A Robust Iterative Learning Control Approach for Linear Continuous Time Processes Based on 2D System Theory

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Abstract: In this paper, a robust iterative learning control (RILC) will be designed to improve the transient response and the tracking performance of differential linear iterative processes which are a distinct class of two dimensional (2D) systems of both processes theoretic and applications interest. Based on H_∞ setting, new sufficient conditions will be developed to demonstrate the effectiveness and the originality of our proposed scheme. The simulation results carried out on servo flexible system will be presented to prove the monotonic convergence of linear continuous time 2D systems.

Keywords: two dimensional systems theory; monotonic convergence; uncertain systems iterative learning control; $H\infty$ approach; linear matrix inequality techniques.

1. Introduction

In the recent decades, the two dimensional (2D) continuous-discrete systems have received important research interest [1,2,3], due to their extensive applications in different practical areas such as long wall and metal rolling, repetitive processes [4,5], image processing, power transmission lines and iterative learning control (ILC) synthesis [6,7,8]. Two dimensional systems are processes which are defined by two independent variables propagating information in two independent directions, where, the first one can be used to reflect the dynamics of the system in the time domain (continuous) and the second one can be used to reflect the iterative learning dynamics (discrete).

Linear iterative learning processes are a distinct class of 2D systems of both systems theoretical and applications interest. ILC is an efficient technique used to control the systems doing the same task over a fixed time interval. The ILC learns the information from the previous iteration to improve the transient response and the tracking performance of the systems during the actual iteration. Using the error measurements in the previous cycle, the iterative learning control law is updated iteratively after each operation. Owing to its effectiveness and simplicity, ILC has been used in many applications of learning control systems (LCS's) include metal rolling operations and long wall coal cutting [9], IC wafer production [10], motion stage for wire bonding [11,12] and robotics [13].

ILC represents one of principal key to find a solution for the problem of monotonic convergence and robust stability of systems that execute the same task repetitively along the pass. The repetitive error generated by these systems, from trial to trial, will be memorized and used to design a new and simple algorithm which implemented in the next iteration to decrease the error until become null. Different type of iterative learning control law are designed to achieve convergence of systems such as the PID type, D type, PD type, PI type and P type. However, several research works have focused on analytical for nominal systems under assumption that there is no uncertainty in the system. In order to study the convergence of the systems with the model uncertainty, many robust approaches are often considered such as H_∞ setting [14, 15], μ synthesis method [16] and the min-max method using the quadratic performance criterion [17].

The problem of H_∞ control for 2D dynamical systems described by state space model have received important interest by several research works over the last decade. Although H_∞ control theory have been perfectly studied and developed, most approaches have been developed based on state space model. The H_∞ setting is an analytical approach that has been used to prove the robust stability and the monotonic convergence of linear systems. Several approaches are also used to demonstrate the effectiveness of the robust iterative learning controller for tracking performance such as the linear inequality matrix (LMI) technique for solving optimization problems.

The objective of this approach is to develop a new robust iterative learning control for 2D linear systems with parametric uncertainties. This control law is able to drive the system to follow a reference model with zero error. A PD type ILC is proposed and studied to achieve this objective. Monotonic convergence and robust stability are also guaranteed by using this new scheme. Based on H_∞ setting accorded to the resolution of LMI problems, new sufficient conditions will be developed to prove the effectiveness and the originality of our proposed scheme.

1. Problem setup

Robust ILC is an interest approach developed and used to improve the perfect tracking performance and the transient response of dynamical systems. Designing an effective and robust control law means to find an appropriate algorithm that is updated iteratively from trial to trial using the information from previous iteration to compute the new control law in the next iteration. This next one can be implemented to the systems to decrease the error between the desired output and the measured output until becomes zero. This iterative control law must have the capability to drive the system to follow a given reference model.

In this paper, the tracking of an output trajectory of reference model is regarded as a principal goal. Let us consider the reference model described by the following expression:

$$\begin{cases} \dot{x}_d(t) = Ax_d(t) + Bu_d(t) \\ y_d(t) = Cx_d(t) \end{cases} \quad (1)$$

Where $x_d(t) \in R^n$ represents the reference state vector, $u_d(t) \in R^m$ represents the reference control input, $y_d(t) \in R^n$ is the reference output, $A \in R^{n*n}$ is the constant matrix, $B \in R^{n*m}$ is the gain matrix of control input, $C \in R^{m*n}$ is the gain matrix of output, $H \in R^{n*m}$ is the gain matrix of disturbance input and $t \in [0, T]$.

To complete the process description, the specification of the boundary condition is required i.e., the initial iteration profile and the initial state on each iteration. The resetting condition is satisfied at each trial i.e. $x_d(0) = 0$, where $x_d(0)$ represents the initial state of the reference model.

To achieve the objective considered in this paper, we propose to design a robust proportional derivative PD type iterative learning control law. This control law is updated iteratively from trial to trial. The analysis and synthesis of this control law are based on a set of lemmas and property, as is the well identified Schur's complement rule.

Lemma. 1. *(Schur lemma)*

Let Σ_1 and Σ_2 be real matrices of appropriate dimensions. Then for any matrix F satisfying $F^T F \leq I$ and a scalar $\varepsilon \succ 0$ the following inequality holds [18]:

$$\sum_1 F \sum_2 + \sum_2^T F^T \sum_1^T \leq \varepsilon^{-1} \sum_1 \sum_1^T + \varepsilon \sum_2^T \sum_2 \quad (2)$$

Property 1:

Let considers an invertible matrix T:

$$T \in R^{(n*n)}, S \leq 0 \Leftrightarrow T^T S T \leq 0 \quad (3)$$

2. Linear systems stability analysis

Linear differential iterative processes are defined by a state space model described by the following systems:

$$\begin{cases} \dot{x}_k(t) = Ax_k(t) + Bu_k(t) + Hw_k(t) \\ y_k(t) = Cx_k(t) \end{cases} \quad (4)$$

Where $x_k(t) \in R^n$ is the state vector, $u_k(t) \in R^m$ is the control input, $y_k(t) \in R^n$ is the output, $w_k(t) \in R^m$ is the disturbance, $H \in R^{n*m}$ is the gain matrix of disturbance input and k≥0 denotes the number of iteration. The boundary condition is $x(0) = x_0$.

For 2D systems with considerable disturbances of the form considered in the system (4), we propose a proportional derivative PD type iterative learning control of the following expression:

$$\begin{cases} u_k(t) = v_{1,k}(t) + v_{2,k}(t) \\ v_{1,k}(t) = u_d(t) + K_{rob} e_k(t) \\ v_{2,k+1}(t) = v_{2,k}(t) + K_P ey_k(t) + K_D \dot{e}y_k(t) \end{cases} \quad (5)$$

The learning rules $v_{1,k}(t)$ and $v_{2,k}(t)$ represent respectively the robust control and the iterative learning control that is iteratively updated, where K_{rob}, K_P and K_D are the robust gain matrix, the proportional gain matrix and the derivative gain matrix, respectively.

At the first iteration (iteration number 0) the iterative learning control is zero i.e. $v_{2,0} = 0$ then $u_0(t) = v_{1,0}(t)$.

