

Adaptive Integral Sliding Mode Control of MIMO systems

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Abstract— This paper deals with asymptotic output tracking algorithm using the adaptive sliding mode control. The integral sliding mode is used to nullifies the matched uncertainties and nonlinearities and also to avoid the reaching phase, so that the manifold is reached at the first time. The output feedback and feedforward adaptive mode is used to enforce the outputs of the uncertain control system to track the outputs of the reference model. The output error between the uncertain system and the reference model vanishes asymptotically despite uncertainties and modeling error. Simulations examples are given to demonstrate the usefulness of the proposed algorithm and a perturbation is added to show the robustness of the command.

Keywords: Lyapunov Stability, Almost Strictly Positive Real, ASPR, Strictly Positive Real, SPR, Model Reference Adaptive Control, integral sliding mode.

I. INTRODUCTION

The simple MRAC of MIMO plants was first proposed by Sobel, Kaufman and Mabius [1]. This class of algorithms requires neither full state access nor satisfaction of the perfect model following conditions. Asymptotic stability is ensured provided that the plant is almost strictly positive real (ASPR). That means, there exist a constant output feedback so that the closed loop system is SPR Barkana [2]. Many researchers extended the original algorithm, to a class of plants which violates this condition. This approach involved designing a supplementary feedforward filter to be included in parallel with the original plant resulting in a new augmented plant which had to satisfy the same strictly positive real condition, unfortunately, the tracking error was not the true difference between the plant and the model outputs since it included the contribution of the supplementary feedforward filter which leads to an asymptotically stable error [3,4,5,6,7,8]. More advances has been done for non-square systems, Where Fradkov in [9] introduced the concept of passification of non square systems using a fictitious output so that the resulting system is square that means the number of outputs and inputs are equal. The same concept has been used by Barkana for linear time varying systems, he called his theory the mitigation condition [10]. In an attempt to extend the concept of output feedback system to nonlinear systems, Fradkov [11] uses the concept of exponential minimum phase systems, and, based on

this concept one can found an adequate output feedback control so that the closed loop control system is exponentially passive. The introduction of sliding mode [12] helps the resolution of the problem of tracking for unmodelled dynamics and external disturbances. Where a manifold is chosen so that once the trajectory of the control system is on this manifold, the dynamic of the system is independent of the nature of the system but depend only by the form of this manifold called sliding surface. So a good choose of this manifold imposes a good dynamic of the control system. To achieve this, a switching control law must be designed to drive the system to this manifold in finite time. One of the drawbacks of the sliding mode control is the sensibility of the system to uncertainties and modeling error in reaching phase and a rapid oscillation of the control law on the sliding surface which must be avoided in real application. The combination of adaptive and sliding mode has been studied by many researchers [13,14,15], where an adequate law has been found for parameter adaptation of the controller for unknown systems, which in the reaching phase, the discontinuous law moves the trajectory of the control system to the sliding manifold and once the trajectory is on the sliding surface, the equivalent control law [12] compensate the matched uncertainties and enforce the system to reach the equilibrium point in finite time. In order to avoid this phase and to let a robustness from the initial time, the concept of integral sliding mode has been introduced [13,14,15]. The integral sliding mode has find many application in industrial process like robots and electromechanical systems, etc, [16,17]. Besides it's advantage like the classical mode, the integral sliding mode can only remove the matched uncertainty once the system is on the sliding surface, that means the unmatched uncertainty still exist. To overcome this matched uncertainty, one add a robust continuous control like H_∞ [18], and L_2 theory [19]. This paper aims to provide a robust controller for a linear system with parameter variation and unmodelled dynamics, the tracking error between the output of the system and the reference model is asymptotically stable and the robustness against perturbation is guaranteed from the initial time. The new is the application of integral sliding mode outside the sliding surface and the simple adaptive control in the sliding surface. The simple adaptive control in this case requires just the output measurement and not the state measurement and the system order can be higher then the model order besides the system parameter are unknown.

