations. The three models are: Linear immigration rate and constant emigration rate as shown in Figure 2a, constant immigration and linear emigration model as shown in Figure 2b, and both of immigration and emigration are linear as shown in Figure 2c.

- **Model 1: Linear immigration and constant emigration model**

$$\lambda_k = I\left(1-\frac{k}{n}\right) \quad , \quad \mu_k = \frac{E}{2} \qquad (2)$$

From equation 2, we can notice that the emigration rate (μ_k) is independent of number of species k as it is constant, while the immigration rate (λ_k) is changing linearly with the change of species number; if the species number k increases, the λ_k will be decreased.

- **Model 2: Constant immigration and linear emigration model**

$$\lambda_k = \frac{I}{2} \quad , \quad \mu_k = \frac{k}{n}\, E \qquad (3)$$

Equation 3 states that the λ_k is constant and equal to half of I, while the μ_k is changing linearly with the change of species number; if the species number k increases, the μ_k will be increased.

- **Model 3: Linear immigration and emigration models**

$$\lambda_k = I\left(1-\frac{k}{n}\right) \quad , \quad \mu_k = \frac{k}{n}\, E \qquad (4)$$

From equation 4, both the λ_k and μ_k are changing linearly with the change of species number; if the species number k increases, the habitat will be crowded so λ_k will be decreased, while the μ_k will be increased as the species will move to another habitat. Equation 5 was used in the original BBO [1].

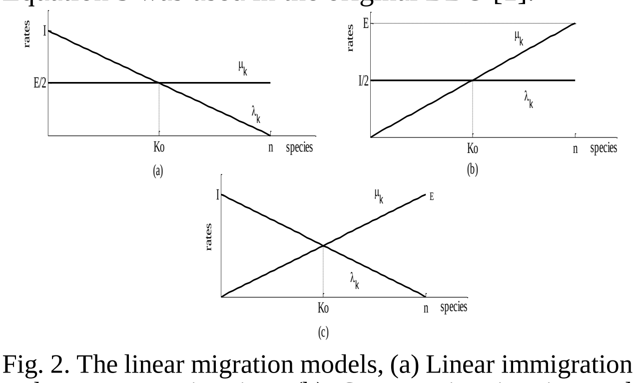


Fig. 2. The linear migration models, (a) Linear immigration and constant emigration, (b) Constant immigration and linear emigration, and (c) Linear immigration and emigration.

4.2. Nonlinear models

The linear models are simpler than the process of migration because a simple change in a part of the system produces complicated changes in the entire system [22]. So three nonlinear migration curves are introduced here, as a quadratic, a sinusoidal, and a sigmoid migration model. The three nonlinear models are shown in Figure 3.

- **Model 4: Quadratic immigration and emigration models**

$$\lambda_k = I\left(1-\frac{k}{n}\right)^2 \quad , \quad \mu_k = \left(\frac{k}{n}\right)^2 E \qquad (5)$$

From equation 5, both the λ_k and μ_k are changing in a quadratic manner, as shown in Figure 3a, with the change of species number; if the island has a small number of species count, the λ_k will be rapidly decreased, while the μ_k will be slowly increased, but if the habitat tends to be filled with species, the λ_k will be slowly decreased while the μ_k will be rapidly increased.

- **Model 5: Sinusoidal immigration and emigration models**

$$\lambda_k = \frac{I}{2}\left(\cos\left(\frac{k\pi}{n}\right)+1\right), \quad \mu_k = \frac{E}{2}\left(\cos\left(\frac{k\pi}{n}+\pi\right)+1\right) \qquad (6)$$

Equation 6 states that both the λ_k and μ_k are sinusoidal functions of the change of species number as shown in Figure 3b; if the habitat has a small or a large number of species count, both of the two rates will be slowly changed, but for a moderate number of species, both of the two rates will be rapidly changed.

- **Model 6: A sigmoid immigration and emigration models**

$$\lambda_k = I\left(1-\frac{1}{1+e^{-a\left(\frac{2k}{n}-1\right)}}\right) \quad , \quad \mu_k = \frac{E}{1-e^{-a\left(\frac{2k}{n}-1\right)}} \qquad (7)$$

From equation 7, both the λ_k and μ_k are sigmoid functions of the change of species number which it has S shape so it is sometimes called S curve, shown in Figure 3c. For the immigration equation, at any new habitat there is a risk from immigration to that place. This may be due to lack of services (*i.e.* lower Habitat Suitability Variables (HSV)), so the immigration is increased slowly, and after reaching a certain value of species number, the HSV will be increased and it will encourage more species to immigrate to that island, and hence the λ_k will be increased rapidly and linearly for intermittent number of species. Finally when the island tends to be filled with species, the λ_k will be increased slowly. The emigration curve is on the contrary of the λ_k behavior. The constant (a) determine the slope and stiffness of the S curve shape; we put its value here to be equal 6.