In this section, An H_∞ setting is proposed to design a robust iterative learning control. Based on linear matrix inequalities LMI technique, a set of sufficient conditions are solved to demonstrate the stability of the system.

We define the tracking error model by the following system:

$$\begin{cases} \dot{e}_k(t) = \dot{x}_d(t) - \dot{x}_k(t) \\ ey_k(t) = y_d(t) - y_k(t) \end{cases} \quad (6)$$

Let consider the state variable defined by:

$$\eta_{k+1} = \int_0^t x_{k+1}dt - \int_0^t x_k dt \tag{7}$$

With the help of (4) and integrating the control law (5), we develop the new state variable described by the following expression:

$$\dot{\eta}_{k+1} = A\eta_{k+1} + B\tilde{u}_{k+1} + H\tilde{w}_{k+1} \tag{8}$$

From (7) and (8) and after substituting (4) from (1) and integrating the control law (5) the error at the iteration k+1 is described by the following expression:

$$ey_{k+1} = -CA\eta_{k+1} - CB\tilde{u}_{k+1} + ey_k - CH\tilde{w}_{k+1} \tag{9}$$

Where

$$\tilde{u}_{k+1} = -K_{rob}\eta_{k+1} + K_P\tilde{e}y_k + K_D ey_k$$

$$\tilde{e}y_k = \int_0^t ey_k\,dt$$

$$\tilde{w}_{k+1} = \int_0^t (w_{k+1} - w_k)dt$$

From (8) and (9), we consider the following new system:

$$\begin{cases} \dot{\eta}_{k+1} = \mathrm{A}\eta_{k+1} + \mathrm{B}u_{k+1} + \mathrm{B}_0 ey_k + \mathrm{B}_{11}\tilde{w}_{k+1} \\ ey_{k+1} = \mathrm{C}\eta_{k+1} + \mathrm{D}u_{k+1} + \mathrm{D}_0 ey_k + \mathrm{B}_{12}\tilde{w}_{k+1} \end{cases} \tag{10}$$

Where:

$$\mathrm{A} = A,\ \mathrm{B} = B,\ \mathrm{B}_0 = 0,\ \mathrm{B}_{11} = H,\ \mathrm{C} = -CA,$$
$$\mathrm{D} = -CB,\ \mathrm{D}_0 = I,\ \mathrm{B}_{12} = -CH$$

It is easy to see the system stability described by (10) along the pass is independent of the influence of the disturbance terms. A lyapunov function interpretation will be also required of this property, where the variable function is taken as:

$$V(k,t) = \eta_{k+1}{}^T P_1 \eta_{k+1} + ey_{k+1}^T P_2 ey_{k+1} \tag{11}$$

Where $P_1 \succ 0$ and $P_2 \succ 0$. The associated increment is:

$$\Delta V(k,t) = \dot{V}_1(k,t) + \Delta V_2(k,t) \tag{12}$$

It is now routine to interpret that stability along the pass is achieved if $\Delta V(k,t) \prec 0$

Definition 1: linear iterative process defined in (10) is said to have H_∞ disturbance attenuation bound $\gamma \succ 0$ if the system is stable along the pass and the induced norm between the output and the disturbance input is bounded by γ.

Theorem 1: a differential linear iterative system defined by (10) is stable along the pass and has H_∞ disturbance norm bound γ if there exist matrices $P_1 \succ 0$ and $P_2 \succ 0$ such that the following LMI holds:

$$\begin{bmatrix} -w_2 & * & * & * & * & * \\ \delta_1 & \delta_2 & * & * & * & * \\ \delta_3 & K_D{}^T \mathrm{B}^T & 0 & * & * & * \\ -N_2^T B^T C^T & N_2^T \mathrm{B}^T & 0 & -w_2 & * & * \\ -H^T C^T & H^T & 0 & 0 & -\gamma^2 I & * \\ 0 & 0 & 0 & w_2 & 0 & -I \end{bmatrix} \prec 0 \tag{13}$$

Where:

$$\delta_1 = -w_1 \mathrm{A}^T \mathrm{C}^T + N_1^T B^T C^T,$$
$$\delta_2 = w_1 \mathrm{A}^T + \mathrm{A}w_1 - N_1^T \mathrm{B}^T - \mathrm{B}N_1,$$
$$\delta_3 = -K_D{}^T B^T C^T + I,$$

Based on the state space model description of the systems dynamics, the conditions which guarantee the robust monotonic convergence and the stability of the system is developed in terms of the feasibility of LMIs.

Proof: introducing the associated Hamiltonian as:

$$\mathbb{H}(k,t) = \Delta V(k,t) + ey_k^T(t)ey_k(t) - \gamma^2 \tilde{w}_{k+1}^T(t)\tilde{w}_{k+1}(t) \tag{14}$$

Now, it easy to show the stability of the system described by the equality (10) along the pass. The system stability and the monotonic convergence are guaranteed along the pass if: $\mathbb{H}(k,t) \prec 0$

The Hamiltonian function can be written as:

$$\mathbb{H} = \begin{bmatrix} \eta_{k+1} \\ ey_k \\ \tilde{e}y_k \\ \tilde{w}_{k+1} \end{bmatrix}^T \Theta \begin{bmatrix} \eta_{k+1} \\ ey_k \\ \tilde{e}y_k \\ \tilde{w}_{k+1} \end{bmatrix} \tag{15}$$

The stability of the system is achieved if $\Theta \prec 0$ where Θ can be written as follow:

$$\Theta = \begin{bmatrix} \hat{A}_1^T P + P\hat{A}_1 + \hat{A}_2{}^T \bar{S}\hat{A}_2 + \bar{L}^T \bar{L} - R & P\hat{B}_1 + \hat{A}_2{}^T \bar{S}\hat{D}_1 \\ \hat{B}_1^T P + \hat{D}_1^T \bar{S}\hat{A}_2 & \hat{D}_1^T \bar{S}\hat{D}_1 - \gamma^2 I \end{bmatrix} \tag{16}$$

Where:

$$\hat{D} = \begin{bmatrix} 0 \\ 0 \\ \mathrm{B} \end{bmatrix}, \hat{B} = \begin{bmatrix} \mathrm{B} \\ 0 \\ 0 \end{bmatrix}, P = \begin{bmatrix} P & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & P \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} \mathrm{A-BK} & \mathrm{B+BK} & \mathrm{BK} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \bar{S} = \begin{bmatrix} P & 0 & 0 \\ 0 & P & 0 \\ 0 & 0 & P \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathrm{C-DK} & \mathrm{D+DK} & DK \end{bmatrix}, \bar{L}\bar{L} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

An obvious application of a successive modified Schur's lemma to the condition (16) followed by replacing the variables with theirs appropriates expressions in the result then applying the property 1 pre and post multiplying the result by $diag\left\{P_4^{-1} \quad P_3^{-1} \quad P_2^{-1} \quad P_1^{-1} \quad I \quad P_2^{-1} \quad I \quad I\right\}$ to eliminate the bilinearity in the matrix.

