Smart Discrete Fourier Transform Algorithm's Comparison with Available Filtering Algorithms for Distance Relay for Transmission Line Protection

***P. Suresh Babu** ****Dr. S. V. Jayaram Kumar**

*Associate Professor in EEE Dept., St. Ann's College of Engineering & Technology CHIRALA-523187 , INDIA E-mail: sureshperli@yahoo.com

**Professor in EEE Dept., JNTU College of Engineering, Kukatpally, Hyderabad-500082, INDIA. E-mail: svjkumar101@rediffmail.com

Abstract: Of all available relays, distance relay attracted more attention for transmission line protection. Whenever fault occurs in power system the actuating quantities contains harmonics, decaying dc components and noise. Faults have to be cleared rapidly in order to prevent damage to the operating devices and personnel. So many filtering algorithms are available for the protection of transmission line but all are having their own merits and demerits. To obtain exact fundamental frequency components for relaying purpose, Smart Discrete Fourier Transform (SDFT) algorithm is proposed. On 17-bus power system, proposed algorithm is tested for speed, accuracy, frequency response, computational burden and capability to distinguish among different types of faults. The results are compared with available filtering algorithms. The power system testing is done with MATLAB and PSCAD/EMTDC environment. The test results shows that proposed SDFT algorithm is better that existing relaying algorithms.

Key words: Filtering Algorithms, Faults, Distance relay, Decaying Dc offset, Harmonics, Power system Protection.

I. INTRODUCTION

According to the historical records, many large scale system-wide blackouts involve relay misoperations. Evaluation and improvement of existing relay algorithms and settings as well as investigation of new techniques for relaying are very important for understanding and mitigating relay misoperations. The most common approach used by many researchers for studying relay algorithm performance is using a simple two-machine system and limited fault scenarios. An algorithm for comprehensive study of different relaying actions and fault analysis under variety of system-wide disturbances is needed.

Numeric transmission line distance protection systems have been widely applied in recent years primarily because of their monitoring and communications capabilities rather than for improved performance of the protection functions. Typical tripping times for digital distance relays range from one to 3 cycles, with state of art filtering algorithm it can offer trip times of one-quarter to one cycle. Recent developments in adaptive algorithms and the use of higher sampling rates combine to provide secure high speed protection not available with previous implementations. The fast and accurate determination of fault location on electric power transmission line is utilized as an aid in the fault analysis and power restoration. At the same time, the fault destination adversely impacts service reliability, operation cost and the quality of power delivery.

Distance relaying techniques have attracted considerable attention for the protection of transmission line. The principle of these techniques measures the impedance at a fundamental frequency between the relay location and the fault point thus determining if a fault is internal or external to a protection zone. Voltage and current signal are used for this purpose and they generally contain the fundamental frequency component in addition to harmonics and the dc offset. With digital technology being ever increasingly adopted in power substation more particularly in the protection field, distance relays have experienced some improvement mainly related to efficient filtering method. During last two decades remarkable work has been demonstrated in the area of distance protection. Many filtering algorithms have been proposed numerical relaying actions. Every algorithm has its own merits and demerits

The aim of most of these filtering algorithms is to extract the fundamental frequency component from the complex post fault voltage and current signals containing a transient dc offset component and harmonic frequency components in addition to the power frequency component. The exponentially decaying dc offset present in the relaying signal gives rise to large errors in the pharos estimates unless the offset terms are removed prior to the execution of the algorithms.

This paper presents a novel Smart Discrete Fourier Transform (SDFT) algorithm with capability to estimate exact fundamental frequency components during faults, that operating signals contain fundamental frequency components in combination with decaying dc components and harmonics. Proposed SDFT algorithm performance is compared in terms of distance relay with existing filtering algorithms regarding their speed, accuracy, computational burden and frequency response.

A 17-bus power system is taken to test the effectiveness of proposed SDFT algorithm. Different types of faults like single line to ground, double line to ground and three lines to ground faults are simulated at different lengths of selected transmission line. The distance relay characteristics are generated using MATLAB and PSCAD/EMTDC output files containing the values of the apparent impedances seen by distance relay placed at bus-3, whose fundamental frequency components extraction is based on proposed SDFT algorithm for faults at selected locations on transmission line L_5.

II. PROPOSED SDFT FILTERING ALGORITHM

The voltage and current signals may contain serious harmonics and decaying dc components during fault interval. The decaying DC seriously decreases the precision and convergence speed of fundamental frequency signal from DFT. In order to overcome the above problems, the proposed digital multifunction relay with SDFT algorithm can estimate the DC offset frequency and phasor from a faulted input operating signals. Since there are several components in a fault current signal, the algorithm first takes DC offset into consideration and uses smoothing windows to eliminate other components in a fault signal.

Consider any fault signal *f(t)* with fundamental frequency components and decaying DC offset components can be expressed as

$$f(t) = F\sin(\omega t + \phi_1) + F\sin(\phi_2)e^{-\alpha t} \qquad \text{----1}$$

Where

F is the amplitude of the faulted signal

ϕ_1 is the phase angle of the faulted signal

ϕ_2 is the fault angle of the signal

$\alpha^{-1} = \tau$ is the time constant of the signal

Suppose $f(t)$ is sampled with a rate of sampling (50*N) Hz to produce the sample set $\{f(k)\}$

$$f(k) = F\sin\left(\omega\frac{k}{50N} + \phi_1\right) + F\sin(\phi_2)\exp\left(-\alpha\frac{k}{50N}\right) \qquad \text{----2}$$

The signal *f(t)* is conventionally represented by phasor complex number $\bar{f}$

$$\bar{f} = Fe^{j\phi_1} = F\cos\phi_1 + jF\sin\phi_1 \qquad \text{----3}$$

Then *f(t)* can be expressed as

$$f(t) = \frac{\bar{f}e^{j\phi_1} - \bar{f}^{*}e^{-j\phi_1}}{2} + F\sin(\phi_2)\exp\left(-\alpha\frac{k}{50N}\right) \qquad \text{----4}$$