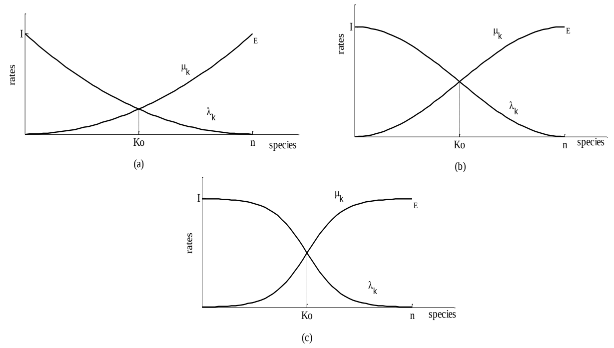


Fig. 3. The nonlinear migration models, (a) Quadratic immigration and emigration, (b) Sinusoidal immigration and emigration, and (c) Sigmoid immigration and emigration.

5. Migration models' comparison

This section consists of two parts. The first part deals with testing the six models with the 23 non-noisy benchmarks that have been introduced in 2005 CEC. The second part deals with testing the six models with seven transfer functions of different orders.

We compare the performance of 23 non-noisy functions that have been accepted for the 2005 Congress of Evolutionary Computation (F_1-F_{25}, excluding F_4 and F_{17}) as F_4 and F_{17} are noisy functions [4]. The following tables are arranged according to the average value of the ranks in the 23 benchmark functions. Table 1 and table 2 are built on the rank of the average values obtained of 25 runs and minimum (best) result of the 25 runs, respectively. The number beside the rounded parentheses is the optimized function value, while the number inside the rounded parentheses is the model rank for that function.

From Table 1, based on the average values obtained from 25 runs, the best model is model 2, while the worst model is model 1. So we can get that the emigration rate (μk) has a higher impact on the optimization performance than the immigration rate (λ_k). Model 5 (sinusoidal) and model 6 (sigmoid) are in the second rank. From Table 2, based on the best value obtained from 25 runs, the best model is model 6 (sigmoid), and model 5 (sinusoidal) is in the second rank. By taking the summation of the average ranks of Table 1 and Table 2, we get that the best one is the sigmoid function, then the sinusoidal and model 2 is in the second place, while model 1 has the worst performance

Table 1

Comparison between the linear and nonlinear models with 23 benchmark functions after 25 Monte Carlo simulations, based on the average values.

Models:	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
F1	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
F2	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
F3	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
F5	0.0035 (3)	0.0035 (3)	0.0033 (2)	0.0055 (5)	0.0056 (6)	0.002 (1)
F6	6.88E-04 (6)	4.03E-05 (2)	6.74E-05 (3)	1.25E-06 (1)	2.40E-04 (4)	4.30E-04 (5)
F7	0.0439 (3)	0.16 (6)	0.1136 (5)	0.091 (4)	0.0337 (2)	0.0108 (1)
F8	20 (1)	20 (1)	20 (1)	20 (1)	20 (1)	20 (1)
F9	8.88E-12 (2)	1.06E-11 (3)	1.44E-11 (5)	1.86E-11 (6)	7.63E-12 (1)	1.27E-11 (4)
F10	29.0698 (6)	14.208 (3)	12.4569 (1)	14.7254 (5)	13.4518 (2)	14.3672 (4)
F11	5.6986 (6)	5.194 (3)	5.142 (2)	4.9995 (1)	5.3579 (5)	5.2768 (4)
F12	0.0263 (6)	0.0102 (4)	6.75E-05 (1)	0.0228 (5)	8.32E-04 (2)	0.0067 (3)
F13	0.3692 (1)	0.4147 (3)	0.4142 (2)	0.4812 (6)	0.4283 (4)	0.4348 (5)
F14	3.4312 (6)	3.4108 (5)	3.3241 (4)	3.2288 (1)	3.2822 (3)	3.2493 (2)
F15	1.57E-12 (3)	2.14E-12 (4)	2.34E-12 (6)	1.29E-12 (2)	2.21E-12 (5)	1.28E-12 (1)
F16	155.4473 (6)	119.3470(1)	124.4194 (2)	135.4592 (5)	129.7935 (3)	135.1532 (4)
F18	707.7532 (2)	640.0447(1)	729.2908 (3)	762.3479 (6)	751.1656 (4)	758.4768 (5)
F19	779.4607 (6)	666.4736(1)	747.3031 (3)	754.1724 (4)	720.6033 (2)	763.2289 (5)

F20	760.5281 (6)	743.2118(4)	644.2817 (1)	711.8090 (2)	746.3468 (5)	715.2827 (3)
F21	436.1320 (1)	476.5827(3)	497.6988 (4)	503.1151 (5)	472.7876 (2)	530.2372 (6)
F22	794.6820 (6)	728.0441(4)	731.6523 (5)	727.1864 (3)	687.6781 (1)	715.6253 (2)
F23	629.5909 (5)	600.5325(3)	641.7982 (6)	613.4627 (4)	597.0560 (2)	590.8697 (1)
F24	219.9607 (6)	204.5532(2)	204.7691 (3)	205.6327 (4)	206.2029 (5)	204.4714 (1)
F25	215.5818 (6)	203.4514(1)	207.3442 (5)	205.1644 (2)	206.2147 (3)	207.1417 (4)
Average rank	3.91304348	2.6086957	2.91304348	3.26086957	2.82608696	2.82608696

Table 2
Comparison between the linear and nonlinear models with 23 benchmark functions after 25 Monte Carlo simulations, based on the best values.