Then setting $N_1 = K_{rob}P_1^{-1}, N_2 = K_P P_2^{-1}, w_1 = P_1^{-1}, w_2 = P_2^{-1},$ $w_3 = P_3^{-1}, w_4 = P_4^{-1}$ in the result. Finally, noting that the result doesn't depend to w_3 and w_4 leads to (13) and the proof is complete:

3. Uncertain linear systems stability analysis

In this section we will extend the results obtained in the previous section to the case where the linear systems presented a parametric uncertainty associated with the process state space model. These uncertainties can arise from different source such as imperfect knowledge of the system dynamics and/or variation of physical parameters over time, leading to only an estimated model. We aim to design in this section the robust iterative learning control law of the previous section to guarantee the system stability along the pass with a given H_∞ disturbance norm level for all tolerable uncertainties.

We suppose that uncertainty is norm bounded in both the pass and state profile updating equations. In this case, the uncertainty is modeled as an additive disturbance to the nominal model state space representation. These differential linear uncertain iterative processes can be presented by the following state space representation:

$$\begin{cases} \dot{x}_{(k,t)} = \left(A + \Delta A\right) x_{(k,t)} + \left(B + \Delta B\right) u_{(k,t)} + H w_{(k,t)} \\ y_{(k,t)} = C x_{(k,t)} \end{cases} \quad (17)$$

Where the admissible uncertainties are assumed to be of the following assumption:

$$\left[\Delta A \quad \Delta B\right] = H_1 F \left[E_1 \quad E_2\right] \quad (18)$$

where H_1, E_1, E_2, are known constant matrices of compatible dimensions. F is unknown matrix with constant entries and satisfies $F^T F \le I$.

With the help of (17) and (7) and integrating the control law (5), we develop the new state variable described by the following expression:

$$\dot{\eta}_{k+1} = \left(A + \Delta A\right)\eta_{k+1} + \left(B + \Delta B\right)\tilde{u}_{k+1} + H\tilde{w}_{k+1} \quad (19)$$

Now, we will determine the error expression at the iteration number k+1:

$$ey_{k+1} = -C\left(A + \Delta A\right)\eta_{k+1} - C\left(B + \Delta B\right)\tilde{u}_{k+1} + ey_k - CH\tilde{w}_{k+1} \quad (20)$$

From (19) and (20), we considered the new system described by the following state representation:

$$\begin{cases} \dot{\eta}_{k+1} = \left(A + \Delta A\right)\eta_{k+1} + \left(B + \Delta B\right)\tilde{u}_{k+1} + B_0 ey_k + B_{11}\tilde{w}_{k+1} \\ ey_{k+1} = \left(C_1 + \Delta C_1\right)\eta_{k+1} + \left(D + \Delta D\right)\tilde{u}_{k+1} + D_0 ey_k + B_{12}\tilde{w}_{k+1} \end{cases} \quad (21)$$

Where:

$B_0 = 0$, $B_{11} = H$, $C_1 = -CA$, $D = -CB$, $D_0 = I$, $B_{12} = -CH$, $\Delta C_1 = -C\Delta A$ and $\Delta D = -C\Delta B$.

Based on (18) the induced uncertainties in the representation (21) verify the following condition:

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C_1 & \Delta D \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F \left[E_1 \quad E_2\right] \quad (22)$$

The resolution of the problem of designing a robust iterative control law (5) to track a reference model is given by the following result.

Theorem 2: Suppose that a robust control law described by (5) is applied to a 2D linear iterative system of the form (21), with uncertainties form modeled by (22). Then, the resulting system is stable along the pass for all tolerable uncertainties and has H_∞ norm bound $\gamma \succ 0$ if there exist matrices $w_1 \succ 0, w_2 \succ 0$, N_1 and N_2 and a scalar $\varepsilon \succ 0$ such that the LMI presented in (23) holds:

$$\begin{bmatrix} \partial_1 & * & * & * & * & * & * & * & * \\ \partial_2 & \partial_3 & * & * & * & * & * & * & * \\ \partial_4 & K_D^T \mathrm{B}^T & 0 & * & * & * & * & * & * \\ -N_2^T B^T C^T & N_2^T \mathrm{B}^T & 0 & -w_2 & * & * & * & * & * \\ -H^T C^T & H^T & 0 & 0 & -\gamma^2 I & * & * & * & * \\ 0 & 0 & 0 & w_2 & 0 & -I & * & * & * \\ 0 & \partial_5 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * \\ 0 & 0 & E_2 K_D & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ 0 & 0 & 0 & E_2 N_2 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} \prec 0 \quad (23)$$

Where:

$$\partial_1 = -w_2 + 4\varepsilon H_2 H_2^{\ T},$$
$$\partial_2 = -w_1 \mathrm{A}^T \mathrm{C}^T + N_1^T B^T C^T + 4\varepsilon H_1 H_2^{\ T},$$
$$\partial_3 = w_1 \mathrm{A}^T + \mathrm{A} w_1 - N_1^T \mathrm{B}^T - \mathrm{B} N_1 + 4\varepsilon H_1 H_1^{\ T},$$
$$\partial_4 = -K_D^{\ T} B^T C^T + I \text{ and } \partial_5 = E_1 w_1 - E_2 N_1$$

If (23) holds, the robust control gain K_{rob} and the proportional control gain K_P are given by $N_1 w_1^{-1}$ and $N_2 w_2^{-1}$, respectively. The derivative control gain K_D can be obtained directly by resolving the above inequality.

Proof: at the beginning interpret (13) in terms of the state space model obtained from application of the robust iterative control law to obtain the following expression:

$$\begin{bmatrix} -w_2 & * & * & * & * & * \\ \ell_1 & \ell_2 & * & * & * & * \\ \ell_3 & \ell_4 & 0 & * & * & * \\ N_2^T \mathrm{D}^T & N_2^T \mathrm{B}^T & 0 & -w_2 & * & * \\ \mathrm{B}_{12}^{\ T} & B_{11}^{\ T} & 0 & 0 & -\gamma^2 I & * \\ 0 & 0 & 0 & w_2 & 0 & -I \end{bmatrix} + \begin{bmatrix} 0 & * & * & * & * & * \\ \ell_5 & \ell_6 & * & * & * & * \\ K_D^{\ T} \Delta \mathrm{D}^T & K_D^{\ T} \Delta \mathrm{B}^T & 0 & * & * & * \\ N_2^T \Delta \mathrm{D}^T & N_2^T \Delta \mathrm{B}^T & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \prec 0 \qquad (24)$$

Where:

$$\ell_1 = w_1 \mathrm{C}_1^{\ T} - N_1^T \mathrm{D}^T,$$

$$\ell_2 = w_1 \mathrm{A}^T + \mathrm{A} w_1 - N_1^T \mathrm{B}^T - \mathrm{B} N_1,$$

$$\ell_3 = K_D^{\ T} \mathrm{D}^T + \mathrm{D}_0^{\ T},$$

$$\ell_4 = K_D^{\ T} \mathrm{B}^T + B_0^{\ T},$$

$$\ell_5 = w_1 \Delta \mathrm{C}_1^{\ T} - N_1^T \Delta \mathrm{D}^T,$$
$$\ell_6 = w_1 \Delta \mathrm{A}^T + \Delta \mathrm{A} w_1 - N_1^T \Delta \mathrm{B}^T - \Delta \mathrm{B} N_1$$

