Dynamic Mathematical Modeling of Hydro Power Plant Turbine

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Abstract- **Power system performance is affected by dynamic characteristics of hydraulic governor-turbines during and following any disturbance, such as occurrence of a fault, loss of a transmission line or a rapid change of load. Accurate modelling of hydraulic System is essential to characterize and diagnose the system response. In this article the mathematical modeling of hydraulic turbine is presented. The model is capable to implement the digital systems for monitoring and control replacing the conventional control systems for power, frequency and voltage. This paper presents the possibilities of modeling and simulation of the hydro power plants and performs an analysis of different control structures and algorithms**

I. INTRODUCTION

Different construction of hydropower systems and different operating principles of hydraulic turbines make difficult to develop mathematical models for dynamic regime, in order to design the automatic control systems. Also, there are major differences in the structure of these models. Moreover, there are major differences due to the storage capacity of the reservoir and the water supply system from the reservoir to the turbine (with or without surge chamber).The dynamic model of the plants with penstock and surge chamber is more complicated than the run-of-the-river plants, since the water feed system is a distributed parameters system. This paper will present several possibilities for the modeling of the hydraulic systems and the design of the control system.

II. MODELING OF THE HYDRAULIC SYSTEM

For run-of-the river types of hydropower plants have a low water storage capacity in the reservoir; therefore the plant operation requires a permanent balance between the water flow through turbines and the river

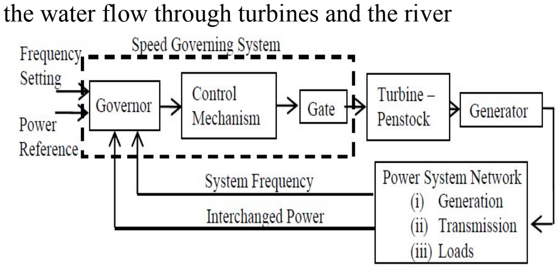

Figure 1: Functional block diagram of hydraulic governor-turbine system interconnected with a power system network.

flow in order to maximize the water level in the reservoir for a maximum efficiency of water use. Next, we will determine the mathematical model for each component of the hydropower system.

A. Mathematical modeling of Hydraulic turbine

The hydraulic turbine can be considered as an element without memory since the time constants of the turbine are less smaller than the time constants of the reservoir, penstock, and surge chamber, if exists, which are series connected elements in the system. As parameters describing the mass transfer and energy transfer in the turbine we will consider the water flow through the turbine Q and the moment M generated by the turbine and that is transmitted to the electrical generator. These variables can be expressed as non-linear functions of the turbine rotational speed N, the turbine gate position Z, and the net head H of the hydro system.

$$Q = Q\,(H, N, Z) \tag{1}$$

$$M = M\,(H, N, Z) \tag{2}$$

Through linearization of the equations (1) and (2) around the steady state values, we obtain:

$$\Delta Q = \frac{\partial Q}{\partial H}\Delta H + \frac{\partial Q}{\partial N}\Delta N + \frac{\partial Q}{\partial Z}\Delta Z \tag{3}$$

$q = a_{11}\, h + a_{12}\, n + a_{13}\, z$

$$\Delta M = \frac{\partial M}{\partial H}\Delta H + \frac{\partial M}{\partial N}\Delta N + \frac{\partial M}{\partial Z}\Delta Z \tag{4}$$

$m = a_{21}\, h + a_{22}\, n + a_{23}\, z$

Where the following notations were used:

$$q = \frac{\Delta Q}{Q_0}, n = \frac{\Delta N}{N_0}, m = \frac{\Delta M}{M_0}, h = \frac{\Delta H}{H_0}, z = \frac{\Delta Z}{Z_0}$$

Which represent the non-dimensional variations of the parameters around the steady state values.

