ANALYSIS OF ADAPTIVE DIGITAL AC BRIDGE EMPLOYING STOCHASTIC GRADIENT SEARCH ALGORITHM

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Abstract: Bridges are the measuring devices used for accurate measurement of impedance. This paper proposes an automatic digital AC Bridge model employing closed loop control system to measure impedance of passive networks for frequencies even up to 1 MHz. In this adaptive digital AC Bridge, balancing is achieved by means of Stochastic Gradient Search Algorithm. Among the various gradient search techniques, the Widrow-Hoff's Least Mean Square (LMS) technique has been chosen, as it involves less computational burden. The digital AC Bridge is balanced by controlling the in-phase and quadrature component of the complex voltages. The stability of the bridge is controlled by the parameter (μ), which also determines the adaptation rate. The system is tested for measuring impedances of pure and impure inductors, capacitors, and a series combination of an inductor and a capacitor. The effects of various parameters like input voltage, reference resistance, learning rate, and sampling rate are simulated and studied using programs written in C++ and the results are reported. The proposed model is advantageous over the traditional heuristic method with higher accuracy, faster convergence, better reproducibility, and reliability.

Key words: Adaptive Digital AC Bridge, Stochastic gradient search algorithm, Accuracy, Impedance measurement, Widrow-Hoff's Least Mean Square technique.

1. Introduction

Measurement of impedance in high frequency passive component for power electronics applications is very much essential to study their characteristics. Multiple circuits, techniques, and instruments have been developed over the past few years to meet the requirements. But the process includes some difficulties in measuring the small real impedance, which poses several unusual challenges. In precision measurements, the impedance is, however, measured by an AC bridge, which is balanced manually and is thus slow in operation, and it also cannot be conveniently combined with automatic test systems.

To rectify these problems, digital bridges based on iterative methods and computer controls, which can easily be combined with automatic systems, have been developed [1]-[9]. Impedance analyzers adequately characterize the values of resistance, inductance, and capacitance present in any components. Although their limitations become insignificant, when it is used for power components [12], the difficulty arises when they are well designed to minimize power loss. Thus, a good power inductor or capacitor will have a real component of impedance that is very small when compared to the imaginary component. Albeit this real component is very small, it becomes much more important since it determines the power loss. Even some small phase angle errors can become critically important. Therefore, it has to be measured very accurately with high resolution. To satisfy the above requirements, a new measuring device is the need of the hour. The digital AC bridges also have the advantages of high accuracy, reproducibility, reliability, and flexibility over conventional AC bridges [1] – [4].

Dutta et al. [1] [5] [14] proposed the idea of controlling an AC bridge by means of a Least Mean Square (LMS) adaptive algorithm for measuring impedances using R-R Bridge at 50 Hz. Award et al. [2] reanalyzed the bridge equation for controlling the parameters of the bridge to obtain faster convergence. In these papers, the AC Bridge is analyzed only for a single passive

component at 50Hz. Moreover, they pointed out that the rate of convergence would deteriorate when some parameters are not appropriately selected or accurately estimated. There is still a lack of measurement range, and fast balance techniques in digital AC bridges, which also have those constraints stated above. In this study, an attempt is made to design an automatic digital AC bridge employing Stochastic Gradient Search Algorithm [13] for balancing the bridge by extending the analysis for different combinations of passive components in frequencies up to 1MHz.

This paper is organized into the following sections. Section-2 proposes Widrow-Hoff's LMS algorithm for balancing digital AC Bridge. Section-3 describes the simulation and analysis of the digital AC Bridge. Conclusions are given in section-4.

2 Widrow-Hoff's LMS Algorithm for Balancing Digital AC Bridge

Fig.1 shows an automatic digital AC bridge, employing the LMS adaptive algorithm. The bridge is balanced by controlling the voltage V_k, according to Widrow–Hoff's LMS algorithm [10,15].
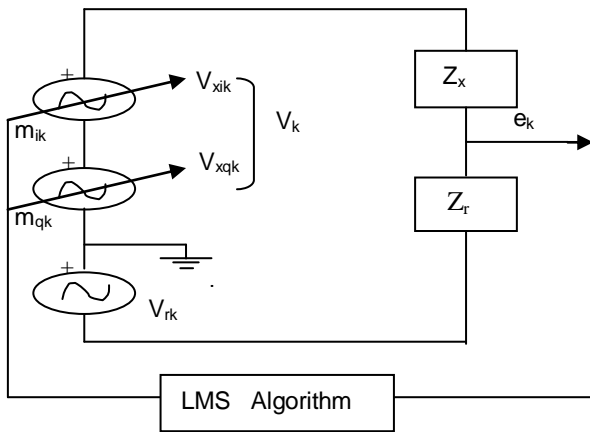

Fig. 1. Automatic Digital AC Bridge employing LMS algorithm.