The second term in the above inequality can be written as: $\bar{H}\bar{F}\bar{E} + \bar{E}^T \bar{F}^T \bar{H}^T$

Where:

$$\bar{H} = \begin{bmatrix} 0 & H_2 & H_2 & H_2 & H_2 & 0 \\ 0 & H_1 & H_1 & H_1 & H_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{F} = diag\{F, F, F, F, F, F\}$$

$$\bar{E} = diag\{0, E_1 w_1 - E_2 N_1, E_2 K_D, E_2 N_2, 0, 0\}$$

An obvious application of lemma 1 followed by an application of Schur's complement lemma yields (25):

$$\begin{bmatrix} \rho_1 & * & * & * & * & * & * & * & * \\ \rho_2 & \rho_3 & * & * & * & * & * & * & * \\ \rho_4 & \rho_5 & 0 & * & * & * & * & * & * \\ N_2^T \mathrm{D}^T & N_2^T \mathrm{B}^T & 0 & -w_2 & * & * & * & * & * \\ \mathrm{B}_{12}^{\ T} & B_{11}^{\ T} & 0 & 0 & -\gamma^2 I & * & * & * & * \\ 0 & 0 & 0 & w_2 & 0 & -I & * & * & * \\ 0 & \rho_6 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * \\ 0 & 0 & E_2 K_D & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ 0 & 0 & 0 & E_2 N_2 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} \prec 0 \qquad (25)$$

Where:

$$\rho_1 = -w_2 + 4\varepsilon H_2 H_2^{\ T},$$
$$\rho_2 = w_1 \mathrm{C}_1^{\ T} - N_1^T \mathrm{D}^T + 4\varepsilon H_1 H_2^{\ T},$$
$$\rho_3 = w_1 \mathrm{A}^T + \mathrm{A} w_1 - N_1^T \mathrm{B}^T - \mathrm{B} N_1 + 4\varepsilon H_1 H_1^{\ T},$$
$$\rho_4 = K_D^{\ T} \mathrm{D}^T + \mathrm{D}_0^{\ T}, \ \rho_5 = K_D^{\ T} \mathrm{B}^T + B_0^{\ T},$$
$$\rho_6 = E_1 w_1 - E_2 N_1$$

Replacing the variables by theirs expression in the result and the proof is complete.

4. Uncertain linear systems with uncertain disturbances

In this section we will extend the results obtained in the previous sections to the case where the uncertain linear systems are affected by perturbations that presented a parametric uncertainty. We aim to design in this section the robust iterative learning control law of the previous sections to guarantee the system stability along the pass with a given H_∞ disturbance norm level for all tolerable uncertainties.

The uncertain linear iterative systems with disturbances parametric uncertainty are defined by a state space model as the following expression:

$$\begin{cases} \dot{x}_{(k,t)} = (A + \Delta A)\, x_{(k,t)} + (B + \Delta B)\, u_{(k,t)} + (H + \Delta H)\, w_{(k,t)} \\ y_{(k,t)} = C x_{(k,t)} \end{cases} \qquad (26)$$

The uncertainties matrices ΔA, ΔB and ΔH are supposed verifying the following assumption:

$$\begin{bmatrix}\Delta A & \Delta B & \Delta H\end{bmatrix} = H_1 F \begin{bmatrix}E_1 & E_2 & E_3\end{bmatrix} \qquad (27)$$

where H_1, E_1, E_2 *and* E_3 are known constant matrices of compatible dimensions. F is unknown matrix with constant entries and satisfies $F^T F \le I$.

With the help of (7) and (26) and integrating the control law (5), we develop the new state variable described by the following expression:

$$\dot{\eta}_{k+1} = (A+\Delta A)\eta_{k+1} + (B+\Delta B)\tilde{u}_{k+1} + (H+\Delta H)\tilde{w}_{k+1} \qquad (28)$$

Now, we will determine the error expression at the iteration number k+1:

$$ey_{k+1} = -C(A+\Delta A)\eta_{k+1} - C(B+\Delta B)\tilde{u}_{k+1} + ey_k - C(H+\Delta H)\tilde{w}_{k+1} \qquad (29)$$

From (28) and (29), we considered the new system described by the following state representation:

$$\begin{bmatrix}\dot{\eta}_{k+1}\\ ey_{k+1}\end{bmatrix} = \left(\begin{bmatrix}A & B_0\\ C_0 & D_0\end{bmatrix} + \begin{bmatrix}\Delta A & 0\\ \Delta C_0 & 0\end{bmatrix}\right)\begin{bmatrix}\eta_{k+1}\\ ey_k\end{bmatrix} + \left(\begin{bmatrix}B\\ D\end{bmatrix} + \begin{bmatrix}\Delta B\\ \Delta D\end{bmatrix}\right)\tilde{u}_{k+1} + \left(\begin{bmatrix}B_{11}\\ D_{11}\end{bmatrix} + \begin{bmatrix}\Delta B_{11}\\ \Delta D_{11}\end{bmatrix}\right)\tilde{w}_{k+1} \qquad (30)$$

Where:

$B_0 = 0,\ B_{11} = H,\ C_0 = -CA,\ D = -CB,\ D_0 = I,$
$D_{11} = -CH,\ \Delta B_{11} = \Delta H,\ \Delta D_{11} = -C\Delta H,$
$\Delta C_0 = -C\Delta A$ and $\Delta D = -C\Delta B.$

Based on (27) the induced uncertainties in the representation (30) verify the following condition:

$$\begin{bmatrix}\Delta A & \Delta B & \Delta B_{11}\\ \Delta C_0 & \Delta D & \Delta D_{11}\end{bmatrix} = \begin{bmatrix}H_1\\ H_2\end{bmatrix} F \begin{bmatrix}E_1 & E_2 & E_3\end{bmatrix} \qquad (31)$$

Where $H_2 = -CH_1$

Theorem 3: Suppose that a robust control law described by (5) is applied to a 2D linear iterative system of the form (30), with uncertainties form modeled by (31). Then, the resulting system is stable along the pass for all tolerable uncertainties and has H_∞ norm bound $\gamma \succ 0$ if there exist matrices $w_1 \succ 0, w_2 \succ 0$, N_1 and N_2 and a scalar $\varepsilon \succ 0$ such that the LMI presented in (32) holds:

$$\begin{bmatrix}
\alpha_1 & * & * & * & * & * & * & * & * & *\\
\alpha_2 & \alpha_3 & * & * & * & * & * & * & * & *\\
\alpha_4 & \alpha_5 & 0 & * & * & * & * & * & * & *\\
\alpha_6 & \alpha_7 & 0 & -w_2 & * & * & * & * & * & *\\
\alpha_8 & H^T & 0 & 0 & -\gamma^2 I & * & * & * & * & *\\
0 & 0 & 0 & w_2 & 0 & -I & * & * & * & *\\
0 & \alpha_9 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * & *\\
0 & 0 & \alpha_{10} & 0 & 0 & 0 & 0 & -\varepsilon I & * & *\\
0 & 0 & 0 & \alpha_{11} & 0 & 0 & 0 & 0 & -\varepsilon I & *\\
0 & 0 & 0 & 0 & E_3 & 0 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix} \prec 0 \qquad (32)$$