Fundamental frequency components of Discrete Fourier transform of {*f(k)*} is calculated from the following equation

$$\hat{f}_r = \frac{2}{N}\sum_{k=0}^{N-1} f(k+r)e^{-j2\pi k/N} \qquad \text{----5}$$

Taking frequency deviation $\omega = 2\pi(50+\Delta f)$ into consideration

$$\hat{f}_r = \frac{\bar{f}}{N}\sum_{k=0}^{N-1} e^{j2\pi(50+\Delta f)\frac{(k+r)}{50N}} e^{-j2\pi\frac{k}{N}} - \frac{\bar{f}^{*}}{N}\sum_{k=0}^{N-1} e^{j2\pi(50+\Delta f)\frac{(k+r)}{50N}} e^{-j2\pi\frac{k}{N}} + \frac{2F\sin(\phi_2)}{N}\sum_{k=0}^{N-1} e^{-\alpha\frac{(k+r)}{50N}} e^{-j2\pi\frac{k}{N}} \qquad \text{----6}$$

We can rearrange the Equn (6) as

$$\hat{f}_r = \frac{\bar{f}}{N} e^{j2\pi\left(1+\frac{\Delta f}{50}\right)\frac{r}{N}} \sum_{k=0}^{N-1} e^{j2\pi\left(\frac{\Delta f}{50}\right)\frac{r}{N}} - \frac{\bar{f}^{*}}{N} e^{-j2\pi\left(1+\frac{\Delta f}{50}\right)\frac{r}{N}} \sum_{k=0}^{N-1} e^{-j2\pi\left(\frac{2+\Delta f}{50}\right)\frac{r}{N}} + \frac{2F\sin(\phi_2)}{N} e^{-\alpha\frac{r}{50N}} \sum_{k=0}^{N-1} e^{-\alpha\frac{k}{50N}} e^{-j2\pi\frac{k}{N}} \qquad \text{----7}$$

Above Equn. (7) can be solved by the following identity

$$\sum_{i=0}^{N-1}\left(e^{j\theta}\right)^{i} = \frac{\sin\frac{N\theta}{2}}{\sin\frac{\theta}{2}} e^{j(N-1)\frac{\theta}{2}} \qquad \text{----8}$$

We can rearrange the Equn. (7) as

$$\hat{f}_r = \frac{\bar{f}}{N} e^{j2\pi\left(1+\frac{\Delta f}{50}\right)\frac{r}{N}} \frac{\sin N\theta_1}{\sin\theta_1} e^{j(N-1)\theta_1} - \frac{\bar{f}^{*}}{N} e^{-j2\pi\left(1+\frac{\Delta f}{50}\right)\frac{r}{N}} \frac{\sin N\theta_2}{\sin\theta_2} e^{j(N-1)\theta_2} + \frac{2F\sin(\phi_2)}{N} \frac{e^{-\frac{\alpha}{50}} - 1}{e^{-\frac{\alpha}{50N} - j\frac{2\pi}{N}} - 1} e^{-\frac{\alpha r}{50N}} \qquad \text{----9}$$

Where

$$\theta_1 = \frac{\pi\Delta f}{50N} \quad \text{and} \quad \theta_2 = -\frac{\pi(2+\frac{\Delta f}{60})}{N}$$

By rearranging Equn. (9) we can get

$$\hat{f}_r^{(50+\Delta f)} = \frac{\bar{f}}{N}\frac{\sin N\theta_1}{\sin\theta_1} e^{j\frac{\pi}{50N}(\Delta f(2r+N-1)+100r)} - \frac{\bar{f}^{*}}{N}\frac{\sin N\theta_2}{\sin\theta_2} e^{j\frac{\pi}{50N}(\Delta f(2r+N-1)+100(r+N-1))} + \frac{2F\sin(\phi_2)}{N} \frac{e^{-\frac{\alpha}{50}} - 1}{e^{-\frac{\alpha}{50N} - j\frac{2\pi}{N}} - 1} e^{-\frac{\alpha r}{50N}} \qquad \text{----10}$$

Let assign

$$\frac{\bar{f}}{N}\frac{\sin N\theta_1}{\sin\theta_1} e^{j\frac{\pi}{50N}(\Delta f(2r+N-1)+100r)} = A_r \qquad \text{----11}$$

$$-\frac{\bar{f}^{*}}{N}\frac{\sin N\theta_2}{\sin\theta_2} e^{j\frac{\pi}{50N}(\Delta f(2r+N-1)+100(r+N-1))} = B_r \qquad \text{----12}$$

$$\frac{2F\sin(\phi_2)}{N} \frac{e^{-\frac{\alpha}{50}} - 1}{e^{-\frac{\alpha}{50N} - j\frac{2\pi}{N}} - 1} e^{-\frac{\alpha r}{50N}} = C_r \qquad \text{----13}$$

Equn. (10) can be re written as

$$\hat{f}_r = A_r + B_r + C_r \qquad \text{----14}$$

So far the development of the algorithm of SDFT is the same as the traditional DFT method. So the SDFT can keep all advantages of DFT such as recursive and half-cycle computing manner. But in the DFT, it doesn't take DC offset into consideration and it assumes that the frequency deviation is small enough to be ignored. It always considers $\hat{f}_r = A_r$, so traditional DFT based methods incur error in estimating frequency and phasor when frequency deviates from nominal frequency (50 Hz) or DC offset is present. If we want to obtain exact solution, we must take *B*, and *C*, into consideration. Then we define

$$a = e^{j(\frac{\pi}{50N}(2\Delta f+100))} \qquad \text{----15}$$

$$b = e^{-\alpha/50N} \quad \text{----16}$$

From Equn (10) following relations are obtained

$$A_{r+1} = A_r.a \quad \text{----17}$$

$$B_{r+1} = B_r.a^{-1} \quad \text{----18}$$

$$C_{r+1} = C_r.b \quad \text{----19}$$

Then

$$\hat{f}_{r+1} = A_{r+1} + B_{r+1} + C_{r+1} = A_r.a + B_r.a^{-1} + C_r.b \quad \text{----20}$$

$$\hat{f}_{r+2} = A_{r+2} + B_{r+2} + C_{r+2} = A_{r+1}.a + B_{r+1}.a^{-1} + C_{r+1}.b \quad \text{----21}$$