In this bridge, V_{rk} and V_k are the two digitally synthesized sinusoidal voltage sources at the kth instance with the same frequency ω, but different amplitudes and phase angles. The reference voltage source, V_{rk}, is of amplitude A and zero phase angle. The in-phase and quadrature voltages V_{xik} and V_{xqk} respectively are combined to give complex voltage V_k, as given below.

$$V_k = V_{xik} + V_{xqk}$$
$$V_k = m_{ik} V_{rk} + m_{qk} V_{qk} \qquad k = 0,1,2...$$
(1)

Where, m_{ik} and m_{qk} represent the coefficients at the kth instant attached to the in-phase and quadrature components of the voltage V_k.

Z_x is the unknown impedance, which may be an inductor or capacitor of pure or impure in kind, or a series combination of pure inductor and capacitor or pure resistor. Z_r is the reference impedance and is chosen to be resistive, i.e., $Z_r = R$. When the bridge is balanced, $e_k = 0$, and the unknown impedance is calculated by:

$$Z_x = m_{ik}R + jm_{qk}R \qquad k = 0,1,2...$$
(2)

Where, $m_{ik}R$ and $m_{qk}R$ are the real and imaginary components of Z_x respectively. Thus the choice of Z_r as pure resistor will reduce the complexity of computation of real and imaginary components of Z_x.

The automatic digital AC Bridge can be looked up as an adaptive error canceller system [16-17] and is shown in Fig. 2.
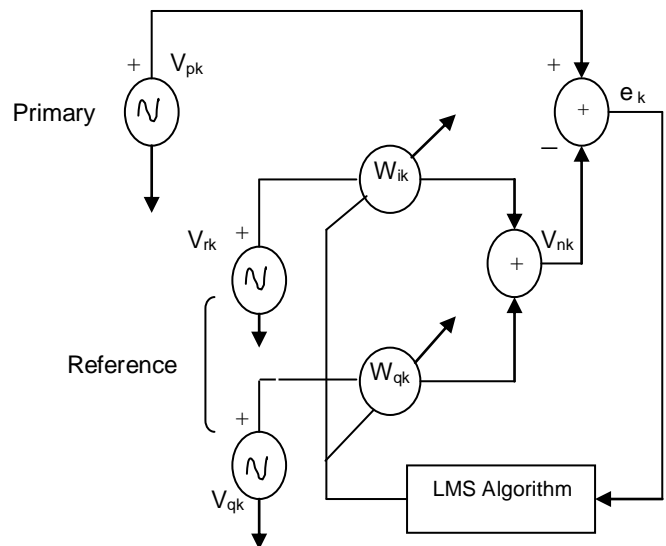

Fig. 2. An automatic digital AC bridge configured as an adaptive error canceller system.

The reference inputs V_{rk} and V_{qk} are assumed to be n–point sinusoids $A\sin(\omega k\tau)$ and $A\cos(\omega k\tau)$ respectively, $k = 0,1,2...$ with sampling frequency $1/\tau$, where τ is the sampling interval and ω is the angular frequency $(2\pi/n\tau)$, n is the number of samples per cycle, and A is amplitude of input voltage. The difference between the primary input signal V_{pk} and the signal V_{nk} is the error e_k, which ultimately adjusts the in -phase weight W_{ik} and quadrature weight W_{qk} through an LMS adaptive algorithm.