Where:

$\alpha_1 = -w_2 + 4\varepsilon H_2 H_2^T,$
$\alpha_2 = -w_1 A^T C^T + N_1^T B^T C^T + 4\varepsilon H_1 H_2^T,$
$\alpha_3 = w_1 A^T + A w_1 - N_1^T B^T - B N_1 + 4\varepsilon H_1 H_1^T,$
$\alpha_4 = -K_D^T B^T C^T + I,$
$\alpha_5 = K_D^T B^T,$
$\alpha_6 = -N_2^T B^T C^T,$
$\alpha_7 = N_2^T B^T,$
$\alpha_8 = -H^T C^T,$
$\alpha_9 = E_1 w_1 - E_2 N_1,$
$\alpha_{10} = E_2 K_D,$
$\alpha_{11} = E_2 N_2$

Proof: First interpret (13) in terms of the state space model obtained from application of the robust iterative control law to obtain the following expression:

$$\begin{bmatrix}
-w_2 & * & * & * & * & *\\
\beta_1 & \beta_2 & * & * & * & *\\
\beta_3 & \beta_4 & 0 & * & * & *\\
N_2^T D^T & N_2^T B^T & 0 & -w_2 & * & *\\
D_{11}^T & B_{11}^T & 0 & 0 & -\gamma^2 I & *\\
0 & 0 & 0 & w_2 & 0 & -I
\end{bmatrix} +
\begin{bmatrix}
0 & * & * & * & * & *\\
\beta_5 & \beta_6 & * & * & * & *\\
K_D^T \Delta D^T & K_D^T \Delta B^T & 0 & * & * & *\\
N_2^T \Delta D^T & N_2^T \Delta B^T & 0 & 0 & * & *\\
\Delta D_{11}^T & \Delta B_{11}^T & 0 & 0 & 0 & *\\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \prec 0 \qquad (33)$$

Where:

$\beta_1 = w_1 C_0^T - N_1^T D^T,$
$\beta_2 = w_1 A^T + A w_1 - N_1^T B^T - B N_1,$

$$\beta_3 = K_D{}^T D^T + D_0{}^T,$$
$$\beta_4 = K_D{}^T B^T + B_0{}^T,$$
$$\beta_5 = w_1 \Delta C_0{}^T - N_1^T \Delta D^T,$$
$$\beta_6 = w_1 \Delta A^T + \Delta A w_1 - N_1^T \Delta B^T - \Delta B N_1$$

The second term in the above inequality can be written as: $\bar{H}\bar{F}\bar{E} + \bar{E}^T \bar{F}^T \bar{H}^T$

Where:

$$\bar{H} = \begin{bmatrix} 0 & H_2 & H_2 & H_2 & H_2 & 0 \\ 0 & H_1 & H_1 & H_1 & H_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{F} = diag\{F, F, F, F, F, F\}$$

$$\bar{E} = diag\{0, E_1 w_1 - E_2 N_1, E_2 K_D, E_2 N_2, E_3, 0\}$$

An obvious application of lemma 1 followed by an application of Schur's complement formula yields (34):

$$\begin{bmatrix} \varphi_1 & * & * & * & * & * & * & * & * & * \\ \varphi_2 & \varphi_3 & * & * & * & * & * & * & * & * \\ \varphi_4 & \varphi_5 & 0 & * & * & * & * & * & * & * \\ \varphi_6 & \varphi_7 & 0 & -w_2 & * & * & * & * & * & * \\ D_{11}{}^T & B_{11}{}^T & 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ 0 & 0 & 0 & w_2 & 0 & -I & * & * & * & * \\ 0 & \varphi_8 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * & * \\ 0 & 0 & \varphi_9 & 0 & 0 & 0 & 0 & -\varepsilon I & * & * \\ 0 & 0 & 0 & \varphi_{10} & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ 0 & 0 & 0 & 0 & E_3 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} \prec 0 \quad (34)$$

Where:

$$\varphi_1 = -w_2 + 4\varepsilon H_2 H_2{}^T$$
$$\varphi_2 = \begin{pmatrix} w_1 C_0{}^T - N_1^T D^T \\ +4\varepsilon H_1 H_2{}^T \end{pmatrix}$$
$$\varphi_3 = w_1 A^T + A w_1 - N_1^T B^T - B N_1 + 4\varepsilon H_1 H_1{}^T$$
$$\varphi_4 = K_D{}^T D^T + D_0{}^T$$
$$\varphi_5 = K_D{}^T B^T + B_0{}^T$$
$$\varphi_6 = N_2^T D^T$$
$$\varphi_7 = N_2^T B^T$$
$$\varphi_8 = E_1 w_1 - E_2 N_1$$
$$\varphi_9 = E_2 K_D$$
$$\varphi_{10} = E_2 N_2$$

Replacing the variables by theirs expression and the proof is complete.

5. SIMULATION EXAMPLE

We use the flexible joint shown in fig 1 to demonstrate the performance of the proposed ILC method. The process consists of a ROTOFLEX Rotary Flexible Joint and a SRV02 Rotary Servo plant [19].

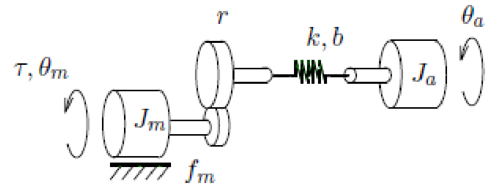

Fig.1. simplified description of the Flexible Joint process.

The system dynamics are represented by:

$$J_m \ddot{\theta}_m + f_m \dot{\theta}_m + \frac{k}{r^2}(\theta_m - r\theta_a) + \frac{b}{r^2}(\dot{\theta}_m - r\dot{\theta}_a) = \tau$$

$$J_a \ddot{\theta}_a - \frac{k}{r}(\theta_m - r\theta_a) - \frac{b}{r}(\dot{\theta}_m - r\dot{\theta}_a) = 0$$

Where:

θ_a, $\dot{\theta}_a$, θ_m and $\dot{\theta}_m$ denote the arm angle, the arm torque, the motor angle and the motor torque respectively. *Jm* and *Ja* denote the moment of inertia of the motor and the arm respectively. The parameters *b* and *k* denote damping and stiffness respectively of the spring, *f* is the viscous friction coefficient of the motor and r is the gear ratio. Finally, the parameter K_T denotes the torque constant and represents the relationship between generated torque and input voltage.

The physical parameters of the system are given by [19]:

TABLE. 1: PHYSICAL PARAMETERS OF THE SYSTEM.