B. The hydraulic feed system

The hydraulic feed system has a complex geometrical configuration, consisting of pipes or canals with different shapes and cross-sections. Therefore, the feed system will be considered as a pipe with a constant cross-section and the length equal with real length of the studied system. In order to consider this, it is necessary that the real system and the equivalent system to contain the same water mass. Let consider m_1, m_2...m_n the water masses in the pipe zones having the lengths $l_1, l_2, ..., l_n$ and

cross-sections A_1, A_2,...,A_n of the real feed system. The equivalent system will have the length $L=l_1+l_2+...+l_n$ and cross-section A, conveniently chosen.
In this case, the mass conservation law in both systems will lead to the equation:

$$A\sum_{i=1}^{n} l_i = \sum_{i=1}^{n} l_i.A_i \qquad (5)$$

Since the water can be considered incompressible, the flow Q_i through each pipe segment with cross-section A_i is identical and equal with the flow Q through the equivalent pipe

$$Q=v.A=Q_i=v_i.A_i \text{ for } i=1, 2,..., n \qquad (6)$$

Where v is the water speed in the equivalent pipe, and v_i is the speed in each segment of the real pipe.
From the mass conservation law it results:

$$v = \frac{Q}{A} = \frac{Q_i \sum l_i}{\sum l_i.A_i} = \frac{\sum l_i}{\sum l_i.A_i}.Q \qquad (7)$$

The dynamic pressure loss can be computed considering the inertia force of the water exerted on the cross-section of the pipe:

$$F_i = -m.a = -L.A.\rho.a = -A\frac{L.\gamma}{g}\frac{dv}{dt} \qquad (8)$$

Where L is the length of the penstock or the feed canal, A is the cross-section of the penstock, γ is the specific gravity of water (1000Kgf/m^3), a is the water acceleration in the equivalent pipe, and g=9.81 m/s^2 is the gravitational acceleration. The dynamic pressure loss can be expressed as:

$$H_d = \frac{F_i}{A} = -\frac{\gamma.L}{g}.\frac{dv}{dt} = -\frac{\gamma.L}{g}\frac{\sum l_i}{\sum l_i.A_i}\frac{dQ}{dt} \qquad (9)$$

Using non-dimensional variations, from (9) it results:

$$\frac{\Delta H_d}{H_{d0}} = \frac{Q_0}{H_{d0}}.\frac{\rho L.\sum l_i}{\sum l_i.A_i}.\frac{d\left(\frac{\Delta Q}{Q_0}\right)}{dt} \qquad (10)$$

Or in non-dimensional form:

$$h_d = -T_w\frac{dq}{dt} \qquad (11)$$

Where T_W is the integration constant of the hydropower system and the variables have the following meaning:

$$h_d = \frac{\Delta H_d}{H_{d0}}, q = \frac{\Delta Q}{Q_0}, T_w = \frac{Q_0}{H_{d0}}\frac{\rho L.\sum li}{\sum liAi}[s] \qquad (12)$$

It must be noted that this is a simplified method to compute the hydraulic pressure loss, which can be used for run-of-the river hydropower plants, with small water head. If an exact value of the dynamic pressure is required, then the formulas presented in [8], sub-chapter 8.4 "The calculation of hydro energy potential" shall be used.
Using the Laplace transform in relation (11), it results:

$$h_d(s) = -sT_w.q(s), and\ q(s) = -\frac{1}{sT_w}h_d(s) \qquad (13)$$

Replacing (13) in (3) and (4) and doing some simple calculations, we obtain:

$$q(s) = \frac{a_{12}}{1+a_{11}T_w s}n(s) + \frac{a_{13}}{1+a_{11}T_w s}z(s) \qquad (14)$$

$$h_d(s) = -\frac{a_{12}T_w s}{1+a_{11}T_w s}n(s) - \frac{a_{13}T_w s}{1+a_{11}T_w s}z(s) \qquad (15)$$

$$m(s) = \left(a_{21} - \frac{a_{12}T_w s}{1+a_{11}T_w s}\right)n(s) + \left(a_{23} - \frac{a_{13}T_w s}{1+a_{11}T_w s}\right)z(s) \qquad (16)$$