The error voltage e_k, at the kth instant is given by:

$$e_k = V_{pk} - V_{nk} \tag{3}$$

The primary input signal at k^{th} instant is given by:

$$V_{pk} = -V_{rk}Z_x/(Z_x + R) \tag{4}$$

Substituting equation (4) for e_k,

$$e_k = -V_{rk}Z_x/(Z_x + R) - V_{nk} \tag{5}$$

From Fig. 2, V_{nk} can be written as:

$$V_{nk} = W_{ik}V_{rk} + W_{qk}V_{qk} \tag{6}$$

Substitution of equation (6) in (5) yields:

$$e_k = -V_{rk}Z_x/(Z_x + R) - (W_{ik}V_{rk} + W_{qk}V_{qk}) \tag{7}$$

For the Widrow–Hoff's LMS algorithm, the expression for digital systems, using this e_k is given by:

$$W_{k+1} = W_k + 2\mu e_k V_k \qquad k = 0,1,2... \tag{8}$$

Where

e_k Error at the kth instant

W_k Weight vector at the kth instant

W_{k+1} Weight vector at the $(k+1)^{th}$ instant

μ Learning rate or convergence factor that controls the stability and the adaptation rate

V_k Complex voltage at k^{th} instant

The LMS adaptive algorithm minimizes the mean square error by recursively altering the weights W_{ik} and W_{qk} through the steepest descent method.

The adaptive in- phase weight W_{ik} is updated according to LMS algorithm as:

$$W_{i(k+1)} = W_{ik} + 2\mu e_k A\sin(\omega k\tau) \tag{9}$$

Where, $\qquad k = 0,1,2...$ and $\tau = 2\pi/n\omega$

Substituting e_k in equation (9) gives us:

$$W_{i(k+1)} = W_{ik} + 2\mu A\sin(\omega k\tau)[-V_{rk}Z_x/(Z_x + R) \\ -(W_{ik}V_{rk} + W_{qk}V_{qk})] \tag{10}$$

Substituting V_{rk} and V_{qk} in equation (10) we have:

$$W_{i(k+1)} = W_{ik} + 2\mu A\sin(\omega k\tau)[-Z_x A\sin(\omega k\tau)/(Z_x + R) \\ -(W_{ik}A\sin(\omega k\tau) + W_{qk}A\cos(\omega k\tau))] \tag{11}$$

Further the equation (11) results in:

$$W_{i(k+1)} = W_{ik}[1 - \mu A^2 + \mu A^2 \cos(2\omega k\tau)] \\ -\mu A^2(Z_x/(Z_x + R))(1 - \cos(2\omega k\tau)) \\ -W_{qk}\mu A^2 \sin(2\omega k\tau) \tag{12}$$

The coefficients m_{ik} and m_{qk} can also be expressed in terms of weights W_{ik} and W_{qk} by a scale factor and are given by:

$$m_{ik} = (-(R + Z_x)/R)W_{ik} \qquad k = 0,1,2... \tag{13}$$

$$m_{qk} = (-(R + Z_x)/R)W_{qk} \qquad k = 0,1,2... \tag{14}$$

From equations (12), (13) and (14), the updating of m_{ik} can be written as:

$$m_{i(k+1)} = m_{ik}[1 - \mu A^2 + \mu A^2 \cos(2\omega k\tau)] + \mu A^2(Z_x/R) \\ (1 - \cos(2\omega k\tau)) - m_{qk}\mu A^2 \sin(2\omega k\tau) \tag{15}$$

The procedure for updating quadrature weight W_{qk} according to LMS algorithm is:

$$W_{q(k+1)} = W_{qk} + 2\mu e_k A\cos(\omega k\tau) \tag{16}$$

Where, $k = 0,1,2...$ and $\tau = 2\pi/n\omega$

Substituting the value of e_k, V_{rk} and V_{qk} in equation (16) we have:

$$W_{q(k+1)} = W_{qk}[1 - \mu A^2 - \mu A^2 \cos(2\omega k\tau)]$$
$$- \mu A^2(Z_x/(Z_x + R))\sin(2\omega k\tau) - W_{ik}\mu A^2 \sin(2\omega k\tau)$$
$$(17)$$

From equations (13), (14), and (17), the updating of m_{qk} can be arrived as:

$$m_{q(k+1)} = m_{qk}[1 - \mu A^2 - \mu A^2 \cos(2\omega k\tau)] + \mu A^2(Z_x/R)$$
$$\sin(2\omega k\tau) - m_{ik}\mu A^2 \sin(2\omega k\tau)$$
$$(18)$$

The algorithm operating on the instantaneous error e_k, as the input, adaptively generates the weights, W_{ik} and W_{qk}. The net result of the algorithmic process is to make the error e_k to approach zero.