Jm (*Kg.m²*)	*Ja* (*Kg.m²*)	K_T (*m.V⁻¹*)	*k*	*f*	*b*
0.0021	0.0991	0.122	35.1	0.0713	0.0924

The model of the plant is defined by the following state space representation:

$$\begin{cases} \dot{x}_k(t) = A x_k(t) + B u_k(t) + H w_k(t) \\ y_k(t) = C x_k(t) \end{cases}$$

Where:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_a} & -\frac{b}{J_a} & \frac{k}{J_a r} & \frac{b}{J_a r} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_m r} & \frac{b}{J_m r} & -\frac{k}{J_m r^2} & -\frac{f}{J_m} - \frac{b}{J_m r^2} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{K_\tau}{J_m} \end{bmatrix}^T$$

$$H=\begin{bmatrix}0 & 0 & 0 & \frac{K_\tau}{J_m}\end{bmatrix}^T$$

$$C=\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}$$

The state variable x is defined by:

$$x=\begin{pmatrix}\theta_a & \dot{\theta}_a & \theta_m & \dot{\theta}_m\end{pmatrix}^T$$

The desired input and the disturbance are:

$$u_d = 2\left|\sin(2\pi t)\right|$$

$$w = 0.2\left|\sin(2\pi t)\right|$$

The robust gain matrix K_{rob}, the proportional iterative gain matrix K_P and the derivative iterative gain matrix K_D can be computed by solving the LMI (13), there are given by:

$K_{rob}=N_1 w_1^{-1}, K_P = N_2 w_2^{-1}$ and K_D can be determined directly from the resolution of the LMI (13).

A feasible solution of the LMI (13) is given by

$K_D = 0.0172$, $K_P = 3.8135\times 10^{-5}$ and

$$K_{rob}=\begin{bmatrix}57.5418 & 0.1515 & -11.5087 & -0.6139\end{bmatrix}$$

The simulation results are obtained for the initial state vector zero. Fig.2, Fig.3 and Fig.4 demonstrate the simulation results of the desired trajectory and the output trajectory at the first iteration, second iteration and the iteration number 20 respectively. Fig.5 and Fig.6 show the errors trajectories at the first iteration and the iteration number 20. Fig.7 shows the error norm trajectory and the maximum error norm trajectory during the 100 iterations.

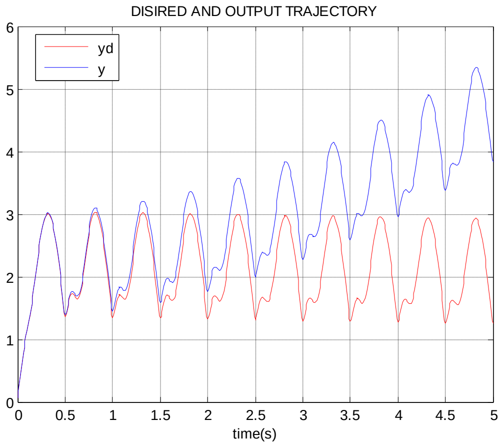

Fig.2. Simulation result of the first scheme to the system: desired trajectory and the output trajectory at the first iteration.

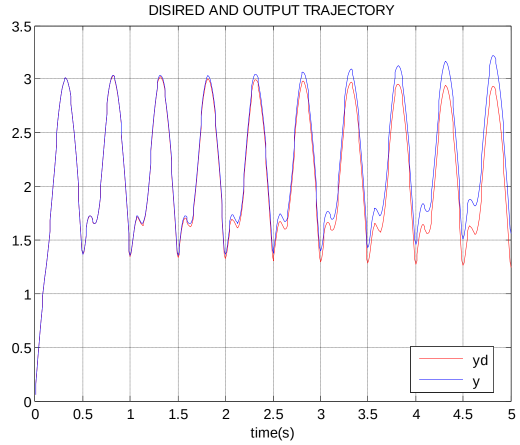

Fig.3. Simulation result of the first scheme to the system: desired trajectory and the output trajectory at the second iteration.

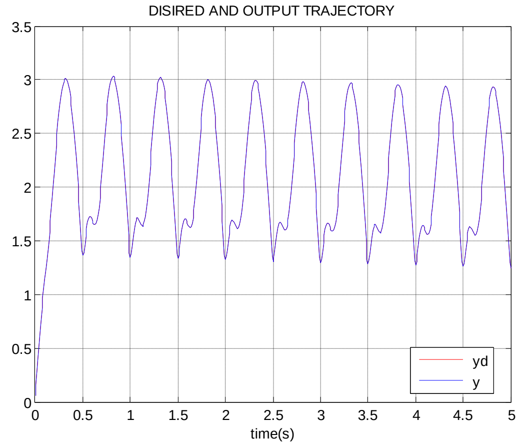

Fig.4. Simulation result of the first scheme to the system: desired trajectory and the output trajectory at the iteration number 20.

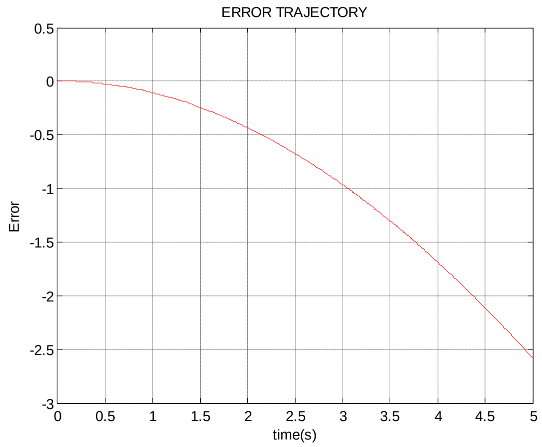

Fig.5. Simulation result of the first scheme to the system: error trajectory at the first iteration.

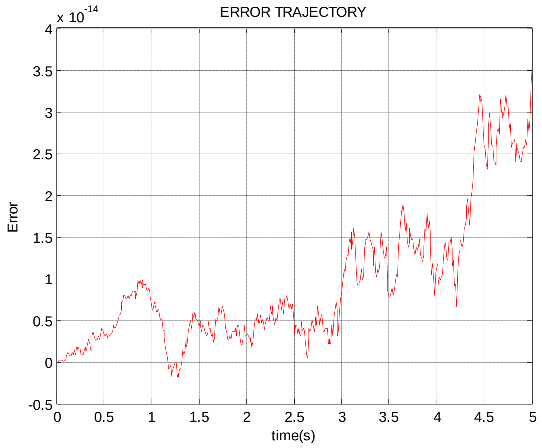

Fig.6. Simulation result of the first scheme to the system: error trajectory at the iteration number 20.

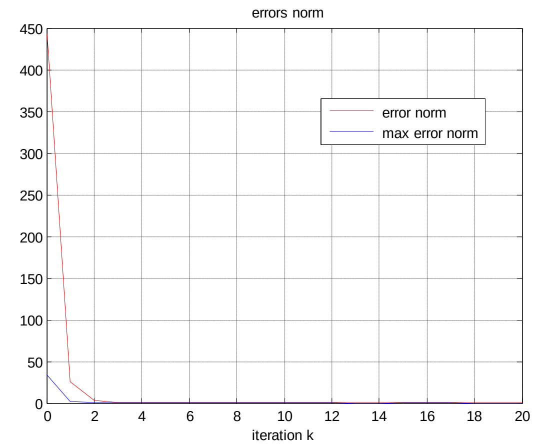

Fig.7. Simulation result of the first scheme to the system: output error norm $\|e(t,k)\|_2$ and maximum output error norm $\|e(t,k)\|$ versus iteration k.

Now, we will add an uncertainty in the parameter of the system in which it will be described by the state space model defined in (17).