The mechanical power generated by the turbine can be calculated with the relation P=η.γ.Q.H, which can be used to obtain the linearized relations for variations of these values around the steady state values:

$$p = \eta.g.\ Q_0.h + \eta.g.H_o.q \qquad (17)$$

Where η is the turbine efficiency, and γ, Q, and H were defined previously.
On the other hand, the mechanical power can be determined using the relation P=Mω=2πM.N, which can be used to obtain the linearized relations for variations of these values around the steady state values:

$$n = \frac{2\pi N_0}{P_0}p - \frac{(2\pi N_0)^2}{P_0}m \qquad (18)$$

Where $P_0 = M_0.w_0$ is the steady state power generated by the turbine for a given steady state flow Q_0 and a steady state head H_0, and N_0 is the steady state rotational speed.
Using these relations, the block diagram of the hydraulic turbine, for small variation operation around the steady state point, can be determined and is presented in Figure 2, where the transfer functions for different modules are given by the following relation:

$$H_{qn}(s) = \frac{a_{12}}{1+a_{11}T_w s}, H_{qz}(s) = \frac{a_{13}}{1+a_{11}T_w s}, H_{hn}(s) = \frac{a_{12}\,T_w s}{1+a_{11}T_w s},$$
$$H_{hz}(s) = \frac{a_{13}}{1+a_{11}T_w s}, H_{mn}(s) = a_{21} - \left(\frac{a_{12}T_w s}{1+a_{11}T_w s}\right), H_{mz}(s) = a_{23} - \left(\frac{a_{13}T_w s}{1+a_{11}T_w s}\right) \qquad (19)$$

For an ideal turbine, without losses, the coefficients a_{ij} resulted from the partial derivatives in equations (12 - 16) have the following values: a_{11}=0.5; a_{12}=a_{13}=1; a_{21}=1.5; a_{23}=1. In this case, the transfer functions in the block diagram are given by the following relation:

$$H_{qn}(s) = \frac{1}{1+0.5.T_w s}, H_{qz}(s) = \frac{1}{1+0.5.T_w s}, H_{hn}(s) = -\frac{T_w s}{1+0.5.T_w s} \qquad (20)$$

$$H_{hz}(s) = \frac{T_w s}{1+0.5.T_w s},\quad H_{mn}(s) = \left(1,5 - \frac{T_w s}{1+0.5\ T_w s}\right) \qquad (21)$$

$$H_{mz}(s) = \left(1 - \frac{T_w s}{1+0.5\ T_w s}\right) = \frac{1-0.5.T_w\,s}{1+0.5.T_w\,s} \qquad (22)$$

III. SIMULATION RESULTS

Example. Let consider a hydroelectric power system with the following parameters:

-Water flow (turbines): Q_N=725 m^3/s;
-Water level in the reservoir: H_N=30 m;
-The equivalent cross-section of the penstock A=60m^2;
-Nominal power of the turbine P_N=178MW;
-Turbine efficiency η=0.94;
-Nominal rotational speed of the turbine=
N=71.43 rot/min;
-The length of the penstock l=Σl_i=20m;

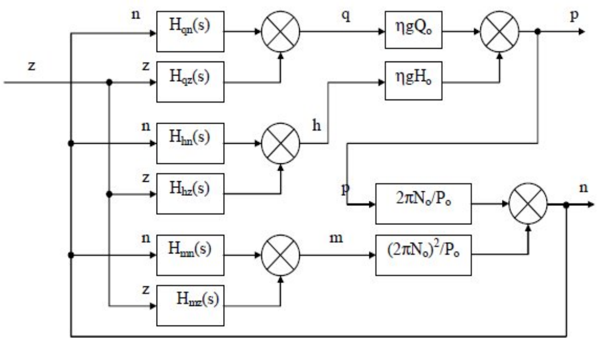

Fig. 2. The block diagram of the hydraulic turbine.

It shall be determined the variation of the time constant T_W for the hydro power system.
For the nominal regime, using relation (12), where Σl_i=20m, the time constant of the system is:

$$T_w = \frac{725}{30} \cdot \frac{20}{9{,}81.60} = 0{,}82s \quad (23)$$

Next we will study the variation of the time constant due to the variation of the water flow through the turbine for a constant water level in the reservoir, H=30m, as well as the variation due to the variable water level in the reservoir for a constant flow Q=725 m^3/s.

In table I, column 3 and figure 3 a) are presented the values and the graphical variation of the time constant T_W for the variation of the water flow between 500 m^3/s and 110 m^3/s, for a constant water level in the reservoir, H=30m. In table 1. column 4 and figure 3 b) are presented the values and the graphical variation of the time constant T_W for the variation of the water level in the reservoir, for a constant water flow, Q=725m^3/s.