Models:	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
F1	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
F2	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
F3	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)	0 (1)
F5	5.84E-07 (1)	6.57E-07 (4)	7.36E-07 (6)	6.74E-07 (5)	5.89E-07 (2)	6.39E-07 (3)
F6	5.17E-10 (4)	6.35E-10 (5)	2.12E-10 (1)	2.55E-10 (2)	8.54E-10 (6)	3.34E-10 (3)
F7	2.51E-09 (6)	3.88E-10 (4)	9.58E-10 (5)	3.63E-10 (3)	1.62E-10 (1)	1.79E-10 (2)
F8	20 (1)	20 (1)	20 (1)	20 (1)	20 (1)	20 (1)
F9	5.68E-13 (2)	9.09E-13 (3)	1.25E-12 (4)	2.10E-12 (6)	3.41E-13 (1)	1.65E-12 (5)
F10	7.9597 (6)	5.9698 (4)	3.9798 (2)	5.9698 (4)	2.9849 (1)	3.9798 (2)
F11	3.182 (3)	3.3354 (4)	2.7917 (2)	1.7002 (1)	3.5688 (5)	3.8688 (6)
F12	6.39E-10 (6)	2.77E-10 (3)	2.17E-10 (2)	4.65E-10 (5)	3.81E-10 (4)	1.76E-10 (1)
F13	0.1814 (5)	0.1595 (4)	0.1102 (2)	0.2299 (6)	0.09 (1)	0.1401 (3)
F14	2.6399 (6)	2.4264 (5)	2.0457 (1)	2.2926 (4)	2.1506 (2)	2.177 (3)
F15	4.26E-14 (3)	2.84E-14 (2)	5.68E-14 (5)	8.53E-14 (6)	4.26E-14 (3)	0 (1)
F16	130.7916 (6)	107.5881(5)	91.4026 (2)	105.6195 (4)	103.9375 (3)	77.9622 (1)
F18	359.4750 (3)	300 (1)	425.1664 (4)	353.8716 (2)	489.3521 (6)	463.3241 (5)
F19	357.8895 (2)	357.2726(1)	383.6435 (3)	390.3390 (4)	399.5492 (5)	433.3985 (6)
F20	423.1574 (6)	397.9806(5)	353.8609 (3)	300 (1)	357.8806 (4)	300 (1)
F21	200 (1)	200 (1)	200 (1)	300 (6)	200 (1)	200 (1)
F22	772.540 (6)	300 (1)	300 (1)	300 (1)	300 (1)	300 (1)
F23	553.9921 (5)	554.0070(6)	553.9470 (4)	478.0971 (3)	463.0759 (2)	425.1725 (1)
F24	204.9248 (6)	200.7292(4)	200.5771 (3)	200.3649 (1)	200.8344 (5)	200.5497 (2)
F25	202.9860 (6)	200.4006(1)	201.0543 (4)	200.9487 (2)	201.6704 (5)	201.0236 (3)
Average rank	3.78260870	2.9130435	2.56521739	3.04347826	2.69565217	2.34782609

Table 3 shows the seven transfer functions (plants) which will be used in the second test for the six migration models. We change only the migration models, while all other LBBO parameters will not be changed. The selected objective function to be minimized is the Integral of Square Error (ISE) for a step response of a process which is controlled by a PID controller as shown in Figure 4, by tuning the proportional gain (K_p), integral gain (K_i), and differential gain (K_d) using MATLAB/SIMULINK. The PID controller transfer function is given by equation 8 as:

$$G_c(S) = \frac{U(S)}{E(S)} = K_p + \frac{K_i}{S} + K_d S \quad (8)$$

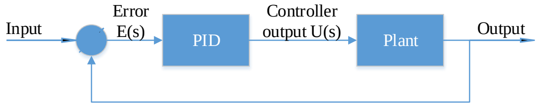


Fig. 4: Block diagram of the seven tested transfer functions

Table 3
Transfer functions that will be used in the second test

Plant Number	Plants' transfer functions
1	$\frac{5}{S^4 + 3S^3 + 7S^2 + 5S}$
2	$\frac{S + 5}{S^4 + 17S^3 + 60S^2 + 10S}$
3	$\frac{300(S + 100)}{S(S + 10)(S + 40)}$
4	$\frac{6}{S^4 + 3S^3 + 4S^2 + 3S + 1}$
5	$\frac{250S + 500}{S^3 + 12S^2 + 100S + 10}$
6	$\frac{S + 5}{S^4 + 17S^3 + 60S^2 + 5S + 5}$
7	$\frac{1}{S^2 + 0.1S + 1}$

The current comparison between the six migration models is split into two main parts as shown in Table 4 and Table 5. From Table 4 we see that model 3 is the best model compared to the other two models for 4 plants (plants 2, 3, 5, and 6) of the seven transfer functions, but model 2 performs the best for two transfer functions (plants 1, 4), and model 1 is the best one only in plant 7 . If we compare between model 1 vs. model 2, model 2 vs. model 3, and model 1 vs. model 3, we can notice that emigration rate has more effect than immigration rate on LBBO performance.
Table 5 summarizes the results of model 3, as it was the best model of the linear migration models, and the three nonlinear models for the seven tested transfer functions. We can notice that model 6 performs better than the other models for 4 transfer functions (plants 2, 4, 5, 7), and model 3 performs the best in two transfer functions (plants 3, 6), while model 4 is the best one only in plant 1. If we compare the six models one by one for the seven transfer functions by giving a high score to the best optimized function value and a low score for the worst, we can find that model 6 is the best and model 5 is in the second place, while model 1 is the worst one at all. The results show that sigmoid model was better than the other models and has the best performance on optimization functions.