Suppose that:

$J_m = J_{m0} + \Delta J_m = J_{m0}(1+\xi)$

With

$\zeta = \xi/(1-\xi)$

$$\Delta A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -334.29\zeta & -0.88\zeta & 66.86\zeta & 3.57\zeta \end{bmatrix}$$

$\Delta B = \begin{bmatrix} 0 & 0 & 0 & 5.81\zeta \end{bmatrix}^T$

For $\xi = 0.1$ applying the decomposition procedure given by (22), we get $H_1 = \begin{bmatrix} 0 & 0 & 0 & -47.62 \end{bmatrix}^T$

$H_2 = 47.62$,

$E_1 = \begin{bmatrix} 7.02 & 0.0185 & -1.4040 & -0.0750 \end{bmatrix}$,

$E_2 = -0.122$ and $F = \zeta$.

Then a feasible solution of the LMI (23) is given by $\varepsilon = 2.5345\times10^{-4}$,

$K_D = 0.0172, K_P = 3.7420\times10^{-4}$ and

$K_{rob} = \begin{bmatrix} 57.5484 & 0.1497 & -11.5095 & -0.5948 \end{bmatrix}$.

Fig.8, Fig.9 and Fig.10 demonstrate the simulation results of the desired trajectory and the output trajectory at the first iteration, fifth iteration and tenth iteration. Fig.11 and Fig.12 show the errors trajectories at the first iteration and the iteration number 20. Fig.13 shows the error norm trajectory and the maximum error norm trajectory during the 20 iterations.

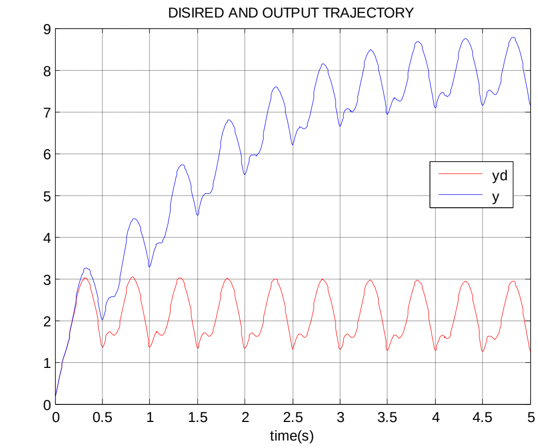

Fig.8. Simulation result of the second scheme to the system: desired trajectory and the output trajectory at the first iteration

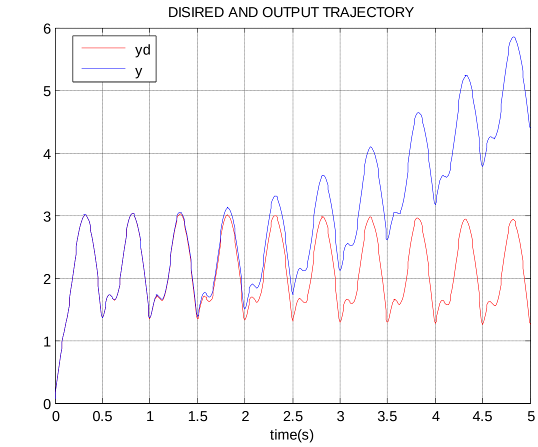

Fig.9. Simulation result of the second scheme to the system: desired trajectory and the output trajectory at the fifth iteration

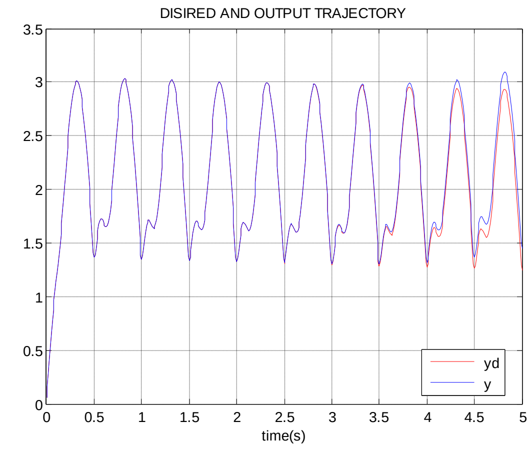

Fig.10. Simulation result of the second scheme to the system: desired trajectory and the output trajectory at the tenth iteration

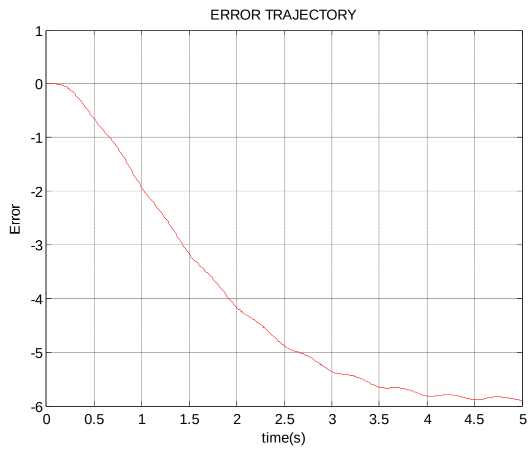

Fig.11. Simulation result of the second scheme to the system: error trajectory at the first iteration.

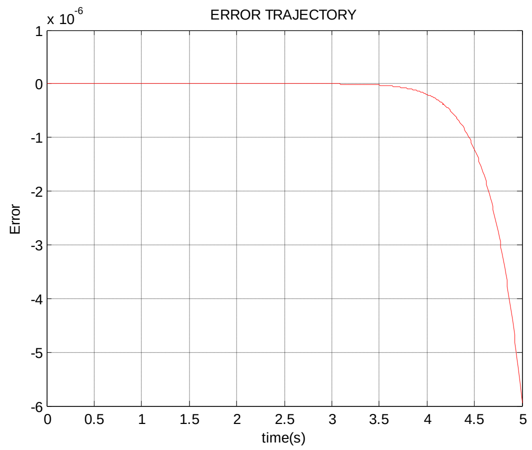

Fig.12. Simulation result of the second scheme to the system: error trajectory at the iteration number 20.

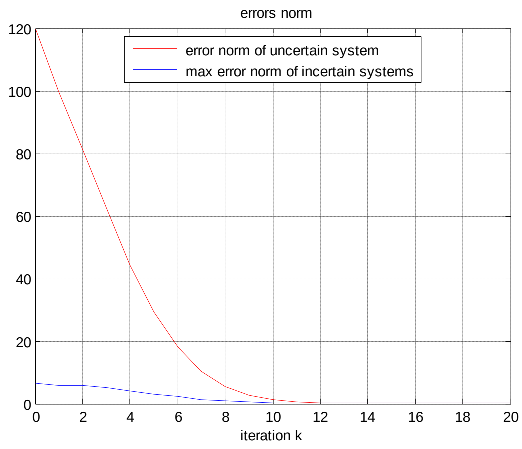

Fig.13. Simulation result of the second scheme to the system: output error norm $\|e(t,k)\|_2$ and maximum output error norm $\|e(t,k)\|$ versus iteration k.

In the next simulation, we will add an uncertainty in the parametric of the disturbance where the system will be described by the state model defined in (26).

With: $E_3 = -0.122$.