The unknown impedance is calculated from the adaptive weights by using the equation (2).

The phase angle θ is calculated by

$$\theta = \tan^{-1}(X_x/(R + R_x)) \qquad (19)$$

Where, R_x and X_x are real and imaginary parts of Z_x.

2.1 Choice of Initial Weights

Since the choice of initial weights influence the net to reach a global (or local) minimum of the error in terms of the speed of convergence, the random initialization technique is used to initialize the weights in the range of 0 to 100, with a condition that both W_{ik} and W_{qk} will not be zero at the initialization.

2.2 Choice of Learning Rate

Adaptation rate can be controlled by adjusting the learning rate (or convergence factor) µ, which is greater than zero but less than the reciprocal of the largest Eigen value $\lambda_{\max}$ of the matrix R_m [4] given by:

$$R_m = \begin{bmatrix} A\sin(\omega k\tau)A\sin(\omega k\tau) & A\sin(\omega k\tau)A\cos(\omega k\tau) \\ A\cos(\omega k\tau)A\sin(\omega k\tau) & A\cos(\omega k\tau)A\cos(\omega k\tau) \end{bmatrix}$$

Again,

$\lambda_{\max}$ is given by Glover and Wang [11] as

$$\lambda_{\max} \leq A^2$$
$$(20)$$

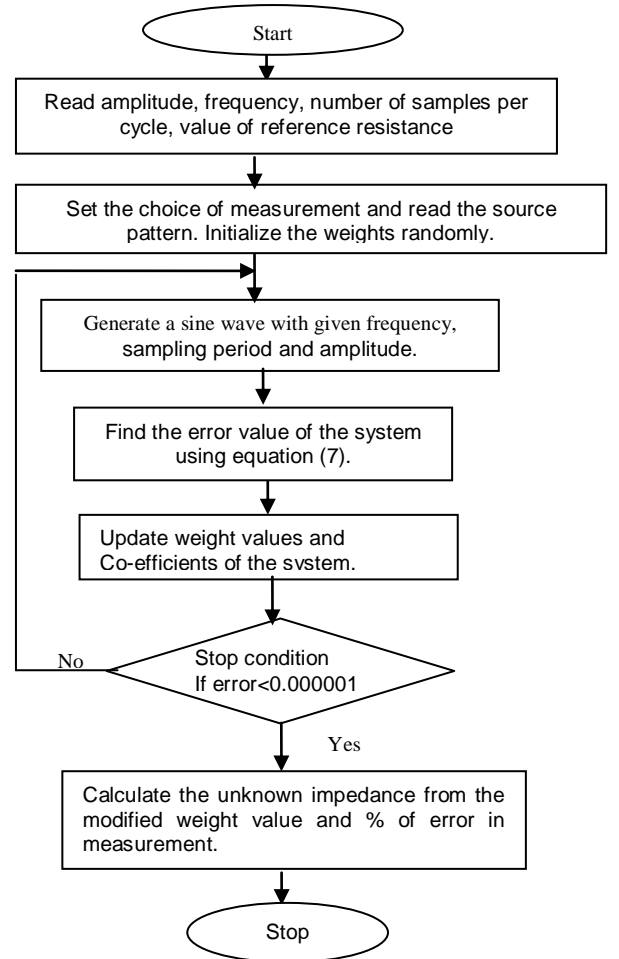Where, 'A' is the amplitude of the sinusoid.

Fig. 3. Flow chart for balancing of digital AC Bridge.

3 Simulations and Analysis of Digital AC Bridge

The computer simulation of the digital AC Bridge has been performed using a program written in C++. The flowchart of the program is shown in Fig.3.The simulation tests for following six combinations of the passive components, namely, pure resistor, pure inductor, pure capacitor, impure inductor (pure inductor associated with a small resistive component in series), impure capacitor (pure capacitor associated with a small resistive component in series), a series combination of pure inductor and capacitor have been conducted. The system was tested for all the six cases over a wide range of impedances and frequencies. The relationship between learning rate and magnitude error, reference resistance and magnitude error, and reference resistance and phase angle error are also analyzed and results in detail are summarized asinTable1through Table 4.The relationship between the number of cycles and m_{ik} value, number of cycles and m_{qk} value for the pure R-L bridge, number of samples per cycle and magnitude error, number of samples per cycle and phase angle error, input voltage amplitude, and magnitude and phase angle error, and learning rate and number of cycles, are analyzed. The graphical representations are given in Fig. 4 to Fig. 7.