The paper is organized as follows. In session II, we give an overview of the integral sliding mode control. In session III we analyses the model reference adaptive control for system with uncertainty and modeling error. The stability proof is investigated in section IV. Simulations results are given in section V, and we finish by a conclusion in section VI

II. INTEGRAL SLIDING MODE CONTROL

The design concept of Integral sliding mode control ISMC is that a discontinuous term is added to the existing feedback controller for a nominal plant model to ensure the desired performance despite parametric uncertainty and external disturbances. The design procedure of ISMC [13] is described briefly as follow.

An actual system with uncertainty conditions is

$$\dot{x}_p = f(x_p) + B(x_p)u + h(x_p, t) \quad (1.1)$$

$$y_p(t) = C_p x_p(t) \quad (1.2)$$

and the ideal closed-loop system is given by

$$\dot{x}_0 = f(x_0) + B(x_0)u_0 \quad (2)$$

where $x_p \in R^n$ is the state vector and $u \in R^m$ is the control input vector, $y \in R^m$ is the output vector, $B(x_p)$ is an appropriate matrix with $\text{rank}(B(x_p)) = m$, $h(x_p, t)$ represents uncertainty conditions such as parameter variations, unmodelled dynamics and external disturbances, and x_0 is the state trajectory of the ideal system under ideal feedback control u_0 . In order to construct the robust control law, we make the following assumptions

Assumption 1

$h(x_p, t)$ is a matched uncertainties, that means

$$h(x_p, t) = B h_1(x_p, t)$$

Assumption 2

The uncertainty $h_1(x_p, t)$ is bounded and is of the form:

$$|h_1(x_p, t)| \leq h_{\max}$$

where h_{\max} is the upper bound which is an known positive constant.

To ensure the sliding surfaces from the initial time instant, that means $s = 0$ for $t = 0$, the control law is given as follow

$$u = u_0 + u_d \quad (3)$$

Where, the first control $u_0 \in R^m$ is to ensure the ideal system trajectory chosen by classic control theory like pole placement, linear quadratic regulator or other methods, and the second control part $u_d \in R^m$ is chosen to reject the perturbation and lead the state of the control system to reach the sliding surface in finite time, and once the state is on the manifold, the control system looks like the ideal one driven by the ideal command. The sliding surface is constructed by two terms. The first term

$s_0(x_p)$ is a function of the state x_p and, in general, it can be chosen to be a linear combination of the state, this first surface is used to construct the second sliding mode by the following manner.

$$s = s_0(x_p) + z \quad (4)$$

with $s, s_0(x_p), z \in R^m$. To enforce the sliding mode, we must

have $\dot{s} = \dot{s} = 0$. The derivation of the variable s is given by:

$$\begin{aligned} \dot{s} &= \dot{s}_0(x_p) + \dot{z} \\ &= \frac{\partial s_0}{\partial x} \left[f(x_p) + B(x_p)u_0 + B(x_p)u_d \right] + \dot{z} \end{aligned} \quad (5)$$

The integral item is defined to meet the requirement

$$\dot{z} = -\frac{\partial s_0}{\partial x} [f(x_p) + B(x_p)u_0], \quad z(0) = -s_0(x_p(0)) \quad (6)$$

With the initial condition $s(t=0) = 0$,

So, equaling \dot{s} to zero, and taking into account that on the sliding mode, one must replace u_d by the equivalent control u_{equ} [12], one gets:

$$\dot{s} = \frac{\partial s_0}{\partial x} [B(x_p)u_{equ} + h(x_p, t)] = \frac{\partial s_0}{\partial x} B(x_p)[u_{equ} + h_1(x_p, t)] = 0$$

Then, the equivalent control is given by

$$u_{equ} = -h_1(x_p, t) \quad (7)$$

The control law u_d is defined to enforce sliding mode along the manifold and is given be,

$$u_d = -M(x_p) \frac{s}{\|s\|} \quad (8)$$

Where $\|(\cdot)\|$ stands for Euclidian norm and $M(x_p)$ is the control gain, selected as constant or varying matrix. In the case where the bound of the uncertainties is unknown, one can choose an adaptive form for $M(x_p)$ which estimates this upper bound. In order to study the stability of the variable s we choose the Lyapunov function candidate as:

$$V = \frac{1}{2} s^T s \quad (9)$$

And the control gain $M(x_p)$ is chosen so that the time derivative of V is negative definite, so the sliding manifold is equal to zero in finite time which is our objective. The time derivative of V along (5) is given by:

$$\dot{V} = s^T \dot{s} = s^T \left\{ \frac{\partial s_0}{\partial x} \left[f(x_p) + B(x_p)u_0 + B(x_p)u_d \right] + \dot{z} \right\} \quad (10)$$