Table 4
PID tuned parameters and the optimization function's value for the linear models (best results are shown in boldface)

plant 1	Model 1	Model 2	Model 3
Kp	1.517460827	1.518880692	1.437167572
Ki	0.005972866	0.01145678	0.008686865
Kd	1.358622967	1.297813451	1.162700932
ISE	1.305258063	**1.305009207**	1.305122625
plant 2	Model 1	Model 2	Model 3
Kp	30	30	30
Ki	0	0	0
Kd	19.20623701	19.20542551	19.20623226
ISE	0.653970518	0.653970936	**0.653970518**
plant 3	Model 1	Model 2	Model 3
Kp	21.27602538	24.70688425	21.20804084
Ki	15.46202525	1.895481651	2.24041869
Kd	0.26229168	0.240983173	0.186195418
ISE	0.012093108	0.011603301	**0.011443951**
plant 4	Model 1	Model 2	Model 3
Kp	0.31324168	0.110093094	0.340474119
Ki	0.270471295	0.233891592	0.099351231
Kd	0.576890196	1.942287867	0.526741553
ISE	1.925869526	**1.925358516**	1.925384631
plant 5	Model 1	Model 2	Model 3
Kp	8	7.851594128	10.00850851
Ki	4	4.355789208	26.59184716
Kd	0.129801341	0.13130008	0.192964879
ISE	0.023948409	0.023940106	**0.023938111**
plant 6	Model 1	Model 2	Model 3
Kp	15	15	15
Ki	0.792527769	0.819524869	0.649174145
Kd	14.12555978	13.86057469	13.72593001
ISE	0.915350976	0.915481103	**0.91374047**
plant 7	Model 1	Model 2	Model 3
Kp	10	10	10
Ki	2.12734617	2.414544195	2.414646718
Kd	3.150329035	2.87137054	2.879315506
ISE	**0.302194136**	0.302371172	0.303081096

Table 5
PID tuned parameters and the value of the optimization function for the model 3 and the nonlinear models (best results are shown in boldface)

plant 1	Model 3	Model 4	Model 5	Model 6

Kp	1.437167	1.538309	1.510413	1.587400
Ki	0.008686	0.012148	0.004504	0.005263
Kd	1.162700	1.428397	1.163757	1.312629
ISE	1.305122	**1.305094**	1.305219	1.305162
plant 2	Model 3	Model 4	Model 5	Model 6
Kp	30	30	30	30
Ki	0	0	0	0
Kd	19.20623	19.20762	19.20766	19.20810
ISE	0.653974	0.653973	0.653971	**0.653969**
plant 3	Model 3	Model 4	Model 5	Model 6
Kp	21.20804	21.92717	21.26515	22.65442
Ki	2.240418	1.158579	0.480784	1.865495
Kd	0.186195	0.224218	0.194982	0.234753
ISE	**0.011443**	0.011559	0.011471	0.011610
plant 4	Model 3	Model 4	Model 5	Model 6
Kp	0.340474	0.329289	0.131805	0.060414
Ki	0.099351	0.154206	0.232231	0.090207
Kd	0.526741	0.528091	0.905909	3.139490
ISE	1.925384	1.925350	1.925338	**1.925336**
plant 5	Model 3	Model 4	Model 5	Model 6
Kp	10.00850	10	8.233459	9.989182
Ki	26.59184	24.72926	2.925314	8.430907
Kd	0.192964	0.185724	0.127003	0.154204
ISE	0.023938	0.023938	0.023933	**0.023921**
plant 6	Model 3	Model 4	Model 5	Model 6
Kp	15	15	15	15
Ki	0.649174	0.819523	0.622282	0.792093
Kd	13.72593	13.86057	13.83564	14.12654
ISE	**0.913740**	0.915481	0.913765	0.915352
plant 7	Model 3	Model 4	Model 5	Model 6
Kp	10	10	9.922937	10
Ki	2.414646	2.416684	2.461179	2.140948
Kd	2.879315	2.883789	3.118592	3.150024
ISE	0.303081	0.303041	0.303043	**0.302162**

We compared the performance of 23 non-noisy functions that have been accepted for the 2005 CEC (F_1-F_{25}, excluding F_4 and F_{17}) with LBBO-S. The algorithms that have been introduced in CEC 2005 (in alphabetical order) were:

(ADE) adaptive differential evolution algorithm, (BLX-GL50) two-sex genetic algorithm with unique crossover operators, (BLX-MA) adaptive memetic algorithm, (CMA-GA-PSO) hybrid covariance matrix adaptation, genetic algorithm, and particle swarm optimization, (DE) differential evolution, (CoEVO) co-evolutionary algorithm Differential evolution, (DMS-L-PSO) multi-swarm particle swarm optimization, (EDA) estimation of distribution algorithm, (EvLib) self-adaptive algorithm that combines a variety of EAs, (FEA) flexible evolutionary algorithm, (G-CMA-ES) covariance matrix adaptation evolution strategy, (IPOP-CMA-ES) variant of the CMA-ES that uses a saying population size, (K-PCX) amalgamation of various EA strategies, (L-CMA-ES) another covariance matrix adaptation evolution strategy, (L-SaDE) adaptive differential evolution algorithm, (PLES) parameter-less evolution strategy, (PSO-CMA-ES) hybrid particle swarm optimization, covariance matrix adaptation, and evolution strategy, (RMA) region-based memetic algorithm, (SPC-PNX) continuous genetic algorithm,(STS) combination of scatter search and tabu search

Some of these algorithms do not include data for all of the tested benchmark functions, so we record results that are reported in the previous algorithms. We will not include the algorithms that do not achieve any success in a benchmark function. F_1-F_7, F_9-F_{12} and F_{15} have known solutions, and F_{16}-F_{25} are unsolved. If we get a solution within 10^{-6} of the global minimum, functions F_1-F_5will be considered to be solved, while F_6-F_7, F_9-F_{12}, and F_{15} are considered to be solved if we get 10^{-2} of the global minimum.

The following section split the 23 benchmark functions (F_1-F_{25}, excluding F_4 and F_{17}) to 3 subsets: solved unimodal functions (F_1-F_3, and F_5-F_6), solved multimodal functions (F_7, F_9-F_{12}, and F_{15}) and unsolved multimodal functions (F_8, F_{13}-F_{14}, F_{16}, and F_{18}-F_{25}). For the first and second subsets, we order the performance of the algorithms according to the rate of success, i.e. we will rank the results according to number of success; the highest percentage of success will be at the top, while the 3rd subset will be arranged according to the average of their ranks.

Table 6 shows the 10-dimentional unimodal functions. Success rate shows the percentage of success achieved by getting the global minimum within certain accuracy. The number in the rounded parentheses is the number of success runs of 25 independent runs, while the number beside the rounded parentheses shows the average number of function evaluation divided by the success rate for that benchmark divided by the best CEC 2005 algorithm. For example, K-PCX is the best algorithm for function F_1 as it solved it 25 out of 25 with an average number of function evaluations equal to 1000, also LBBO-S solved it 25 out of 25 independent runs with an average number of function evaluations equal to 1700, so the number outside the rounded parentheses is

$$\frac{1700/(\frac{25}{25})}{1000/(\frac{25}{25})} = 1.7$$

For LBBO-S, functions F_2 and F_3 are lower than one, which means that these algorithms have better performance than the best one of 2005 CEC. If an algorithm does not achieve any success at all, the number inside the square brackets shows its rank based on the average values.

Table 6
Comparison between LBBO-S (shown in bold-italic font) and 16 other algorithms on 5 ten-dimensional unimodal benchmark functions after 25 runs, all the algorithms are sorted from best to worst based on the success rate.

	No. of solved functions	Success rate (%)	F_1	F_2	F_3	F_5	F_6
G-CMA-ES	5	100	1.6 (25)	1 (25)	1 (25)	1 (25)	1.5 (25)
L-CMA-ES	5	100	1.7 (25)	1.7 (25)	1.1 (25)	1 (25)	1.3 (25)
CMA-GA-PSO	5	100	1.7 (25)	1.1 (25)	1.2 (25)	1.2 (25)	1.5 (25)
EDA	5	96	10 (25)	10 (25)	2.5 (23)	4.2 (25)	9.6 (22)
DMS-L-PSO	5	96	12 (25)	12 (25)	1.8 (25)	18.6 (20)	7.7 (25)
DE	5	95	29 (25)	29 (25)	18.5 (20)	6.9 (25)	6.6 (24)
LBBO-S	***5***	***83***	***1.7 (25)***	***0.9 (25)***	***0.4 (25)***	***68.4 (4)***	***1.3 (25)***
BLX-GL50	4	80	19 (25)	19 (25)	[12]	4.7 (25)	7.3 (25)
SPC-PNX	3	60	6.7 (25)	6.7 (25)	[15]	6.8 (25)	[13]
CoEVO	3	60	23 (25)	23 (25)	6.8 (25)	[13]	[12]
PSO-CMS-ES	3	60	21.4 (25)	21.4 (25)	[11]	[16]	11.4 (25)
L-SaDE	5	59	10 (25)	10 (25)	8 (16)	[11]	6.8 (25)
K-PCX	3	58	1 (25)	1 (25)	[10]	[15]	1 (22)
EvLib	3	53	6.7 (25)	6.7 (25)	[13]	[14]	11.5 (16)
PLES	3	41	6 (25)	6 (25)	[16]	166 (1)	[14]
BLX-MA	2	40	12 (25)	12 (25)	[14]	[12]	[11]
FEA	2	38	18.5 (25)	18.5 (23)	[17]	[17]	[15]

Table 7
Comparison between LBBO-S (shown in bold-italic font) and 16 other algorithms on 6 ten-dimensional multimodal benchmark functions after 25 runs, all the algorithms are sorted from best to worst based on the success rate.

