

NUMERICAL MODELING FOR DESIGNING MAGNETOSTRICTION IN INDUCTION MACHINES

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Abstract: These papers present a macroscopic model for computing the deformation of ferromagnetic material due to magnetic forces and magnetostriction by using finite element method, in particular two dimensional case and weak coupling. A virtual work method based on the derivative of the magnetic energy is used to calculate the forces distribution on the stator core of an electrical machine. A deformation of teeth of stator is also calculated.

Key words: magnetostriction, magnetic forces, induction machine, finite element method.

1. Introduction

Magnetostriction is a coupled magneto-mechanical phenomenon [1]. For rotating electrical machines, magnetostriction is one of a potential cause of noise and vibrations. The mechanical deformation is also cause changes in the air-gap contribution to generation of harmonic and additional noise.

The magnetic and magneto-elastic properties of magnetic materials differ from one material to other. Some properties, such as magnetization of iron are related to the applied mechanical stress. Magnetostriction is an even function [2], we can defined an analytic model depend on the magnetic flux density and quasi-independent of applied mechanical stress, the magnetostriction material characteristic ε is a function of square of magnetic flux density B .

The method developed in this paper is applied to calculate magnetic and magnetostriction forces in stator core of an induction machines, and its contribution to deformation of the stator core.

1. Analytically Study

For deformable materials, a phenomenon how coupled magnetic and mechanic behavior induced energy due to magnetic and mechanical contributions, a total potential energy density is given by:

$$w(B, \varepsilon) = H \cdot B + \sigma \cdot \varepsilon \quad (1)$$

With H the magnetic field, B the magnetic flux density, σ the stress tensor and ε the strain tensor.

From magneto-elastic coupling coefficient [3], it is possible to write the magneto-mechanical behavior law under this expression:

$$H(B, \varepsilon) = \int_0^\varepsilon \frac{\partial \sigma}{\partial B} d\varepsilon + H(B, 0) \quad (2)$$

$H(B, 0)$ is the magnetic field at null stress.

By analogy of thermo-elastic behavior [4,5], it is possible to suppose a magnetostrictive and mechanical deformation due to direct effect of magnetostriction and a mechanical strain state of material, neglecting thermal deformation; a total deformation is given by:

$$\varepsilon = \varepsilon^\mu(B) + \varepsilon^e(\sigma) \quad (3)$$

Where ε^μ is the magnetostrictive deformation tensor and ε^e an elastic deformation tensor.

This hypothesis allows expressing a mechanical behavior law (Hooke's law):

$$\sigma(B, \varepsilon) = C \cdot (\varepsilon - \varepsilon^\mu(B)) \quad (4)$$

C is the matrix of elastic constants.

A magneto-mechanical behavior law expression can be determined from (2) and (4) as:

$$H(B, \varepsilon) = H(B, 0) + \frac{\partial \varepsilon^\mu(B)}{\partial B} C \cdot \varepsilon \quad (5)$$

2. Choice of deformation model

From experimental test of deformation [6,7], Fig. 1. shows that magnetostriction deformation is an even function.

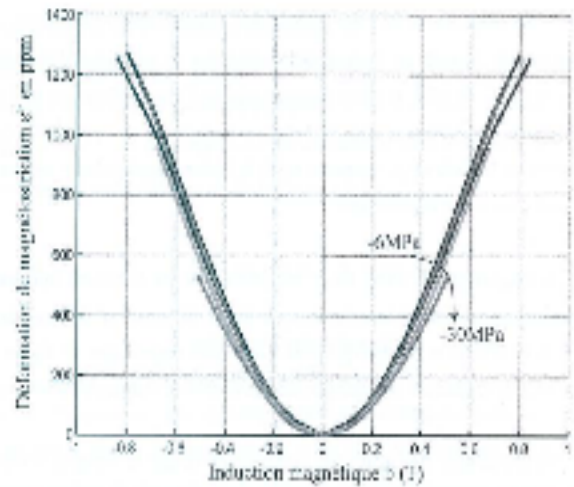


Fig. 1. Magnetostriction characteristics of Terfenol-D

Now we can defined an analytic model depend on the magnetic flux density and quasi-independent of applied mechanical stress, it is defined by:

$$\varepsilon_{II}^{\mu}(B) = \beta_0 \cdot B_{II}^2 \quad (6)$$

Where ε_{II}^{μ} is the deformation of magnetostriction in a direction parallel to that of the material rod, β_0 is a coefficient of the material and B_{II} the magnetic flux density in a direction parallel to that of the material rod.

In magnetic induction referential $(B_{II}, B_{\perp 1}, B_{\perp 2})$, where components $B_{\perp 1}$ and $B_{\perp 2}$ are orthogonal to the direction of B_{II} and under these hypotheses:

1. The magnetic material is isotropic.
2. A principal magnetostriction deformation carried out at constant volume $\varepsilon_{11}^{\mu} + \varepsilon_{22}^{\mu} + \varepsilon_{33}^{\mu} = 0$.
3. Magnetostriction deformations following orthogonal axes to the direction of B are same and its values are half compared with that parallel to the direction of B (in order to confirm the hypothesis 2).

A quadratic macroscopic model of a principal deformation of magnetostriction can be written at tonsorial form [6]:

$$[\varepsilon^{\mu}(B)] = \begin{bmatrix} \varepsilon_{11}^{\mu} & 0 & 0 \\ 0 & \varepsilon_{22}^{\mu} & 0 \\ 0 & 0 & \varepsilon_{33}^{\mu} \end{bmatrix}_{(B_{II}, B_{\perp 1}, B_{\perp 2})} \quad (7)$$

From (6) and (7), the expression of magnetostriction deformation becomes:

$$[\varepsilon^{\mu}(B)] = \beta_0 \cdot B^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}_{(B_{II}, B_{\perp 1}, B_{\perp 2})} \quad (8)$$

Until here, tonsorial model is expressed in cause's referential how give it product and not that of the material, moreover, our magnetostriction deformations were expressed in a referential different of that the material. Thus, it is important to write last expression in local material referential. According to Euler's rotation theorem (Euler angles), we can write a deformation tensor components in total material base as:

$$[\varepsilon^{\mu}(B)] = \beta_0 \begin{bmatrix} (B_1)^2 \frac{(B_2)^2}{2} \frac{(B_3)^2}{2} & \frac{3}{2} B_1 \cdot B_2 & \frac{3}{2} B_1 \cdot B_3 \\ \frac{3}{2} B_2 \cdot B_1 & (B_2)^2 \frac{(B_3)^2}{2} \frac{(B_1)^2}{2} & \frac{3}{2} B_2 \cdot B_3 \\ \frac{3}{2} B_3 \cdot B_1 & \frac{3}{2} B_3 \cdot B_2 & (B_3)^2 \frac{(B_1)^2}{2} \frac{(B_2)^2}{2} \end{bmatrix} \quad (9)$$

Where B_1, B_2 and B_3 are the component of magnetic

induction in material referential, they are depended to direction cosines.

In 2D case, the component of magnetic flux density B_3 is null. Thus, the expression of magnetostriction tensor in this case becomes:

$$[\varepsilon^{\mu}(B)] = \beta_0 \begin{bmatrix} (B_1)^2 \frac{(B_2)^2}{2} & \frac{3}{2} B_1 \cdot B_2 & 0 \\ \frac{3}{2} B_2 \cdot B_1 & (B_2)^2 \frac{(B_1)^2}{2} & 0 \\ 0 & 0 & (B_2)^2 \frac{(B_1)^2}{2} \end{bmatrix} \quad (10)$$

3. The magneto-mechanical system

The magnetic and displacement fields are given by the following differential equations [3]:

$$\begin{cases} \text{rot } H = J \\ \text{div } \sigma = -F^{\Omega} \end{cases} \quad (11)$$

Where H is the magnetic field, J the current density,

σ the stress tensor and F^{Ω} the volume force density.