Taking into account that the uncertainty $h(x_p, t)$ is matched (assumption 1), then putting (6) in the relation (10) gives:

$$\dot{V} = s^T \frac{\partial s_0}{\partial x} B(x_p) [u_d + h_1(x_p, t)] \quad (11)$$

If we replace the expression of u_d given by (8), in (11) one gets:

$$\dot{V} = s^T \frac{\partial s_0}{\partial x} B(x_p) \left[-M(x_p) \frac{s}{\|s\|} + h_1(x_p, t) \right] \quad (12)$$

Which can be written as:

$$\dot{V} = -s^T \frac{\partial s_0}{\partial x} B(x_p) M(x_p) \frac{s}{\|s\|} + s^T \frac{\partial s_0}{\partial x} B(x_p) h_1(x_p, t) \quad (13)$$

Let's take $Q = \frac{\partial s_0}{\partial x} B(x_p) M(x_p)$, so (13) becomes:

$$\begin{aligned} \dot{V} &= -s^T Q \frac{s}{\|s\|} + s^T \frac{\partial s_0}{\partial x} B(x_p) h_1(x_p, t), \text{ so} \\ \dot{V} &\leq -\lambda_{\min}(Q) \|s\| + s^T \frac{\partial s_0}{\partial x} B(x_p) h_1(x_p, t) \\ &\leq -\lambda_{\min}(Q) \|s\| + \|s\| \left\| \frac{\partial s_0}{\partial x} B(x_p) \right\| \|h_1(x_p, t)\| \\ &\leq -\lambda_{\min}(Q) \|s\| + \|s\| \left\| \frac{\partial s_0}{\partial x} B(x_p) \right\| h_{\max} \\ &\leq -\lambda_{\min}(Q) + \left\| \frac{\partial s_0}{\partial x} B(x_p) \right\| h_{\max} \leq 0 \end{aligned}$$

So, $\dot{V} \leq 0$, if and only if

$$\lambda_{\min}(Q) \geq \left\| \frac{\partial s_0}{\partial x} B(x_p) \right\| h_{\max} \quad (14)$$

A necessary condition for (14) to be achieved is that $\lambda_{\min}(Q)$ must be positive, so the matrix Q must be positive definite. For the SISO linear system, where $m=1$ and for a choose of the variable $s_0 = Cx$, the relation (14) is verified for $Q = CBM \geq |CB| h_{\max}$. Then, for $CB > 0$, and in order to have a non-increasing Lyapunov function, one must have:

$$M \geq h_{\max} \text{ for } CB > 0 \quad (15)$$

For the MIMO linear system, and with the variable $s_0 = Cx$ and for a large positive $\lambda_{\min}(Q)$, the relation (14) is verified.

Or $Q = \frac{\partial s_0}{\partial x} BM = CBM$ then

$$\lambda_{\min}(Q) \leq \lambda(Q) \leq \sigma_{\max}(Q) = \|Q\| \leq \|CB\| \|M\| = \|CB\| \sigma_{\max}(M)$$

The relation (14) is then verified if $\|CB\| \sigma_{\max}(M) \geq \|CB\| h_{\max}$

Then, for the MIMO linear system, the sliding surface is attractive and the Lyapunov function is negative definite if and only if the control gain M is chosen so that the inequality bellow is verified.

$$\sigma_{\max}(M) \geq h_{\max} \text{ with } \lambda_{\min}(Q) = \lambda_{\min}(CBM) > 0 \quad (16)$$