	No. of solved functions	Success rate (%)	F_7	F_9	F_{10}	F_{11}	F_{12}	F_{15}
CMA-GA-PSO	6	96	1 (25)	2.6 (25)	1.1 (24)	0.1 (25)	2.1 (25)	2.8 (20)
PSO-CMS-ES	6	75	5.1 (25)	0.4 (25)	0.2 (25)	0.4 (10)	3.4 (25)	27.2 (2)
G-CMA-ES	5	63	1 (25)	4.5 (19)	1.2 (23)	1.4 (6)	4 (22)	[10]
LBBO-S	***4***	***57***	***29.7 (11)***	***0.21 (25)***	***[14]***	***[13]***	***3.64 (24)***	***0.56 (25)***
L-SaDE	4	53	36.2 (6)	1 (25)	[6]	[12]	3.9 (25)	1 (23)
DMS-L-PSO	4	47	126 (4)	2.1 (25)	[5]	[11]	6.6 (19)	1.7 (22)
K-PCX	3	40	[14]	2.9 (24)	1 (22)	[17]	1 (14)	[17]
EvLib	3	33	[17]	0.1 (25)	[15]	[15]	35.3 (5)	1.7 (19)
DE	5	30	255 (2)	10.6 (11)	[19]	1 (12)	8.8 (19)	75.8 (1)
FEA	2	28	[15]	0.5 (25)	[16]	[18]	[16]	0.4 (17)
L-CMA-ES	2	25	1.2 (25)	[17]	[20]	[8]	11.6 (12)	[11]
BLX-GL50	3	17	12.3 (9)	10 (3)	[6]	[7]	12.1 (13)	[16]
BLX-MA	2	15	[13]	5.7 (18)	[9]	[9]	[14]	8.5 (5)
EDA	3	9	404 (1)	[14]	[8]	2.9 (3)	4.3 (10)	[14]
SPC-PNX	2	1	383 (1)	[13]	[10]	5.8 (1)	[15]	[12]
PLES	1	1	[16]	[15]	[17]	[19]	182 (1)	[15]
CoEVO	0	0	[12]	[16]	[18]	[18]	[17]	[13]

Table 8
Comparison between LBBO-S (shown in bold-italic font) and 14 other algorithms on 12 ten-dimensional unsolved multimodal functions after 25 runs, all the algorithms are sorted from best to worst based on the average values.

	Av. Rank	F_8	F_{13}	F_{14}	F_{16}	F_{18}	F_{19}	F_{20}	F_{21}	F_{22}	F_{23}	F_{24}	F_{25}
G-CMA-ES	1.92	[1]	[2]	[4]	[3]	[1]	[2]	[1]	[2]	[1]	[1]	[1]	[4]

LBBO-S	***4.17***	***[1]***	***[1]***	***[1]***	***[9]***	***[2]***	***[1]***	***[2]***	***[11]***	***[2]***	***[12]***	***[7]***	***[1]***
IPOP-CMA-ES	4.50	[9]	[7]	[2]	[1]	[9]	[9]	[8]	[2]	[3]	[1]	[1]	[2]
RMA	5.67	[10]	[6]	[7]	[7]	[8]	[6]	[12]	[2]	[4]	[1]	[1]	[4]
G-CMA-ES	5.92	[5]	[5]	[8]	[2]	[9]	[9]	[8]	[2]	[5]	[1]	[13]	[4]
EDA	6.08	[10]	[15]	[12]	[11]	[4]	[4]	[4]	[2]	[7]	[1]	[1]	[2]
BLX-MA	6.58	[8]	[9]	[4]	[14]	[5]	[5]	[5]	[2]	[10]	[8]	[1]	[8]
L-CMA-ES	7.00	[1]	[4]	[15]	[4]	[6]	[8]	[6]	[1]	[5]	[7]	[15]	[12]
STS	7.25	[7]	[3]	[3]	[10]	[7]	[6]	[7]	[2]	[12]	[9]	[11]	N/A
SPC-PNX	7.58	[10]	[8]	[10]	[6]	[12]	[12]	[11]	[2]	[9]	[1]	[1]	[9]
BLX-GL50	8.33	[15]	[11]	[4]	[8]	[9]	[9]	[8]	[2]	[8]	[10]	[12]	[4]
K-PCX	8.58	[1]	[14]	[14]	[5]	[3]	[3]	[3]	[15]	[15]	[13]	[8]	[9]
DE	11.67	[10]	[10]	[12]	[13]	[13]	[13]	[13]	[12]	[11]	[11]	[9]	[13]
FEA	11.92	[6]	[12]	[9]	[12]	[14]	[14]	[14]	[14]	[13]	[14]	[10]	[11]
CoEVO	13.67	[10]	[13]	[11]	[15]	[15]	[15]	[15]	[13]	[14]	[15]	[14]	[14]

From tables 6, 7, and 8 we can conclude the following points:

1) LBBO-S has a moderate performance for the solved unimodal functions as it ranked 7th out of 17 algorithms.
2) A better performance is obtained in the solved multimodal functions, as LBBO-S ranked 4th out of 17 algorithms.
3) LBBO-S has its best performance for the unsolved multimodal performance as it ranked 2nd out of 15 algorithms.
4) Our proposed algorithm is more suitable for the multimodal problems, especially the difficult high dimension multimodal functions.
5) LBBO-S is better than the best algorithm for functions F_2 and F_3 in the solved unimodal functions, F_9 and F_{15} in the solved multimodal functions and it was the best algorithm for F_8, F_{13}, F_{14}, F_{19}, and F_{25} in the unsolved multimodal functions. It is ranked 2nd in F_{18}, F_{20}, and F_{22}.