Then a feasible solution of the LMI (32) is given by $\varepsilon = 2.5023\times10^{-4}, K_D = 0.0172, K_P = 4.6798\times10^{-4}$ and $K = [57.5484 \quad 0.1497 \quad -11.5094 \quad -0.5947]$. Fig.14, Fig 15 and Fig.16 demonstrate the simulation results of the desired trajectory and the output trajectory at the first iteration, fifth iteration and the tenth iteration. Fig.17 and Fig.18 show the errors trajectories at the first iteration and the iteration number 20. Fig.19 shows the error norm trajectory and the maximum error norm trajectory during the 20 iterations.

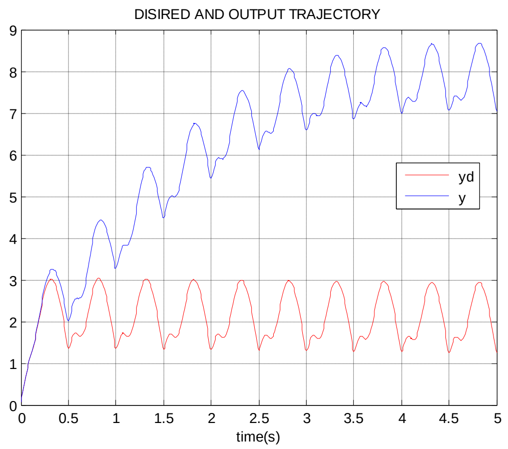

Fig.14. Simulation result of the third scheme to the system: desired trajectory and the output trajectory at the first iteration

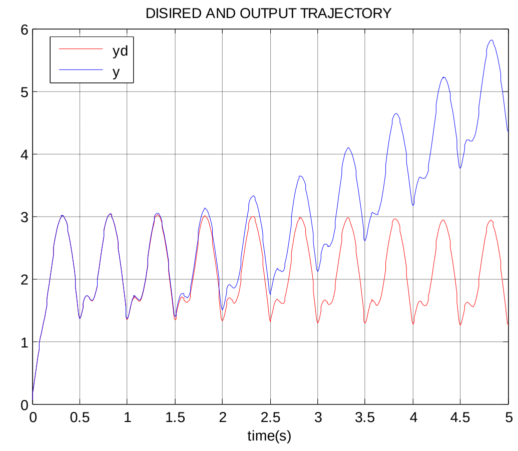

Fig.15. Simulation result of the third scheme to the system: desired trajectory and the output trajectory at the fifth iteration

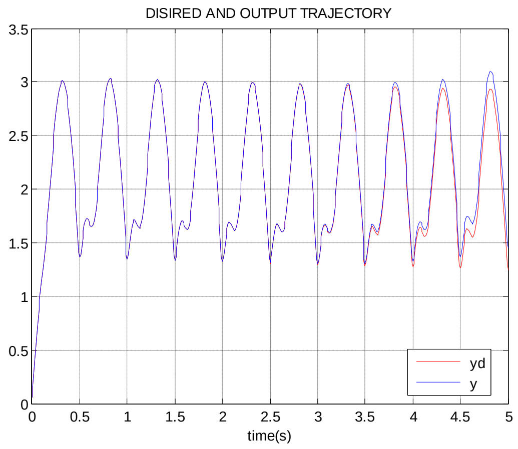

Fig.16. Simulation result of the third scheme to the system: desired trajectory and the output trajectory at the tenth iteration

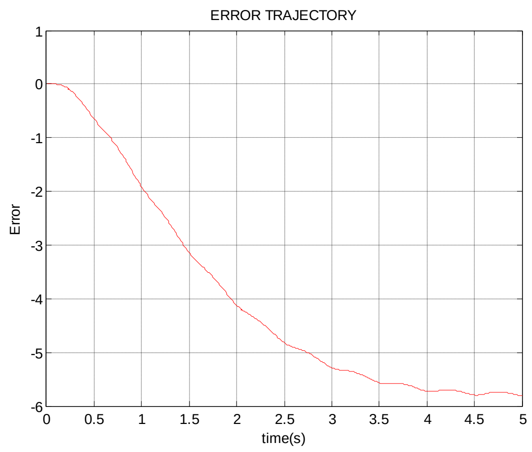

Fig.17. Simulation result of the third scheme to the system: error trajectory at the first iteration.

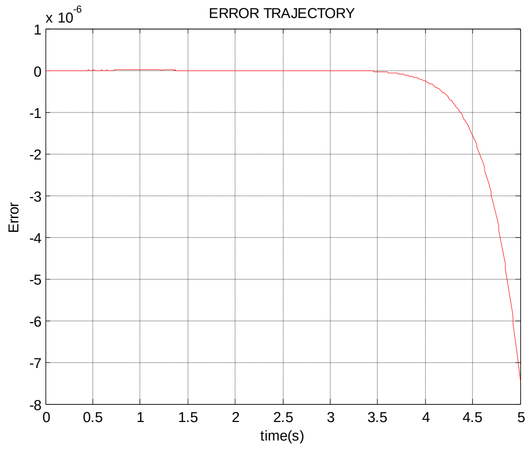

Fig.18. Simulation result of the third scheme to the system: error trajectory at the iteration number 20.

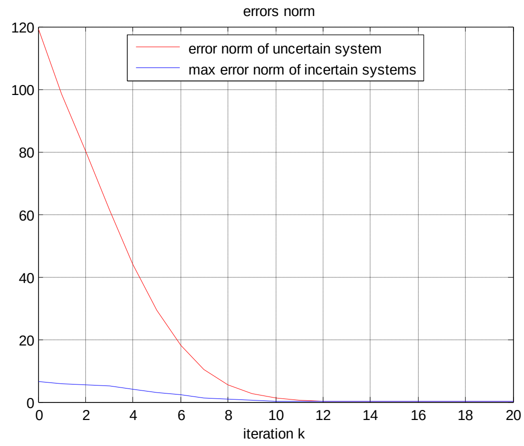

Fig.19. Simulation result of the third scheme to the system: output error norm $\|e(t,k)\|_2$ and maximum output error norm $\|e(t,k)\|$ versus iteration k.

It's very clear that the system follows the model along the pass. In the first scheme, the maximum error is very largest but it quickly converges to zero at the iteration number 2 (fig. 7). In the second and third schemes, we add an uncertainty in the system parametric; the monotonic convergence is also demonstrated (Fig 13 and Fig. 19). The error decrease from iteration to iteration until becomes zero at the iteration number 10. The example shows that the proposed algorithms are robust with respect to disturbances of system parameters and the tracking error is very small after little trials. The proposed controller guarantees system stability and monotonic convergence along the pass and provides that the H_∞ norm bound is never greater than γ .

6. Conclusion

In this paper, the guaranteed monotonic convergence problem for differential iterative systems with the presence of norm-bounded uncertainty has been solved. Important new results on the relatively open problem of the control of iterative processes which are a distinct class of 2D linear systems have been developed. These results are physically based ILC laws in an H_∞ norm bound setting where the required computations are LMI based. Furthermore, the proposed control methodology formulated in terms of LMI gives perfect tracking performance of reference model based on H_∞ setting. The simulation results, carried out on a servo flexible model, have clearly exhibited the excellent output-tracking performance and the efficiency of the designed approach.

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