So, the control law u_d (8) leads the system's state to the sliding surface. One simple choose of M is $M = MI_{m \times m}$, M is a positive scalar. The inequality (16) becomes $M \geq h_{\max}$ with $M \lambda_{\min}(CB) > 0$, so the Lyapunov function is negative definite if and only if

$$M \geq h_{\max} \text{ and } CB > 0 \quad (17)$$

III. DIRECT MODEL REFERENCE ADAPTIVE CONTROL

The linear time invariant model reference adaptive control is considered for the plant:

$$\dot{x}_p(t) = A_p x_p(t) + B_p(t) u_p(t) \quad (18)$$

$$y_p(t) = C_p x_p(t)$$

where $x_p(t)$ is the $(n \times 1)$ state vector, $u_p(t)$ is the $(m \times 1)$ control vector, $y_p(t)$ is the $(q \times 1)$ plant output A_p , B_p are matrices with appropriate dimensions. The range of the plant parameters is assumed to be known and bounded with

$$a_{-ij} \leq a_p(i, j) \leq \bar{a}_{ij}, i, j = 1, \dots, n \quad (19)$$

$$b_{-ij} \leq b_p(i, j) \leq \bar{b}_{ij}, i, j = 1, \dots, n \quad (20)$$

The objective is to find, without explicit knowledge of A_p and B_p , the control $u_p(t)$ such that the plant output vector $y_p(t)$ follows the reference model

$$\dot{x}_m(t) = A_m x_m(t) + B_m(t) u_m \quad (21)$$

$$y_m(t) = C_m x_m(t)$$

where $x_m(t)$ is the $(n_m \times 1)$ state vector, u_m is the $(q \times 1)$ control vector, $y_p(t)$ is the $(q \times 1)$ plant output. The output y_m is the desired response to the set point command u_m . The model incorporates the desired behavior of the plant, but its choice is not restricted. In particular, the order of the plant may be much larger than the order of the reference model. The ideal control law that generates perfect output tracking and ideal state trajectories is assumed to be a linear combination of the model states and model input, i.e. [20,21].

$$\begin{bmatrix} x_p^* \\ u_p^* \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} x_m(t) \\ u_m(t) \end{bmatrix} \quad (22)$$

Where the S_{ij} matrices satisfy

$$\begin{aligned} S_{11} A_m &= A_p S_{11} + B_p S_{21} \\ S_{11} B_m &= A_p S_{12} + B_p S_{22} \\ C_m &= C_p S_{11}; \\ C_p S_{12} &= 0 \end{aligned} \quad (23)$$

Then the adaptive control law based on the command generator tracker (CGT) approach is given as: [6,21]

$$u_p(t) = K_e(t)e_y(t) + K_x(t)x_m(t) + K_u(t)u_m(t) \quad (24)$$

Where $e_y(t) = y_m(t) - y_p(t)$

and $K_e(t)$, $K_x(t)$ and $K_u(t)$ are adaptive gains and concatenated into the matrix $K(t)$ as:

$$K(t) = [K_e(t) \ K_x(t) \ K_u(t)] \quad (25)$$

Defining the vector $r(t)(n_r \times 1)$ where $n_r = q + n_m + m$ as:

$$r(t) = [(y_m(t) - y_p(t))^T \ x_m^T(t) \ u_m^T(t)]^T \quad (26)$$

The control $u_p(t)$ is written in a compact form as

$$u_p(t) = K(t)r(t) \quad (27)$$

where $K(t) = K_p(t) + K_i(t)$ (28)

$$K_p(t) = [y_m(t) - y_p(t)]r^T(t)T_p, T_p \geq 0 \quad (29)$$

$$\dot{K}_I(t) = [y_m(t) - y_p(t)]r^T(t)T_i, T_i = T_i^T > 0 \quad (30)$$

In the case where there is an output perturbation like noise measurement, one can use an modified version of (30) called the σ modification and given by :

$$\dot{K}_I(t) = [y_m(t) - y_p(t)]r^T(t)T_i - \sigma K_I(t)$$

σ is a positive scalar

IV. STUDY OF THE STABILITY

The first step of the demonstration is to design a positive definite quadratic form in the state variable $e_x(t) = x_p^*(t) - x_p(t)$ and $K_I(t)$ of the adaptive system.