6. Application, Simulation, and Discussion

The aim of this part is to apply the LBBO algorithm with sigmoid migration (LBBO-S) and make a comparison among its performance with PSO, BBO, and MBBO. The selected objective function is ISE for a step response which is controlled by a PID controller for the seven plants stated before.

Table 9

PID tuned parameters' values obtained by LBBO-S, MBBO, BBO and PSO

Plant No.	LBBO-S			MBBO			BBO			PSO		
	Kp	Ki	Kd	Kp	Ki	Kd	Kp	Ki	Kd	Kp	Ki	Kd
1	1.587	0.005	1.313	1.38	0.005	1.04	1.003	0	1.00	0.56	0	0.62
2	30	0	19.208	29.68	0	19.07	25.85	0	12.65	4.36	0	14.41
3	22.654	1.865	0.235	24.20	0.99	1.00	24.99	0.286	10.8	0.17	0	0.03
4	0.06	0.09	3.139	0.259	0.11	0.40	0.21	0.1	0.4	0.1	0.48	15.92
5	9.989	8.43	0.154	4.23	3.34	0.129	0.99	1.03	0	0.44	0.18	0.21
6	15	0.792	14.126	14.99	0.82	13.98	13.98	1.00	13.85	3.78	0.12	13.00
7	10	2.14	3.15	6.05	1.11	5.02	4.01	0.97	3.00	0.72	1.11	3.58

Table 10

Parameters' values for LBBO-S, MBBO, BBO, and PSO

Parameter	LBBO	MBBO	BBO	Parameter	PSO
No. of Islands	50	50	50	S: Birds' no. in the Population	100
No. of Generation	25	40	50	Nc: No. of Generation	100
No. of SIVs per Island	3	3	3	n: Search Space's Dimension	3
Mutation Probability	0.005	0.005	0.005	C2: PSO probability	0.12

Table 11

The optimized functions' values obtained by LBBO-S, MBBO, BBO, and PSO (best results is in boldface).

Plant No.	Minimum Cost Function			
	LBBO-S	MBBO	BBO	PSO
1	**1.305**	1.360	1.475	1.952
2	**0.654**	0.664	0.751	2.240
3	**0.012**	0.021	0.216	0.147
4	**1.925**	2.031	2.150	21.760
5	**0.024**	0.031	0.066	0.370
6	**0.915**	0.918	0.951	2.560
7	**0.302**	0.512	0.537	1.533

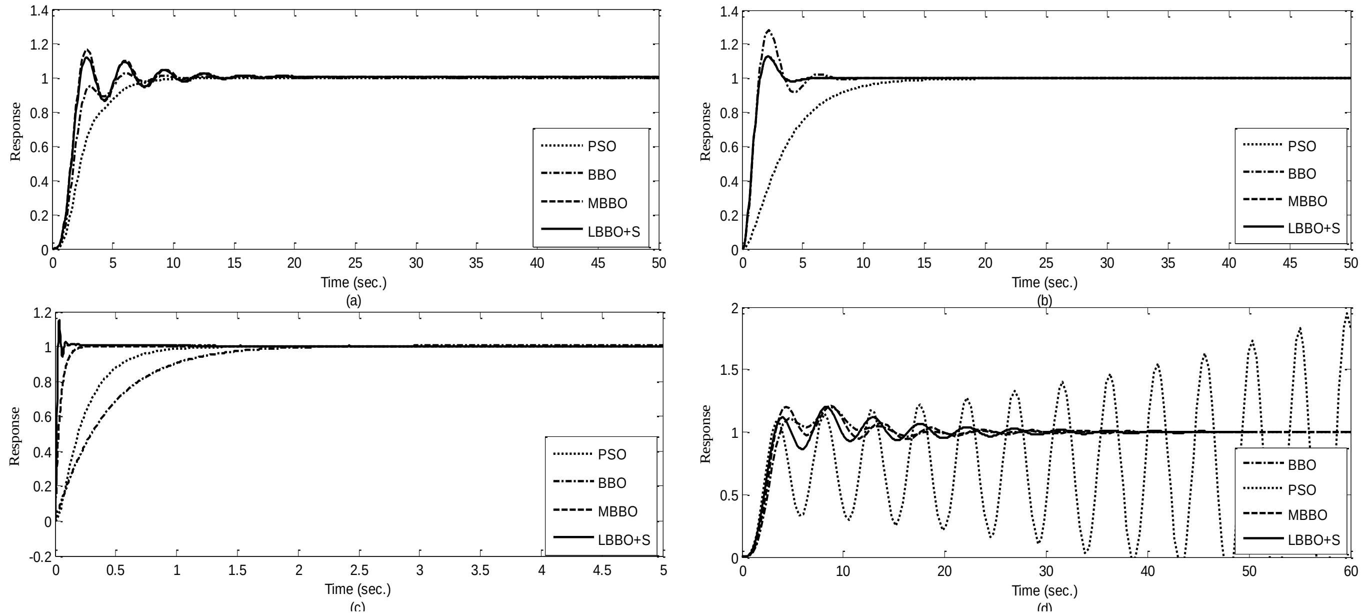


Fig. 5. Step response of the tested plants, (a) plant no. 1, (b) plant no. 2, (c) plant no. 3, and (d) plant no. 4.