Table 1. The Effect of Learning Rate (μ) on Impure R-C Bridge

Learning rate (μ)	Effective Series Resistance (ESR) in Ω	Capacitance (in Farad)	Theoretical value of impedance		Observed value of impedance		Observed value of Effective Series Resistance (ESR) in Ω	Absolute Error percentage	
			Magnitude in Ω	Phase angle in degrees	Magnitude in Ω	Phase angle in degrees		Magnitude	Phase angle
0.001563	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.330065	6.96E-09	2.59E-08
0.015625	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.329913	9.56E-08	3.44E-08
0.023650	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.329884	9.18E-08	4.60E-08
0.174650	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.327538	2.13E-06	9.80E-07
0.257300	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.323872	1.03E-06	2.46E-06
0.362500	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.326292	5.13E-06	1.47E-06
0.500000	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.321265	5.48E-06	3.49E-06
0.650000	0.33	1.0E-12	159155	-89.6399	159155	-89.6399	0.320183	8.01E-06	3.91E-06

Input voltage Amplitude (A) = 0.5 V; Frequency = 1MHz; Reference resistance (R) = 1000Ω;
Number of samples per cycle (n) = 4000

Table 2. The Effect of Reference Resistance on Impure R-L Bridge

Reference Resistance in Ω	Effective Series Resistance (ESR) in Ω	Inductance (in Henry)	Theoretical value of impedance		Observed value of impedance		Observed value of Effective Series Resistance (ESR) in Ω	Absolute Error percentage	
			Magnitude in Ω	Phase angle in degrees	Magnitude in Ω	Phase angle in degrees		Magnitude	Phase angle
2000	1.5	0.1273	799850	89.8566	799850	89.8566	1.49992	2.14E-09	6.31E-09
4000	1.5	0.1273	799850	89.7134	799850	89.7134	1.49980	3.81E-08	1.65E-08
6000	1.5	0.1273	799850	89.5701	799850	89.5701	1.49996	5.67E-09	3.34E-09
8000	1.5	0.1273	799850	89.4268	799850	89.4268	1.49971	4.32E-09	2.36E-08
10000	1.5	0.1273	799850	89.2836	799850	89.2836	1.49977	2.82E-08	1.85E-08

Input voltage Amplitude = 0.5 V; Frequency = 1MHz; Number of samples per cycle (n) = 4000;
Learning rate (μ) = 0.001563

Table 3. R-R Bridge

Resistance in Ω	Theoretical value of impedance		Observed value of impedance		Absolute Error percentage	
	Magnitude in Ω	Phase angle in degrees	Magnitude in Ω	Phase angle in degrees	Magnitude	Phase angle
0.001000	0.001000	0	0.000997	-3.35E-07	0.307760	-3.35E-07
0.010000	0.010000	0	0.009996	-2.78E-07	0.044804	-2.78E-07
0.100000	0.100000	0	0.099994	-1.65E-07	0.006482	-1.65E-07
1.000000	1.000000	0	0.999997	-3.38E-07	0.000314	-3.38E-07
10000.00	10000.00	0	10000.00	-3.56E-07	5.12E-07	-3.56E-07
560000.0	560000.0	0	560000.0	-3.91E-07	1.63E-07	-3.91E-07
10000000	10000000	0	10000000	-2.28E-07	4.51E-07	-2.28E-07

Input voltage Amplitude = 0.5 V; Frequency = 50 Hz; Number of samples per cycle (n) = 512;
Reference Resistance (R) = 1000Ω; Learning rate (μ) = 0.001563