Before doing this, it is assumed that T_i^{-1} is a symmetric positive definite matrix. Then an appropriate choice of the Lyapunov function is:

$$V = e_x^T P e_x + Tr \left[S(K_I - \tilde{K})T_i^{-1}(K_I - \tilde{K})^T S^T \right] \quad (31)$$

where Tr : represents the trace of a matrix

Its time derivative is given by:

$$\dot{V} = \dot{e}_x^T P e_x + e_x^T P \dot{e}_x + 2Tr \left[S(K_I - \tilde{K})T_i^{-1} \dot{K}_I^T S^T \right] \quad (32)$$

Where P is a symmetric positive definite matrix of size

$n \times n$, \tilde{K} is a matrix of dimension $m \times n_r$ and S is a non-singular matrix of dimension $m \times m$.

since the matrix \tilde{K} appears only in the function V and not in the control algorithm, it is called fictitious gain matrix, it has the same dimension as K where

$$\tilde{K}r = \tilde{K}_e C_p e_x + \tilde{K}_x x_m + \tilde{K}_u u_m \quad (33)$$

And the three gains \tilde{K}_e , \tilde{K}_x and \tilde{K}_u are as \tilde{K} fictitious.

The equation of the error e_x to given by:

$$\begin{aligned} \dot{e}_x &= A_p x_p^* + B_p u_p^* - A_p x_p - B_p u_p \\ &= A_p [x_p^* - x_p] + B_p [u_p^* - u_p] \\ &= A_p e_x + B_p [u_p^* - u_p] \end{aligned} \quad (34)$$

Substituting u_p^* from (22) and u_p from (24), one gets:

$$\dot{e}_x = A_p e_x + B_p [S_{21}x_m + S_{22}u_m - K_e C_p e_x - K_x x_m - K_u u_m] \quad (35.a)$$

$$\dot{e}_x = A_p e_x + B_p [S_{21}x_m + S_{22}u_m - K_I r - C_p e_x r^T T_p r] \quad (35.b)$$

Then the adaptive system is described by:

$$\dot{e}_x = A_p e_x + B_p [S_{21}x_m + S_{22}u_m - K_I r - C_p e_x r^T T_p r] \quad (36)$$

$$\text{or} \quad \dot{K}_I = C_p e_x r^T T_i \quad (37)$$

Substituting (36), (37) in (32), one gets:

$$\begin{aligned} \dot{V} &= [A_p e_x + B_p (S_{21}x_m + S_{22}u_m - K_I r - C_p e_x r^T T_p r)]^T P e_x \\ &+ e_x^T P [A_p e_x + B_p (S_{21}x_m + S_{22}u_m - K_I r - C_p e_x r^T T_p r)] + \\ &2Tr \left[S(K_I - \tilde{K})T_i^{-1} (C_p e_x r^T T_i)^T S^T \right] \end{aligned} \quad (38)$$

We can write it as:

$$\begin{aligned} \dot{V} &= e_x^T A_p^T P e_x + (x_m^T S_{21}^T B_p^T + u_m^T S_{22}^T B_p^T - r^T K_I^T B_p^T \\ &- r^T T_p^T r e_x^T C_p^T B_p^T) P e_x + e_x^T P A_p e_x + e_x^T P B_p (S_{21}x_m + \\ &S_{22}u_m - K_I r - C_p e_x r^T T_p r) + \\ &2Tr \left[S(K_I - \tilde{K})T_i^{-1} T_i^T r e_x^T C_p^T S^T \right] \end{aligned} \quad (39)$$

Knowing that for two vectors $U(l,1)$ and $V(1,l)$ then $Tr[UV] = V.U$

Therefore

$$\begin{aligned}\dot{V} = & e_x^T (PA_p + A_p^T P)e_x + e_x^T PB_p S_{21}x_m + e_x^T PB_p S_{22}u_m \\ & - e_x^T PB_p K_I r - e_x^T PB_p C_p e_x r^T T_p r + x_m^T S_{21}^T B_p^T P e_x + \\ & u_m^T S_{22}^T B_p^T P e_x - r^T K_I^T B_p^T P e_x - r^T T_p^T r e_x^T C_p^T B_p^T P e_x \\ & + 2e_x^T C_p^T S^T (K_I - \tilde{K})r\end{aligned}\quad (40.a)$$

That means

$$\begin{aligned}\dot{V} = & e_x^T (PA_p + A_p^T P)e_x + 2e_x^T PB_p (S_{21}x_m + S_{22}u_m) \\ & - 2e_x^T PB_p C_p e_x r^T T_p r + 2e_x^T [C_p^T S^T S - PB_p] K_I r \\ & - 2e_x^T C_p^T S^T \tilde{K} r\end{aligned}\quad (40.b)$$