Equn.(14) is multiplied both sides with 'b' and subtract from Equn (20) gives

$$\hat{y}_r = \hat{f}_{r+1} - \hat{f}_r b = A_r(a-b) + B_r(a^{-1} - b) \quad \text{----22}$$

$$\hat{y}_{r+1} = \hat{f}_{r+2} - \hat{f}_{r+1} b = A_{r+1}(a-b) + B_{r+1}(a^{-1} - b) \quad \text{----23}$$

$$\hat{y}_{r+2} = \hat{f}_{r+3} - \hat{f}_{r+2} b = A_{r+2}(a-b) + B_{r+2}(a^{-1} - b) \quad \text{----24}$$

We can rearrange Eq.(22), (23) and (24) as

$$\hat{y}_{r+1}a - \hat{y}_r = A_r(a^2-1)(a-b) \quad \text{----25}$$

$$\hat{y}_{r+2}a - \hat{y}_{r+1} = A_{r+1}(a^2-1)(a-b) \quad \text{----26}$$

Equn (25)/ equn (26) gives

$$a = \frac{A_{r+1}}{A_r} = \frac{\hat{y}_{r+2}a - \hat{y}_{r+1}}{\hat{y}_{r+1}a - \hat{y}_r} = \frac{\hat{y}_{r+3}a - \hat{y}_{r+2}}{\hat{y}_{r+2}a - \hat{y}_{r+1}} \quad \text{----27}$$

Put Equn.(23) & Equn.(24) in Equn.(27)

$$[\hat{f}_{r+2}(\hat{f}_r + \hat{f}_{r+2}) - \hat{f}_{r+1}(\hat{f}_{r+1} + \hat{f}_{r+3})]b^2 + [\hat{f}_{r+3}(\hat{f}_r + \hat{f}_{r+2}) - \hat{f}_{r+1}(\hat{f}_{r+2} + \hat{f}_{r+4})]b + [\hat{f}_{r+3}(\hat{f}_{r+1} + \hat{f}_{r+3}) - \hat{f}_{r+2}(\hat{f}_{r+2} + \hat{f}_{r+4})] = 0 \quad \text{----28}$$

Solve Equn.(28) to obtain 'b'. From the definition of 'b' in Equn (16) we can obtain the exact solution of the time constant.

$$\tau = \frac{1}{50N \log b} \quad \text{----29}$$

Equn.(27) can be rearranged as

$$\hat{y}_{r+1}a^2 - (\hat{y}_r + \hat{y}_{r+2})a + \hat{y}_{r+1} = 0 \quad \text{----30}$$

Solve Equn.(30) to obtain 'a'. From the definition of 'a' in Equn. (15) we can get the exact solution of the frequency.

$$f = 50 + \Delta f = \cos^{-1}\{\mathrm{Re}(a)\}\frac{50N}{2\pi} \quad \text{----31}$$

From Equn.(29) and Equn.(3I), it is observed that SDFT can provide exact time constant and frequency using $\hat{f}_r$, $\hat{f}_{r+1}$, $\hat{f}_{r+2}$, $\hat{f}_{r+3}$ and $\hat{f}_{r+4}$ in the absence of noise. Moreover, we can estimate phasor and fault angle after getting exact time constant and frequency by the following equations:

$$A_r = \frac{\hat{y}_{r+1}a - \hat{y}_r}{(a^2-1)(a-b)} \quad \text{----32}$$

$$F = abs\left(A_r \frac{N \sin\theta_1}{\sin(N\theta_1)}\right) \quad \text{----33}$$

$$\phi_1 = angle(A_r e^{-j\theta_1(N-1)}) \quad \text{----34}$$

$$C_r = \frac{a^2 \hat{f}_{r+1} - a(\hat{f}_r + \hat{f}_{r+2}) + \hat{f}_{r+1}}{(a-b)(ab-1)} \quad \text{----35}$$

$$\phi_2 = \sin^{-1}\left(\frac{C_r N}{2X} \frac{e^{-\frac{\alpha}{50N} - j\frac{2\pi}{N}} - 1}{e^{-\frac{\alpha}{50}} - 1}\right) \quad \text{----36}$$

Furthermore, we take noise into consideration and use smoothing windows to filter noise. Consider a sampled set *{f(k) }* to be a filtered set { *z(k)*} by a smoothing window $\{SW_{(m)}\big|_{s_1, s_2, s_3 \ldots\ldots s_m}$ with window size 'm'.

$$z(k) = \sum_{i=1}^{m} s_i f(k+i-1) \quad \text{----37}$$

Moreover, the DFT of { *z(k)*} is given by

$$\hat{z}_r = \frac{2}{N}\sum_{k=0}^{N-1} z(k+r)e^{-j\frac{2\pi k}{N}}$$

$$= \frac{2}{N}\sum_{k=0}^{N-1}\left[\sum_{i=1}^{m} s_i f(k+r+i-1)\right]e^{-j\frac{2\pi k}{N}}$$

$$= \sum_{i=1}^{m} s_i \left[\frac{2}{N}\sum_{k=0}^{N-1} f(k+r+i-1)e^{-j\frac{2\pi k}{N}}\right]$$

$$= \sum_{i=1}^{m} s_i \hat{f}_{r+i-1} \quad \text{----38}$$

From the definition of Equn.(14), we can obtain:

$$\hat{z}_r = A_r(s_1 + s_2 a + \ldots\ldots + s_m a^{m-1}) + B_r(s_1 + s_2 a^{-1} + \ldots\ldots + s_m a^{-(m-1)}) + C_r(s_1 + s_2 b + \ldots\ldots + s_m b^{(m-1)}) \quad \text{----39}$$