We associate to these equations the flux conservation law and the mechanical geometric law [3]:

$$\begin{cases} \text{div } B = 0 \\ \varepsilon_{i,j} = \frac{1}{2} (U_{i,j} + U_{j,i}) \end{cases} \quad (12)$$

With B the magnetic flux density, ε the strain tensor and U the displacement [8].

Both magneto-static and elasticity finite element methods are based upon the minimization on an energy function [9]. The total energy E of the electromechanical system consist of the elastic energy stored in a body with deformation, the magnetic energy stored in magnetic system with vector potential A and the work of magnetic and mechanical sources [9]:

$$E(B, \varepsilon) = W(B, \varepsilon) - T \quad (13)$$

Where W the magneto-elastic energy it is given by:

$$W(B, \varepsilon) = \int_{\Omega} \left(\int_0^B H(B, \varepsilon) dB + \int_{\varepsilon^{\mu}}^{\varepsilon} \sigma(B, \varepsilon) d\varepsilon \right) d\Omega \quad (14)$$

T is the work of magnetic and mechanical sources

$$T = \int_{\Omega} A \cdot J^s d\Omega + \int_{\Gamma} A \cdot (H \times n) d\Gamma + \int_{\Omega} U \cdot F^{\Omega} d\Omega \quad (15)$$

Thus, we can obtain the following functional energy to be minimized.

$$E(B, \varepsilon) = \int_{\Omega} \left(\int_0^B H \cdot dB \right) d\Omega + \int_{\Omega} A \cdot J^s d\Omega + \int_{\Omega} \frac{1}{2} \varepsilon^t \cdot \sigma d\Omega - \int_{\Omega} U \cdot F^{\Gamma} d\Omega - \int_{\Gamma^{\sigma}} U \cdot F^{\Gamma} d\Gamma \quad (16)$$

Minimization of the functional energy $E(B, \varepsilon)$ versus A and U leads to the following system of algebraic matrix equation in terms of the magnetic

vector potential A and the displacement U [3,6]:

$$[S] \cdot [A] = [J^s] + [J^\mu] \quad (17)$$

$$[K] \cdot [U] = [F] + [F^\mu] + [F^m] \quad (18)$$

Where A and U are the unknowns, $[J^s]$ is the excitation current density and $[J^\mu]$ the magnetizing current induced by mechanical stress (inverse effect). $[S]$ is the magnetic stiffness matrix, which is given by:

$$\begin{aligned} [S_{i,j}]^e &= \int_{\Omega} \nu (\text{grad } N_i) \cdot (\text{grad } N_j) d\Omega \\ &= \int_{\hat{\Omega}} \nu (\text{grad } N_i) \cdot (\text{grad } N_j) \det(J) d\hat{\Omega} \end{aligned} \quad (19)$$

N_i is the shape function related to the node i , ν the reluctivity of material in the element under consideration, Ω the area of the element, $\hat{\Omega}$ the area of the reference element and J the Jacobean matrix for the transformation from the reference frame to the global one [10].

$[K]$ is the mechanical stiffness matrix [8,11], it is given by:

$$[K]^e = \int_{\Omega^e} [DN]^t [P]^t [C] [P] [DN] d\Omega \quad (20)$$

Where $[C]$ is the matrix of elastic constants, it is given for two cases [7,12]:

a) For plan stress

$$[C] = \frac{E}{1-\lambda^2} \begin{bmatrix} 1 & \lambda & 0 \\ \lambda & 1 & 0 \\ 0 & 0 & \frac{1-\lambda}{2} \end{bmatrix} \quad (21)$$

b) For plan strain

$$[C] = \frac{E}{(1+\lambda) \cdot (1-2\lambda)} \begin{bmatrix} 1-\lambda & \lambda & 0 \\ \lambda & 1-\lambda & 0 \\ 0 & 0 & \frac{1-2\lambda}{2} \end{bmatrix} \quad (22)$$

E is the Young's modulus and λ the Poisson's ratio.

$$[DN] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \frac{\partial N_3}{\partial y} \end{bmatrix} \quad (23)$$

$[P]$ is the permutation matrix.