H	Q	Tw(H=30m)	Tw(Q=725mc/s
17	1135,46	1,286065	1,449101
20	965,14	1,093155	1,231736
23	839,25	0,950569	1,071075
26	742,42	0,8408882	0,947489
29	665,62	0,7538998	0,849473
32	603,21	0,6832217	0,769835
35	551,51	0,6246598	0,703849
38	507,97	0,5753446	0,648282

Table I. Variations of the time constant of the hydro system

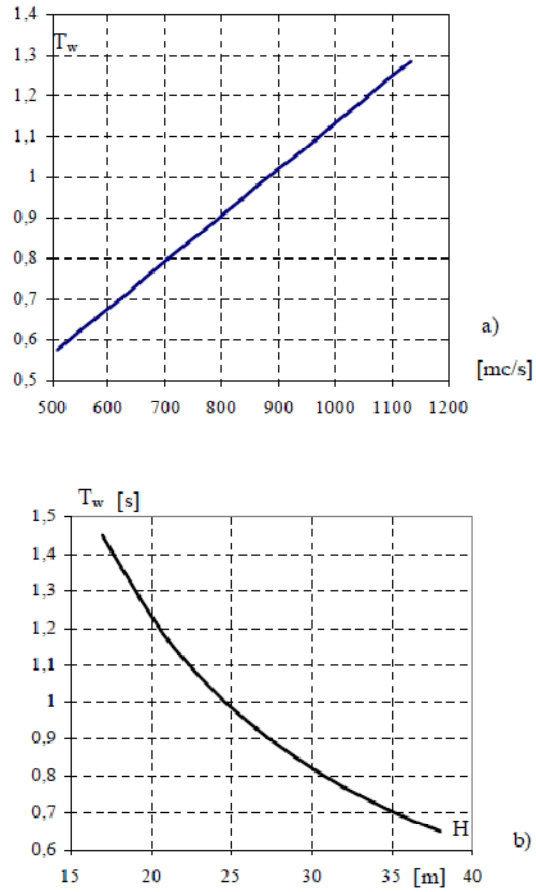

Figure 3. Variation of the integral time constant Tw: a) by the flow Q, b) by the water level H

It can be seen from the table or from the graphs that the time constant changes more than 50% for the entire operational range of the water flow through the turbine or if the water level in the reservoir varies. These variations will create huge problems during the design of the control system for the turbine, and robust control algorithms are recommended.

In figure 4 the block diagram of the turbine's power control system, is presented using a secondary feedback from the rotational speed of the turbine. It can be seen from this figure that a dead-zone element was inserted in series with the rotational speed sensor in order to eliminate the feedback for ±0.5% variation of the rotational speed around the synchronous value. This oscillation has no significant influence on the performance of the system but would have lead to permanent perturbation of the command sent to the turbine gate.

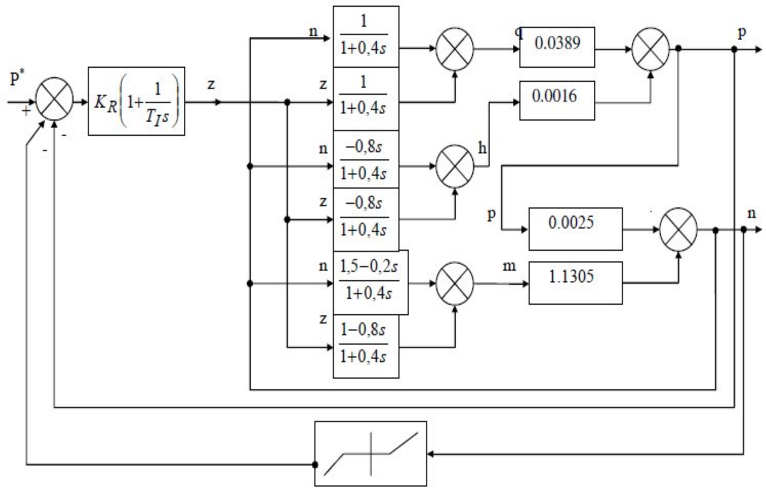

Figure 4. Block Diagram of the control system for hydraulic turbines

The constants of the transfer functions had been computed for a nominal regime T_W=0.8s. The optimal parameters for a PI controller are: K_R=10, T_I=0.02s. The results of the turbine simulation for different operational regimes are presented in figure 5, for a control system using feedbacks from the turbine power and rotational speed, with a dead-zone on the rotational speed channel for ±0.5% variation of the rotational speed around the synchronous value (a) Power variation with 10% around nominal value, b) Rotational speed variation for power control).