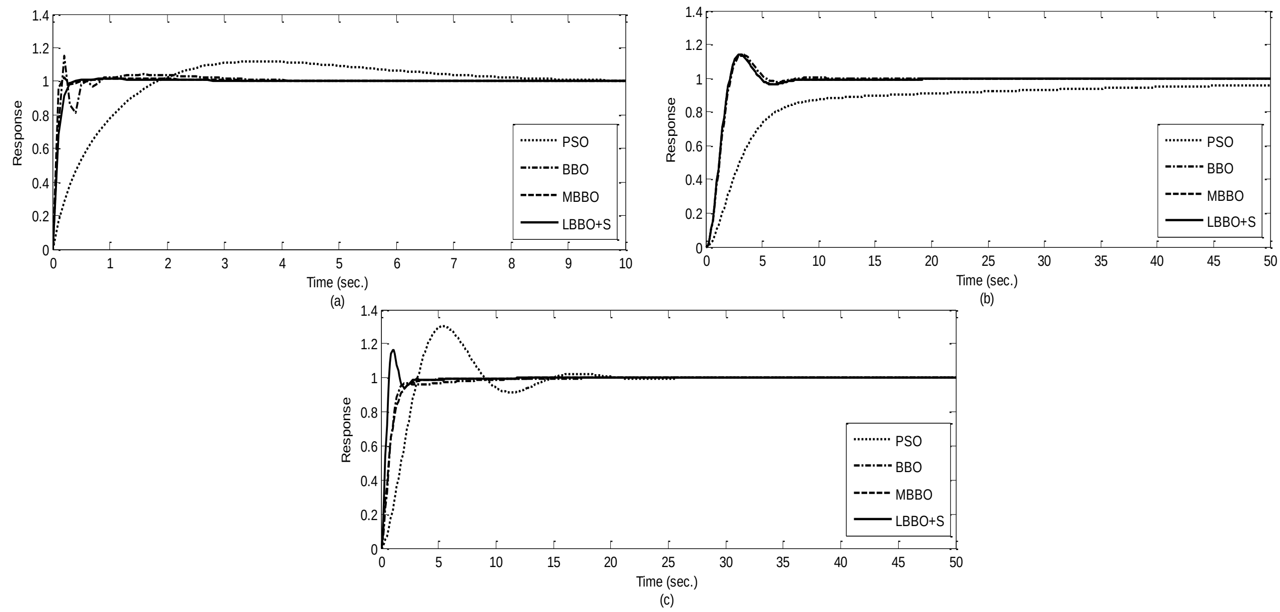


Fig. 6. Step response of the tested plants, (a) plant no. 5, (b) plant no. 6, and (c) plant no. 7.

Figure 5 shows the unit step response for the first 4 tested plants using the four optimization algorithms. For plant no. 1, shown in Figure 5a, we notice that LBBO-S has the lowest settling time, but its overshoot was greater than conventional BBO. Figure 5b shows plant no. 2. Here LBBO-S has the lowest settling time and lowest overshoot while PSO resulted in the plant to be over-damped. Plant no. 3 is shown in Figure 5c. LBBO-S has the fastest settling time but it has an overshoot. Figure 5d shows plant no. 4. We notice that LBBO-S has the lowest settling time, and lowest overshoot. Here PSO made the plant to be unstable. Figure 6 shows the unit step response for the last three tested plants. Figure 6a shows plant no. 5. We notice that LBBO-S performance was the best one as it has the lowest settling time and lowest overshoot. Figure 6b shows plant no. 6. LBBO-S has approximately the same performance of MBBO, with both algorithms having the fastest settling time, and PSO here has a steady state error. Finally, Figure 6c shows the step response of plant 7. LBBO-S has the fastest settling time, but with an overshoot.

7. CONCLUSIONS

Linearized Biogeography-Based Optimization (LBBO) algorithm had been presented. We also explored and analyzed six migration models. To study the effect of those migration models on LBBO performance, twenty-three benchmark functions and seven transfer functions were employed. Simulation results showed that different migration models in LBBO result has a sensible performance. Also LBBO migration models, which are closer to nature (that is, nonlinear), are significantly better than linear models for most of the functions that were examined in this paper. It is also found that the sigmoid function was the best one and the sinusoidal function was in the second rank.

The proposed algorithm LBBO with the sigmoid migration (LBBO-S) had been compared with conventional BBO, MBBO, and PSO. It was clear that the LBBO-S had a better performance than MBBO, BBO, and PSO; as it had the lowest cost value of the seven tested plants, the lowest settling time, the lowest number of oscillations, and the lowest value of overshoot, except plants no. 1, 3 and 7. Also it was the fastest algorithm as it required the lowest number of generations to reach the optimal value.

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