The relations of Equn. (17), Equn. (18) and Equn.(19) are still kept in Equn.(39). Therefore, the same steps from Equn.(20) to Equn.(33) can be used in Equn.(39). Hence we can estimate time constant and frequency without modifying equations, but we have to do some change in Equn.(32) and Equn.(35) when we estimate phasor and fault angle.

$$A_r = \frac{\hat{y}_{r+1}a - \hat{y}_r}{(a^2-1)(a-b)(s_1 + s_2 a + \ldots\ldots + s_m a^{m-1})} \quad \text{----40}$$

Where

$\hat{y}_r = \hat{z}_{r+1} - \hat{z}_r b$

$\hat{y}_{r+1} = \hat{z}_{r+2} - \hat{z}_{r+1} b$

$$C_r = \frac{a^2 \hat{f}_{r+1} - a(\hat{f}_r + \hat{f}_{r+2}) + \hat{f}_{r+1}}{(ab-1)(a-b)(s_1 + s_2 b + \ldots\ldots + s_m b^{m-1})} \quad \text{----41}$$

The phasor obtained from Equn. (40) and fault angle obtained from Equn.(41) will allay the phase shift and amplitude decay caused by smoothing windows.

III. FILTERING ALGORITHMS AVAILABLE

i) Infinite Impulse Response Algorithm

In traditional method, transfer of analog filter to digital filter is practiced to get the desired requirements.

Transformation method used is impulse variance that uses s-plane conversion to z-plane of differential equations to difference equations and then to direct synthesis.

Analog prototype low pass filter is given as

$$H_N(s) = \frac{1}{s^2 + \sqrt{2}s + 1} \qquad \text{----42}$$

Analog prototype can be converted into digital by bilinear transformation, it can be expressed as

$$s \to \frac{2}{T}\frac{1-Z^{-1}}{1+Z^{-1}} \qquad \text{----43}$$

T is sampling time

The bilinear transformed equation that is 's' replaced by right hand term of equation results in

$$H_N(s) = (1+2Z^{-1}+Z^{-2})\left\{\left(\frac{4}{T^2}+\frac{2\sqrt{2}}{T}+1\right)+\left(\frac{-2}{T^2}\right)Z^{-1}\right\}^{-1}$$

$$+(1+2Z^{-1}+Z^{-2})\left\{\left(\frac{4}{T^2}-\frac{2\sqrt{2}}{T}+1\right)Z^2\right\}^{-1} \qquad \text{----44}$$

The fundamental component present in the signal *x(t)* will be extracted by designing band pass filter at 50Hz frequency can be obtained from below expression

$$Y_{50}(N) = 0.523\,X(N-1) - 0.365\,X(N-2) + 0.734\,X(N-3)$$
$$-0.649X(N-4) + 0.632Y(N-1) - 0.643Y(N-2) +$$
$$0.879Y(N-3) - 0.369Y(N-4) \qquad \text{----45}$$

ii) Least Square Curve Fitting Algorithm:

Here it is assumed that the inrush current contains decaying DC and no more than five harmonics, then in a certain time interval. The inrush signal can be represented as

$$x(t) = P_0 e^{-\lambda t} + \sum_{k=1}^{t} P_k \sin(k\omega_0 t + \theta_k) \qquad \text{----46}$$

Where x(t) is instantaneous differential signal sampled at a time t

P_0 is decaying Dc component.

λ is inverse time decay time constant of Dc component

P_k is peak component of the k^{th} harmonic differential signal

ω_0 is fundamental frequency

θ_k is Phase angle of k^{th} harmonic

$$x(t) = P_0 - P_0\lambda t + \sum_{k=1}^{5} P_k \cos\theta_k \sin(k\omega_0 t)$$
$$+\sum_{k=1}^{5} P_k \sin\theta_k \cos(k\omega_0 t) \qquad \text{----47}$$

In matrix form the above equation can be written as

$$[X] = [A][Y] \qquad \text{----48}$$

Least square components obtained from

$$[Y][(A^T A)^{-1} A^T] = [B][X] \qquad \text{----49}$$

It can be shown that

$$[B] = (A^T A)^{-1} A^T \qquad \text{----50}$$

As the matrix A has known elements, matrix B can be found easily. Matrix b can then be used to compute the vector X from the sampled signal. The Fourier sine and cosine components of fundamental frequency can be obtained from

$$C_1 = P_1 \cos(\theta_1 t_n) = \sum_{n=1}^{16} B(3,n)x(t_n) \qquad \text{----51}$$

$$S_1 = P_1 \sin(\theta_1 t_n) = \sum_{n=1}^{16} B(4,n)x(t_n) \qquad \text{----52}$$

iii) Kalman Filtering Algorithm:

The Kalman filtering algorithm has been proven to be the optimal linear estimator even in noisy environment once a signal is represented by a state variable equation of the form

$$X_{K+1} = FX_K + W_K \qquad \text{----53}$$
$$Z_K = HX_K + V_K$$

The covariance matrices for W_k and V_k vectors are given as follows.

$$E[W_K\ W_i^T] = Q \qquad i = k$$
$$= 0 \qquad i \neq k$$
$$E[V_K\ V_i^T] = R \qquad i = k$$
$$= 0 \qquad i \neq k$$
$$E[W_K\ V_i^T] = 0 \qquad \text{for all k and } I \qquad \text{----54}$$

Where E denotes the expected values.