$[F]$ is the matrix of external forces [13], it is given by:

$$[F]^e = \int_{\Omega^e} [M]^t [F^\Omega] d\Omega + \int_{\Omega^e} [M]^t [F^s] d\Omega_\sigma \quad (24)$$

Where:

$$[M] = \begin{bmatrix} N_1 & N_2 & 0 & 0 \\ 0 & 0 & N_1 & N_2 \end{bmatrix} \quad (25)$$

The magnetostriction forces equation is given by:

$$[F^\mu]^e = \int_{\Omega^e} [DN]^t [P] [C] [\varepsilon^\mu] d\Omega \quad (26)$$

The calculation of magnetic forces $[F^m]$ is based on local application of virtual work principle [10]. These models of forces have been calculated on each node as the derivative of the magnetic energy, with respect to the displacement at a constant magnetic flux [10,13].

$$F^m = - \int_0^A A^t \frac{\partial S}{\partial u} dA \quad (27)$$

From (19), the magnetic stiffness matrix can be written as:

$$[S_{ij}] = \frac{\nu}{4\Delta} [b_i b_j + c_i c_j] \quad (28)$$

Where:

$$\begin{cases} c_i = x_k - x_j \\ b_i = y_i - y_j \end{cases} \quad (29)$$

x_i, y_i with $i=1, 2, 3$ are the coordinates of nodes i of the element and the index i, j and k are circular indices (1,2,3,1...), Δ is the element area.

The derivatives of the stiffness matrix with respect to the x_1 and y_1 coordinates of the nodes of the element are [14]:

$$\frac{\partial S^e}{\partial x_1} = \frac{\nu}{4\Delta} \begin{bmatrix} 0 & c_1 & -c_1 \\ c_1 & 2c_2 & (c_3 - c_2) \\ -c_1 & (c_3 - c_2) & -2c_3 \end{bmatrix} - \frac{b_1}{2\Delta} S^e + \frac{S^e}{\nu} \frac{\partial \nu}{\partial x_1} \quad (30)$$

$$\frac{\partial S^e}{\partial y_1} = \frac{\nu}{4\Delta} \begin{bmatrix} 0 & -b_1 & b_1 \\ -b_1 & -2b_2 & (b_2 - b_3) \\ b_1 & (b_2 - b_3) & -2b_2 \end{bmatrix} - \frac{c_1}{2\Delta} S^e + \frac{S^e}{\nu} \frac{\partial \nu}{\partial y_1} \quad (31)$$

The integral in (27) is going achieve using the coordinate transformation $A = A_0 \cdot t$ detailed in [9,14], so $dA = A_0 \cdot dt$.

Now, if we solve equations (17) and (18), we obtain the magnetic vector potential A and the displacement U . It is easily that equations (17) and (18) are coupled through the magnetic force (the magnetic force dependence on the magnetic vector potential) and the variation of the magnetostrictive strain with magnetic field. The nonlinear equation (17) and (18) will be solved by iterative method, like fixed point [15,16] and Newton-Raphson methods [15,17].

4. Example

We apply this model to 2D example which is presented in the Fig.2, it consists of a four-pole induction machine, the stator core is a silicon (3.5%) steel material called (M19); the induction machine was simulated in static case. The inner and outer diameters of stator are 111 mm and 157.5 mm successively, the three-phase induction machine has 36 slots.