By setting :

$$C_p = GB_p^T P \quad \forall A_p, B_p \quad \text{and} \quad G = (S^T S)^{-1} \quad (41)$$

The derivative of the Lyapunov function becomes:

$$\begin{aligned}\dot{V} = & e_x^T (PA_p + A_p^T P)e_x + 2e_x^T PB_p (S_{21}x_m + S_{22}u_m) \\ & - 2e_x^T PB_p (S^T S)^{-1} B_p^T P e_x r^T T_p r - 2e_x^T C_p^T S^T \tilde{K} r\end{aligned}\quad (41)$$

Substituting $\tilde{K} r = \tilde{K}_e C_p e_x + \tilde{K}_x x_m + \tilde{K}_u u_m$ in the previous equation, one gets:

$$\begin{aligned}\dot{V} = & e_x^T \left[P(A_p - B_p \tilde{K}_e C_p) + (A_p - B_p \tilde{K}_e C_p)^T P \right] e_x \\ & - 2e_x^T PB_p (S^T S)^{-1} B_p^T P e_x r^T T_p r + \\ & 2e_x^T PB_p \left[(S_{21} - \tilde{K}_x)x_m + (S_{22} - \tilde{K}_u)u_m \right]\end{aligned}\quad (43)$$

Thus, if we set $\left[(S_{21} - \tilde{K}_x)x_m + (S_{22} - \tilde{K}_u)u_m \right] = 0$, or

$\tilde{K}_x = S_{21}$ and $\tilde{K}_u = S_{22}$ (none of which is required for implementation), the derivative of V becomes:

$$\begin{aligned}\dot{V} = & e_x^T \left[P(A_p - B_p \tilde{K}_e C_p) + (A_p - B_p \tilde{K}_e C_p)^T P \right] e_x \\ & - 2e_x^T PB_p (S^T S)^{-1} B_p^T P e_x r^T T_p r\end{aligned}\quad (44)$$

This derivative consists of two terms. If T_p it is a positive semi-definite matrix and $PB_p = C^T$ for some $P = P^T > 0$,

then the second term is negative semi-definite in e_x^T . Meanwhile, If the first term is negative semi-definite in e_x^T ,

then the derivative of the Lyapunov function is negative definite in e_x^T . Applying the Lasale theorem, the equilibrium

point is given by $\dot{V}(t) = 0$, so that $e_x(t)$ goes asymptotically to zero. This means that the output of the system approaches asymptotically the output of the reference model. And since $V(e_x(t), K_I(t))$ is non-increasing function, the vector $K_I(t)$ is bounded. We can conclude that the adaptive control is stable if there exist a $P = P^T > 0$ and a $Q = Q^T \geq 0$ so that

$$\begin{aligned}\left[P(A_p - B_p \tilde{K}_e C_p) + (A_p - B_p \tilde{K}_e C_p)^T P \right] &= -Q \quad (44) \\ PB_p &= C^T \quad T_p \geq 0, \quad T_i \geq 0\end{aligned}$$

These relations implies that the feedback system is SPR, so the original system is called ASPR.

V. SIMULATION

Before the simulation, one note that outside the sliding surface the system is described by equation (1) with an additive perturbation and on the sliding surface, the system is given by equation (18) that means without perturbation. So, we must impose that the system goes to the sliding surface that means command's robustness against added perturbation. In the simulation, it is required that the uncertain system track the reference model, which is given by:

$$\dot{x} = Ax + B(u + p(t)), \quad y = Cx$$

$$\dot{x}_m = A_m x_m + B_m u_m, \quad y_m = C_m x_m$$

The matrix A, B are unknown but constants, $p(t)$ is a perturbation with known bound, that means $p(t) \leq \rho$. For the purpose of simulation, the matrix A, B and C are given by:

$$A = \begin{bmatrix} 0 & 1 \\ -0.0009 & -0.0595 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [-0.0431 \quad 1.25]$$

the matrix A_m, B_m and C_m are given by $A_m = -1, B_m = 1, C_m = 1$ so that the transfer function of the reference model is given by $G_m(s) = \frac{1}{s+1}$ which incorporated all the desired performance. The input to the reference model is a square wave of amplitude $u_m = \pm 1$ and period of 120 seconds. The perturbation is selected to be $p(t) = \sin(t)$. The initial condition is taken as