Having a prior knowledge of the initial estimation error covariance matrix Po-, the Kalman gains can be computed recursively as follows.

$$K_k = \overline{R_k} H^T (H\overline{R_k}H^T + R)^{-1}$$
$$R_k = (I - K_k H)\overline{P_k}$$
$$\overline{R_{k+1}} = FR_k F^T + Q \qquad \text{----55}$$

Where

K_k is the Kalman gain matrix at time *tk*.

$\overline{P_k}$ is the estimation error covariance matrix at t_k;

P_k, the error covariance matrix for the updated estimate at t_k

I is the identity matrix.

Having an initial state estimate X_{0-} , the Kalman filter equation, which recursively estimate new values of the state vector, is as follows.

$$\hat{X}_K = \hat{X}_{\overline{K}} + K_K(Z_K - H\hat{X}_{\overline{K}})$$
$$\hat{X}_{K+1} = F\hat{X}_K \qquad \text{----56}$$

Where $\hat{X}_K$ is the estimate of X_k.

The discrete time state space representation of periodic signal having harmonic components on to nth order with samples *Zk* at time t_k can be given.

$$X_{K+1} = FX_K$$
$$Z_K = HX_{\overline{K}} \qquad \text{----57}$$

$$F = \begin{bmatrix} f(1\psi) & & & 0 \\ & f(2\psi) & & \\ & & f(3\psi) & \\ 0 & & & 1 \end{bmatrix}$$

ψ =*wT*, *w* is the fundamental supply frequency in rad/s and is the sampling interval, *s* and

H = [1, 0, 1, 0 ....1, 0,1]

iv) Block Pulse Functions Algorithm:

A set of block pulse functions on a unit time interval (0,1) with N number of samples per cycles defined as

$$\phi_n(t) = 1 \quad \text{for} \quad (n-1)/N < t < n/N \qquad \text{----58}$$

$= 0$ otherwise both for n=1,2,3,…..,N

If there is a function f(t), which is integrable in (0,1) can be approximated using BPF as,

$$f(t) = \sum_{n=1}^{N} a_n \phi_n(t) \qquad \text{----59}$$

where the coefficient an are block pulse function coefficients determined so that the integral square error is minimized.

$$\xi = \int_0^1 f(t) - \sum_{n=1}^{N} a_n \phi_n(t)^2 \, dt \qquad \text{----60}$$

For such a square fit a*n* is given by

$$a_n = N \int_{(n-1)/N}^{n/N} f(t)\,dt \qquad \text{----61}$$

= average value of f (*t*) in the interval (*n*-1)*N*<*t*<*n*/*N*

v) Wavelet Transforms Algorithm:

The wavelet transform translate the time domain function into a representation localized not only in frequency but also in time. Wavelet theory is the mathematics associated with building a model for a non-stationary signal, with a set of components that are small wave called wavelets. There are some conditions that must be met for a function to qualify as a wavelet. They must be oscillatory and have amplitudes that quickly decay to zero. The product of an oscillatory function with a decay function yields the wavelet. A number of different wavelets are used to approximate any given function with each wavelet generated from one original wavelet called a mother wavelet. The new elements, called daughter wavelet are nothing but scaled and translated mother wavelets. Scaling implies that the mother wavelet is either dilated or compressed and translation implies shifting of the mother wavelet in the time domain.

Let a equation for a mother wavelets follows.

$$g(t) = e^{-\alpha t^2} e^{j\omega t}$$

$$g^1(a,b,t) = \left(1/\sqrt{a}\right) g\left((t-b)/a\right) \qquad \text{----62}$$

Where

a is a constant;

b is the time translation factor

a is the scaling factor.

The energy of the scaled daughter wavelet is normalized to keep the energy same, as the energy in the mother wavelet.

For computer implementation, the discrete wavelet transform is used. A discrete wavelet transform results in a finite number of wavelet coefficients depending upon the integer number of the discretization step in scale and translation denoted by m and n, respectively. So any wavelet coefficient can be described by two integer, *m* and *n*. If a_o and b_o are the segmentation step sizes for the scale and translation respectively, the scale and translation in terms of these parameters will be $a = a_0^{\,m}$ and $b = nb_0 a_0^{\,m}$

After discretization in terms of the parameter a_o, b_o, *m* and *n* the mother wavelet can be written as,

$$g^{'}(m,n,t) = \left(1/\sqrt{a_0^{\,m}}\right) g\left((t - nb_0 a_0^{\,m})/a^m\right) \qquad \text{----63}$$

$$g^{'}(m,n,t) = a_0^{-m/2} g(t a_0^{-m} - nb_0) \qquad \text{----64}$$

After discretization, the wavelet domain coefficients are no longer represented by a simple a and b instead they are represented in terms of *m* and *n*. The discrete wavelet coefficient Wgf (m, n) are given by

$$W_{gf}(m,n) = a_0^{m/2} \int_{-\infty}^{\infty} f(t) g(t a_0^{-m} - nb_0)\,dt \qquad \text{----65}$$

The transformation is over continuous time but the wavelets are represented in a discrete fashion like the continuous wavelet transformation these discrete wavelet coefficient represent the correlation between the original signal and wavelets for different combination of *m* and *n*.

DISTANCE RELAY

In general thirteen input signals, namely, three line-to-ground voltages, three line-to-line voltages, three line currents Three differences in line currents and residual current are required to obtain phasor quantities necessary for an impedance relay. In this work all the 13 signals are obtained from simultaneously taken samples of 6 signals, namely, three line-to-ground voltages and three line currents using following relations

$$V_{L1-L2} = V_{L1} - V_{L2} \qquad \text{----66}$$

$$i_{L1-L2} = i_{L1} - i_{L2} \qquad \text{----67}$$

$$i_R = i_{L1} + i_{L2} + i_{L3} \qquad \text{----68}$$

The ground impedance seen at the relay point is calculated as

$$Z_{Li} = \frac{V_{L1}}{I_{L1} + \frac{K-1}{3} I_R} \qquad \text{----69}$$

V_{L1}, I_{L1} *are* the RMS values of the relay voltage and current

I_R is the RMS value of the residual current

K is the degree of compensation, being a ratio of zero to positive sequence impedance of the line that remains constant for all fault locations within the protected line.