Table 4. Series Combination of Pure L and C

Inductance (in Henry)	Capacitance (in Farads)	Theoretical value of impedance		Observed value of impedance		Absolute Error percentage	
		Magnitude in Ω	Phase angle in degrees	Magnitude in Ω	Phase angle in degrees	Magnitude	Phase angle
0.0001	1.00E-09	469.1640	25.1343	469.1640	25.1343	1.08E-07	1.01E-07
0.5000	1.00E-10	3.14E+06	89.9818	3.14E+06	89.9818	7.82E-09	8.94E-09
0.0500	1.00E-12	155004.0	89.6304	155004.0	89.6304	8.13E-09	6.16E-09
0.1000	2.20E-04	628319.0	89.9088	628319.0	89.9088	6.10E-09	4.82E-09
1.0000	1.00E-05	6.28E+06	89.9909	6.28E+06	89.9909	1.02E-08	4.36E-09

Input voltage Amplitude = 0.5 V; Frequency = 1MHz; Number of samples per cycle (n) = 4000;
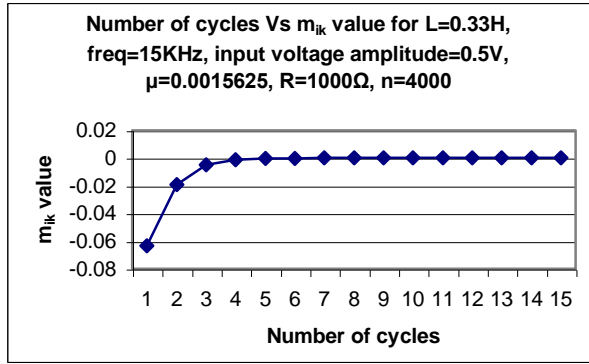Reference Resistance (R) = 1000Ω; Learning rate (μ) = 0.001563
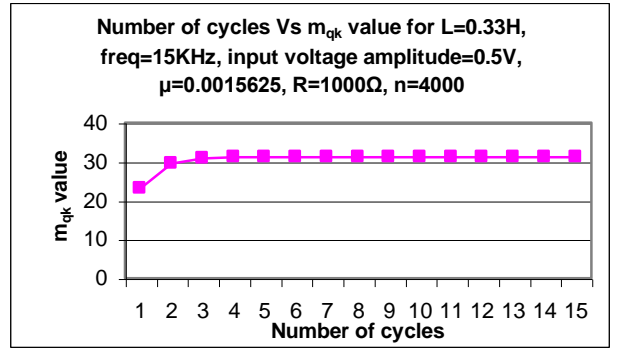
Fig. 4.a

Fig. 4.b

Fig. 4.a, b. Learning curves of m_{ik} and m_{qk} in the case of pure R-L Bridge
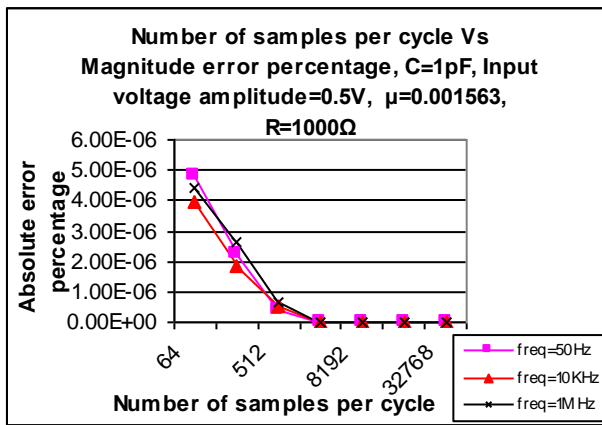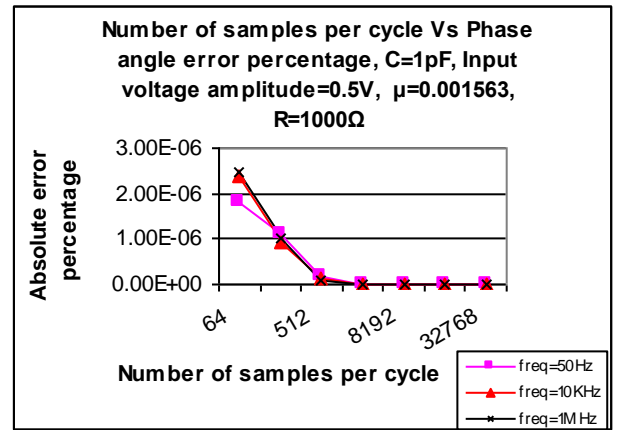