$x_0 = [1 \ 0]^T, x_{m0} = 0.5$. According to (29) and (30), we select $T_p = T_i = 0.1I_3$

Case 1 without sliding mode

In this case, the control input is just the adaptive one given by (27). Figure 1 shows the output of the system and model and we see that the output is stable but it does not track the reference model this is because of the perturbation that the adaptive command can't cancel. Figure 2 depicts the control input when we see the effort of the adaptive command to maintain a good tracking. The figure 3 represents the evolution of the three adaptive gains K_e , K_x and K_u used to construct the adaptive command see (25), (26) and (27).

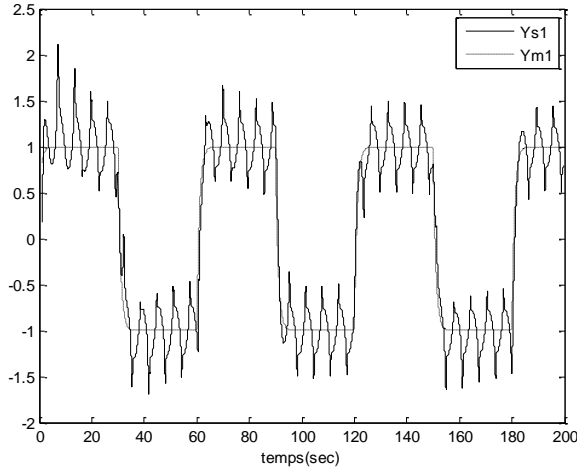


Fig 1 Output of the system and the model without sliding mode

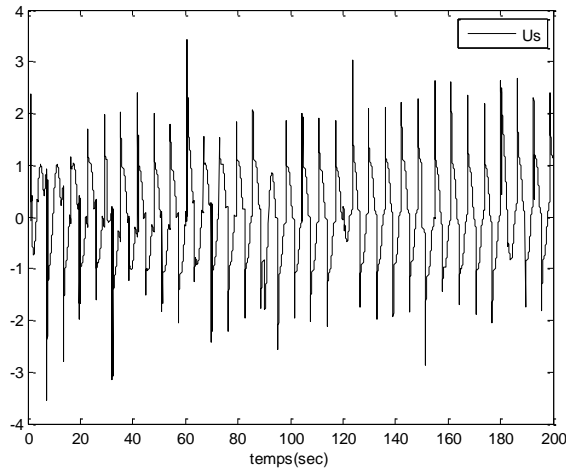


Fig 2. Control input without sliding mode

Case 2 with sliding mode

In this case, the control input is hybrid that means constructed by the adaptive and sliding mode. The adaptive input has been used before and given by equation (27), and we have seen that it does not give a good result in view of a bad tracking between output of the system and the model (see figure 1). In order to overcome this drawback, a sliding mode has been added to the adaptive command. This sliding command is given by equation (8). So, the final hybrid command applied to our system is given by:

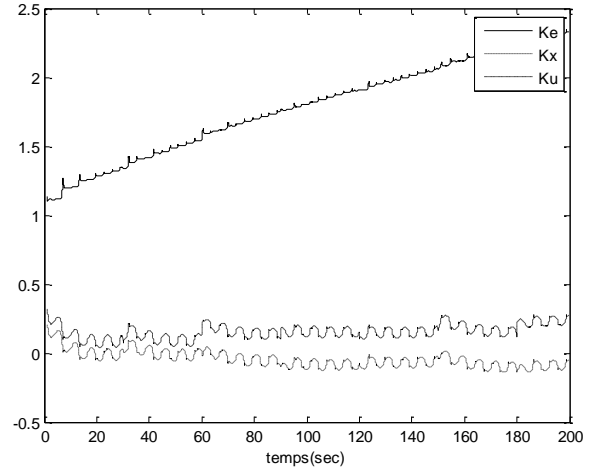


Fig 3. The gains K_e , K_x and K_u without sliding mode

$$u_p(t) = K(t)r(t) + u_d = K(t)r(t) - M \frac{s}{\|s\|}$$

That means, we have used (27) and (8). The gain M is chosen according to (17) and for a perturbation $p(t) = \sin(t)$, one select M to be equal to 1.5.