The phase impedances at the relay point are calculated as

$$Z_{L1-L2} = \frac{V_{L1-L2}}{I_{L1-L2}} \qquad \text{----70}$$

The Function logic supports two commonly used operating Circular and Quadrilateral characteristics of conventional impedance relays. The Function Logic implements a three stepped distance protection by accepting three such characteristics, one for each zone.

TEST SYSTEM AND RESULTS

Proposed filtering algorithm was tested with 17 bus system as shown in fig.1

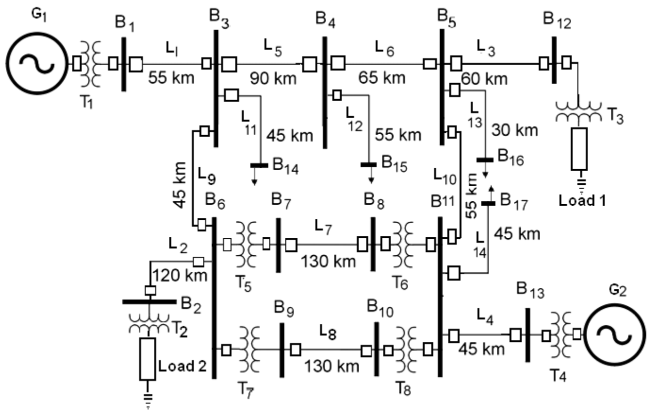


Fig.1 Test System Data

Generator G_1 is rated at 2GW and generator G_2 is rated at 1GW, both are at 15KV, 0.866 power factor lagging with transient reactances of 0.14p.u. Load-1 rated at $462\angle 30^0$ MVA and Load 2 is rated at $577\angle 30^0$ MVA.

Transformers Data

Transformer	MVA rating	Voltage ratio KV/KV
T1 T2	2500 1800	15/500
T3 T4	500 600	500/15
T5	600	500/230
T6 T7	1000 1000	500/230
T8 T9	1000 1000	230/500

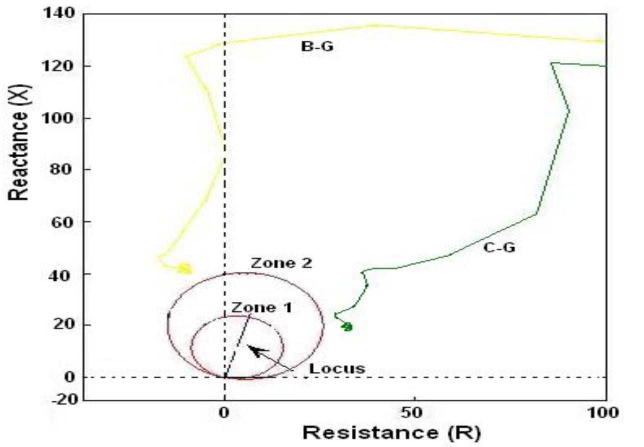
Each transformer has reactance of 0.1p.u. Each transmission line has shunt conductance of $1.0e^{-10}$ mhos per meter. Load connected at bus-14 is 200MW at 500KV, bus-15 is 150MW at 230KV, bus-16 is 150MW at 500KV and bus-17 is 150MW at 500KV.

Zones and Reach adjustment of Distance relay

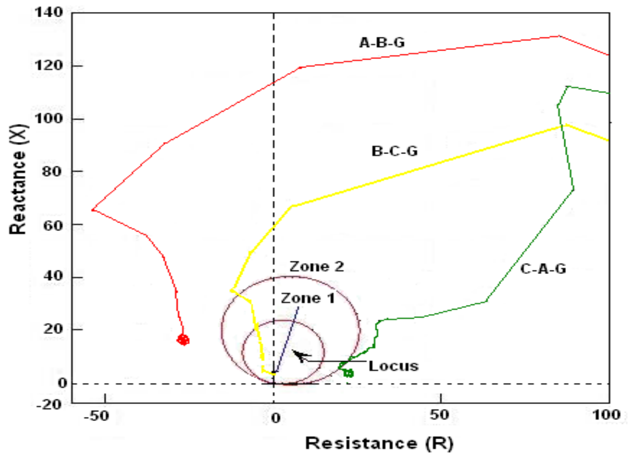
The adjustment of zone-1 was set to protect up to 80% of the impedance of the protected line. Zone-2 was set to protect up to 50% of the shortest impedance of the lines T_6 and T_{13}, emanating from the remote bus. Zone-3 was set to protect up to 100% of the remote line *T6*. Distance relay was placed at bus-3 for the proposed simulation.

Case-1: Simulation of faults at 10% of Line L_5 :

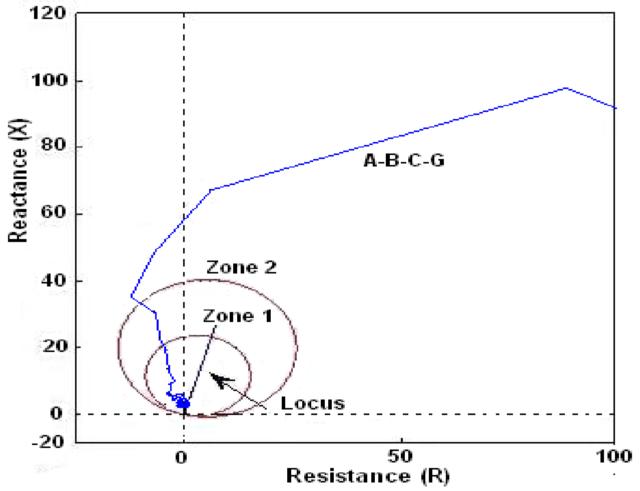
Different types of faults like single line-to-ground, double line-to-ground and three-phase-to-ground faults were simulated at 10% of transmission line L_5 from bus 3. Fault impedances from relay location are calculated by distance relay at 10% of line L_5. The corresponding characteristics are as shown below.