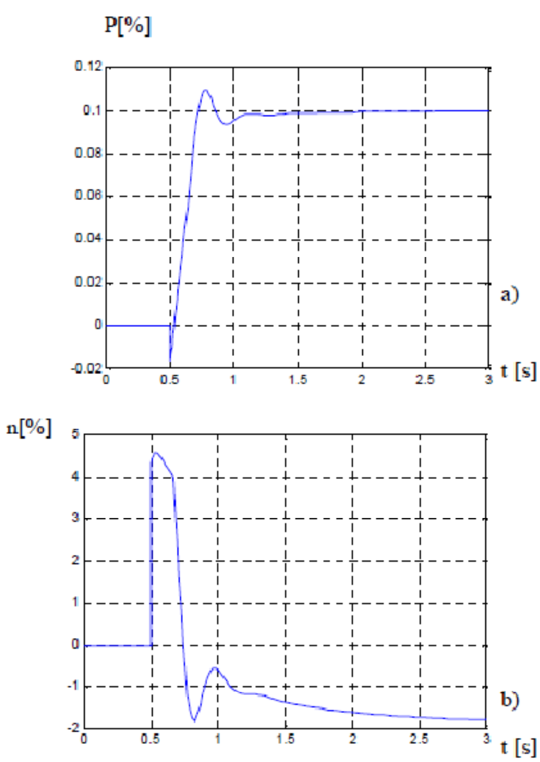

Figure 5 Control structure with feedbacks from turbine power and rotational speed:

In figure 6 the variations of the turbine power (graph a) and rotational speed (graph b) for the control system a feedback from the turbine power but no feedback from the rotational speed are presented.

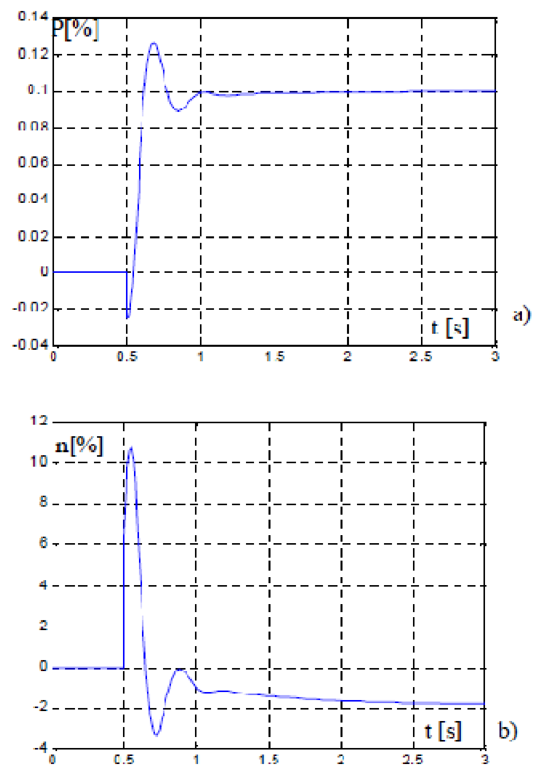

Figure 6 Control structure with only power feedback
a) Power variation with 10% around nominal value
b) Rotational speed variation for power control

I. CONCLUSIONS

The detailed mathematical modelling of hydraulic turbine is vital to capture essential system dynamic behavior .The possibility of implementation of digital systems for monitoring and control for power, frequency and voltage in the cascade hydro power plant was discussed. The simplified mathematical models, capable to accurately describe dynamic and stationary behavior of the hydro units a developed and simulated. These aspects are compared with experimental results. Finally, a practical example was used to illustrate the design of controller and to study the system stability.

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