Figure 4 shows the output of the system and model and we see a good tracking where the error vanishes at about 10s. We see also the effect of the sliding mode input that have been added in comparison with figure 1. In figure 5 we see the chattering phenomena which is inherent to sliding mode which also represent a drawback and we must think to add a boundary layer to avoid this chattering. Figure 6 represents the three adaptive gains K_e , K_x and K_u and we see that these gain converge to a fixed value.

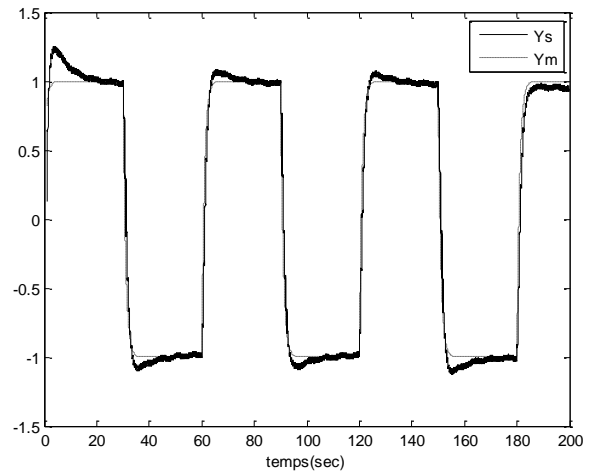


Fig 4 Output of the system and the model with sliding mode

VI CONCLUSION

This paper presents at first the adaptive command applied to a

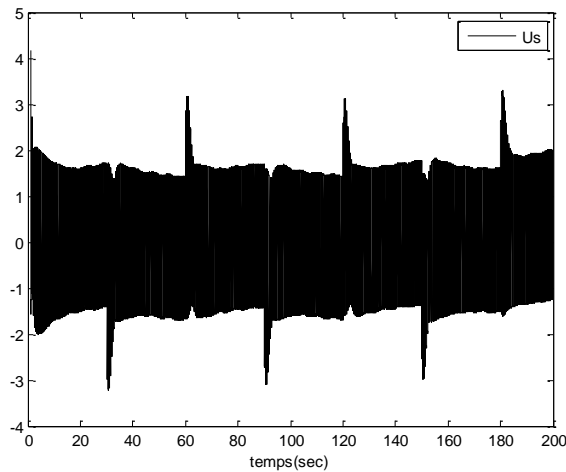


Fig 5. Control input with sliding mode

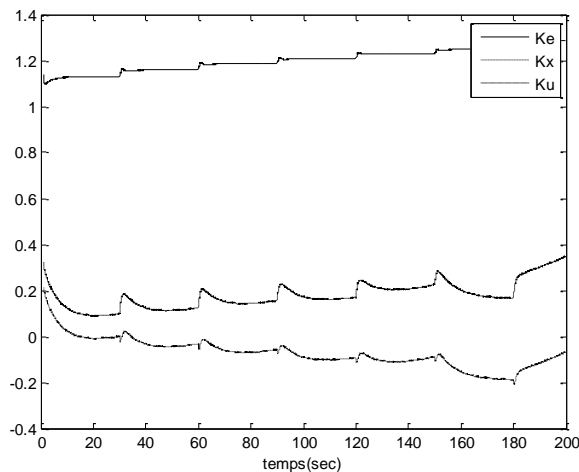


Fig 6. The gains K_e , K_x and K_u with sliding mode

perturbed system and we have seen that this command does not give a good result and in order to overcome these drawbacks, a integral sliding mode have been added in the second stage. The hybrid command constructed by the adaptive and integral sliding gives a good result and we have seen that the error between the system and model goes to zero in finite time. The only drawback of this hybrid command in the existence of chattering phenomena where we can remove them by using a boundary layer around the surface s . It is also known that the system is sensitive to perturbation if one add a boundary layer and the robustness of the sliding mode is affected by adding a boundary layer. This point will be investigated in future work.

VII Reference

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