Single line-to-ground faults at 10% of Line L_5 from Bus 3



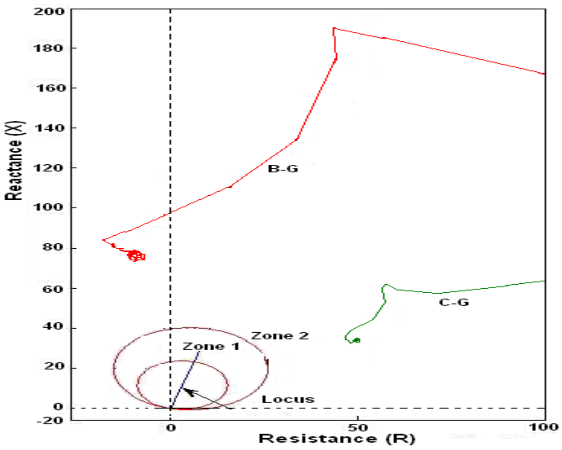
Double line-to-ground faults at 10% of LineL_5 from Bus 3



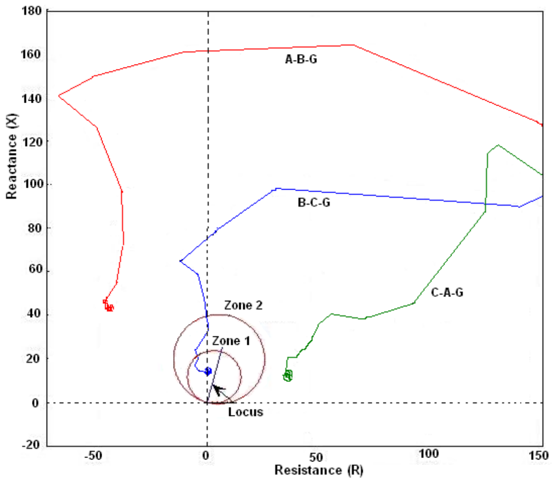
Three-phase-to-ground fault at 10% of Line L_5 from Bus 3

Case-2: Simulation of faults at 50% of line L_5 :

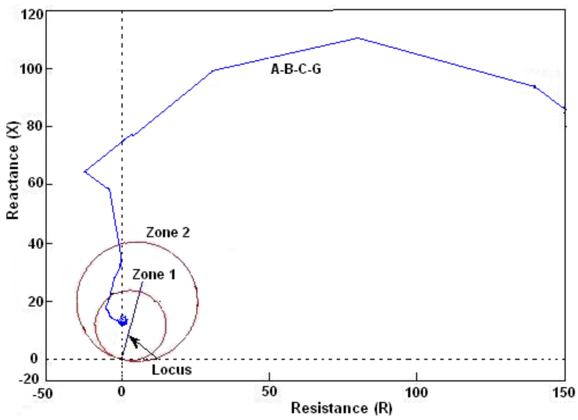
Single line-to-ground, double line-to-ground and three-phase-to-ground faults were simulated at 50% of transmission line L_5 from bus 3. Fault impedances from relay location are calculated by distance relay at 50% of line L_5. The corresponding characteristics are as shown below.



Single line-to-ground faults at 50% of Line L_5 from bus-3



Double line-to-ground faults at 50% of Line L_5 from bus-3



Three phase-to-ground fault at 50% of Line L_5 from bus-3

CONCLUSIONS

A number of algorithms for numerical distance protection have been compared with proposed novel SDFT algorithm to calculate the apparent impedance of the line between the relay's location and the fault point by using digitized samples of voltage and current signals. Each algorithm has certain merits and demerits regarding computational simplicity, speed, accuracy and frequency response. The selection of any particular algorithm depends on the protection requirement. All the various schemes of algorithms proposed advocate a family of wide spectrum of techniques to determine the impedance of the transmission line under fault condition. Different fault types were simulated in two different locations of transmission line L_5. The distance relay's correctness in fault identification, the time of the event they were monitoring and exhibition of required discrimination in their operation were compared between proposed Smart Discrete Fourier Transform algorithm (SDFT) and above said filtering algorithms. The distance relay calculated the apparent impedance of the transmission line, and the plotted apparent impedances in R-X plane supported the correct operation of the relay with the proposed algorithms are compared.

In the Kalman filtering algorithm uses only present sampled signals and does not require any past data to be stored in the memory. The major difference between the Kalman filter and proposed SDFT algorithm is evaluation of filter gains. In SDFT there is no need of evaluating gains but case of Kalman filter requires it. The gains of Kalman filter vary with time as the gains are non-stationary. Being recursive in nature, even though Kalman filter is computationally more efficient but it has limited capability for modeling the decaying dc component. In addition, statistical properties of the signal to be processed are needed for calculating the Kalman gains. So it is concluded that proposed SDFT algorithm is efficient and more acceptable.

One of the most prominent advantages of algorithms using Blocked Pulse Function Algorithm coefficients is that the sampling rate can be any positive integer, where as in other method like in Haar and Walsh transform, one is compelled to use sampling rates that are equal to integral powers of 2. In other algorithms generally 16 samples/cycle are used to give the satisfactory results. For distance protection however, the BPF algorithm uses 12 samples, which requires less multiplication, so less memory space and time but obtained samples contains errors as it is not handling decaying dc component properly. This drawback is completely eliminated in proposed SDFT algorithm.

Least Square Curve Fitting Algorithm can also handles decaying dc offset components to filter from operating signals as it is in proposed SDFT algorithm but it is suffering from drawback that it can only work for the harmonics of order 5. Above which this algorithm outcomes suffers from large errors. But there is no restriction on order of harmonics in SDFT algorithm.