Fig. 5.a

Fig. 5.b

Fig. 5. a, b . Effect of sampling on magnitude and phase angle error in the case of pure R- C Bridge.
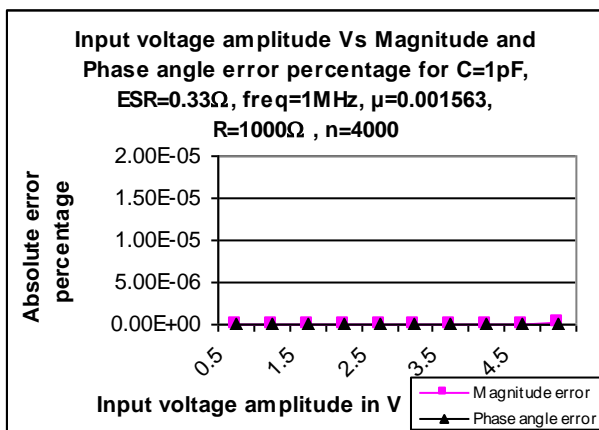

Fig. 6. Effect of amplitude of input voltage on magnitude and Phase angle error in the case of impure R – C Bridge.
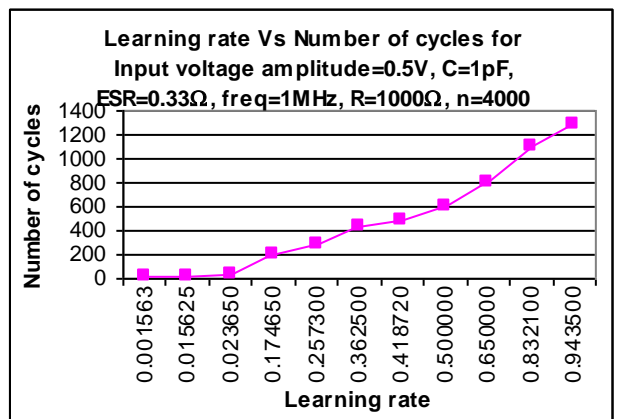

Fig. 7. Effect of learning rate and number of cycles in the case of impure R-C Bridge

From the above simulation results, we can make the following observation:

1. From Fig. 4.a and 4.b, we can see that m_{ik} and m_{qk} converge quickly within five cycles for a pure R-L bridge configuration.
2. When the number of samples per cycle is increased the absolute error is minimized in the simulation. High sampling rates may be chosen to get more accurate result, as shown in Fig. 5.a and b. The figures also show that the measurement accuracy remains the same even at higher frequencies, unlike the conventional measurement technique.
3. From Fig. 6, it can be seen that the input amplitude voltage does not affect the accuracy of the system.
4. From Table1, it is evident that the error is minimum in lower learning rates in the simulation. It can be inferred that more accurate results may be achieved, by choosing lower learning rate and the system converges quickly as shown in Fig. 7.
5. From Table2, it is observed that the system is accurate in measuring the impedance at low as well as high values of reference resistors for impure R-L Bridge. The same is observed while simulating the system with all other different combinations of passive components.
6. The proposed digital AC Bridge may be utilized in measuring resistance for both very low (Effective Series Resistance (ESR) of inductance and capacitance) and very high values accurately which can be referred from Table 3.
7. The simulated system performs the measurement of impedances even for a series combination of pure inductor and capacitor as evidenced in Table 4.

4. Conclusions

In this study, the Stochastic Gradient Search Algorithm has been adopted for the automatic balancing of digital AC Bridge. This impedance-measuring device can be used for measuring impedances of various passive components at frequencies up to 1MHz, accurately, unlike the conventional AC bridges. This paper advocates in extending the usage of the device in accurately measuring the very small real component of impedances that are present even in high frequency inductor and capacitor, which cannot be measured with other conventional systems accurately. The effects of variation of input voltage, reference resistance, sampling rate, and learning rate of the system have also been analyzed. The proposed automatic digital AC Bridge has been analyzed by simulation study and the results are highly feasible in terms of accuracy. Taking into account the effect of various physical parameters, this bridge can be designed and deployed physically for the real time measurement of impedance at a different range of frequencies.

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