Being recursive in nature, an Infinite Impulse Response Algorithm is an accurate and error free algorithm but it needs to transform differential equations from s-plane to z-plane frequently and feedback is desired. Probability of errors in transformation will result with less knowledge in z-transforms. But this feedback and interrelating output with input is not required in SDFT algorithm.

Non-stationary signals, where the interest is only for what spectral components exist in the signal but not interested where these occur, can be easily handled with wavelet transforms. It provides the time frequency representation. Although the discretized Continuous Wavelet Transform(CWT) enables the computation of required CWT by the computer, but it is not a true discrete transform. As a matter of fact, the wavelet series is simply a sampled version of the CWT and the information it provides is highly redundant for the signal concerned. This redundancy, on the other hand, requires a significant large amount of computation time and resources. Time of fault clearance based on fault amplitude computation is more important than the instant where the fault has occurred. Computation time of Wavelet is more than proposed SDFT. More than that, in some limit, SDFT can handle non-stationary signals as wavelets.

The discrete Fourier transform algorithm was used to extract real and imaginary components that involves number of multiplication operations, in addition to summation /subtraction operations, it is less time consuming algorithm. Based on the experimental results, proposed SDFT algorithm is found to be the best algorithms in terms of accuracy and computing time when compared some of existing filtering algorithm.

REFERENCES

[1]. M S Sachdev and M Nagpal. ‘A Recursive Least Error Squares Algorithm for Power System Relaying and Measurement Application.’ IEEE Transactions on Power Delivery, vol 6, no 3, 1991, p 1008.

[2].F Liang and B Jeyasurya. ‘Transmission Line Distance Protection Using Wavelet Transform Algorithm.’ *IEEE Transactions on Power Delivery*, vol 19, no 2, April 2004, p 545.

[3].C Fernandez and F Luis Pagola. ‘Total Least Squares and Discrete Time Models in H V Distance Protection.’ IEEE Transactions on Power Delivery, vol 14, no 1, 1999, p 74.

[4].A S Alfuhaid and M A EI-Sayed. ‘A Recursive Least Squares Digital Distance Relaying Algorithm.’ IEEE Transactions on Power Delivery, vol 14, no 4, October 1999, p 1257.

[5].D C Robertson, O I Camps, J S Mayer and W B Gish. ‘Wavelets and Electromagnetic Power System Transients.’ *IEEE Transactions on Power Delivery*, vol 11, no 2, April 1996, p 1050.

[6].A Girgis and R G Brown. ‘Application of Kalman Filtering in Computer Relaying.’ IEEE Transactions on PAS, vol 100, no 7, 1981, p 3367.

[7].J Hector, F Altuve, V Ismael Diaz and M Ernesto Vazquez. ‘Fourier and Walsh Digital Filtering Algorithms for Distance Protection.’ IEEE Transactions on Power Systems, vol 11, no 1, February 1996, p 457.

[8]Dario D’Amore and Alessandro Ferrero. ‘A Simplified Algorithm for Digital Distance Protection Based on Fourier Techniques.’ IEEE Transactions on Power Delivery, vol 4, no 1, January 1989, p 157.

[9].J. Z Yang and C. W. Liu, “A Smart Method Makes DFT More Precise for Power System Frequency Estimation:’ IEEUPES Winter Meetzng, Feb. 1999, pp. 909-913.

[10].P Pillay and A Bhattacharjee. ‘Application of Wavelets to Model Short Term Power System Disturbances.’ *IEEE Transactions on Power Systems*, vol 11, no 4, November 1996, p 2031.

[11]. A A Girgis and D G Hart. ‘Implementation of Kalman and Adaptive Kalman Filtering Algorithm for Digital Distance Protection on a Vector Signal Processor.’ *IEEE Transactions on Power Delivery*, vol 4, no 1, January 1989, p 141.

[12].J L Pinto de Sa. ‘A New Kalman Filter Approach to Digital Relaying.’ *IEEE Transactions on Power Delivery*, vol 7, no 3, July 1992, p 1652.

[13]. Norman T. Stringer, ‘The Effect of DC Offset on Current Operated relays,” IEEE Trans. on Industry Applications, Vol. 34, No. 1. JanuarylFebruaty 1998, pp. 30-34.

[14]. A H Osman and O P Malik. ‘Transmission Line Distance Protection based on Wavelet Transform.’ *IEEE Transactions on Power Delivery*, vol 19, no 2, April 2004, p 515.

[15.]Lai, T. M., L. A. Snider, E. Lo, 2006. “Wavelet Transform Based Relay Algorithm for the Detection of Stochastic High Impedance Faults”. *Electric Power Systems Research*, vol. 76, no. 8, pp. 626-633.

[16] Sheng, Y. and S. M. Rovnyak, 2004. “Decision Tree-Based Methodology for High Impedance Fault Detection”, *IEEE Transactions on Power Delivery*, vol. 19, no. 2, pp 533-536.

[17] Huang, S. and C. Hsieh, 1999. “High-impedance Fault Detection Utilizing A Morlet Wavelet Transform Approach,” *IEEE Transactions on Power Delivery*, vol. 14, no. 4, pp. 1401-1410.

[18] Kim, C. H., R. Aggarwal, 2001. “Wavelet Transforms in Power Systems,” *Power Engineering Journal* vol. 15, no. 4, pp. 193-202.

[19].Akorede, M. F., H. Hizam, 2009. “Wavelet Transforms: Practical Applications in Power Systems”,*Journal of Electrical & Technology*, vol. 4, no. 2, pp 168-174.

[20] D.V., Coury, H.G.F., Brito: “Digital filters applied to computerrelaying”, *Proc. IEEE International Conference on Power System Technology POWERCOM ’98*, New York, NY, USA, pp. 1062-66, 1998.