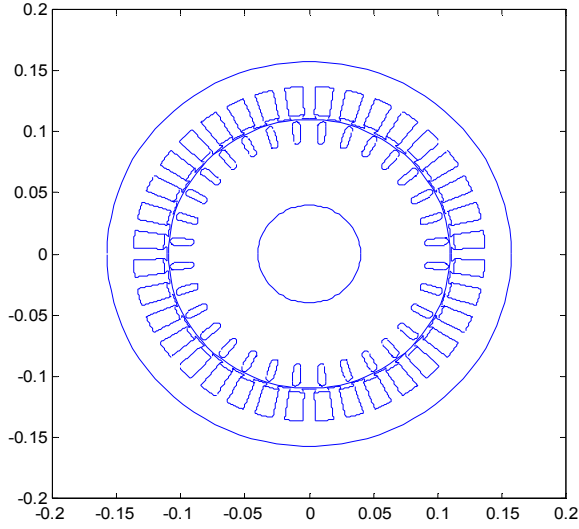


Fig. 2. Geometry of induction machine

The magnetic potentials are nulls at the outer diameter of stator. For elastic problem, only the left and right points of the outer boundary are considered fixed.

Fig.3 shows the BH curve for the material that used, this curve is simulated by cubic spline interpolation, at first approach; we assume that magnetic field is small enough so that the BH curve of a material can be considered as linear (we work at linear zone of BH curve).

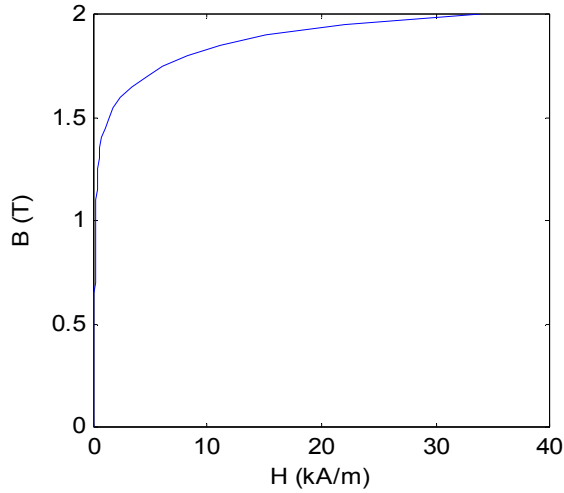


Fig. 3. B-H curve for M19 material

The mechanical property of the magnetostriction material is considered isotropic; the magnetostriction curve $\varepsilon^{\mu}(B)$ produced by the model is shown in Fig. 4, the Young's modulus and Poisson's ratio are respectively 190 MPa and 0.3 (1 ppm = 10^{-6} m).

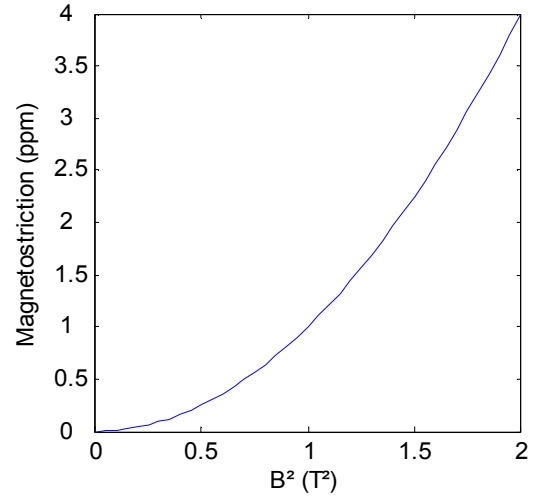


Fig. 4. Magnetostriction versus magnetic flux density

This curve follows the expected behavior of M19 material. This curve represents an even function.

Fig.5. presents a mesh of an induction machine with 16360 elements; the mesh is simulated by finite element method using standard and own routines in a commercial software package (Matlab version 6.5).

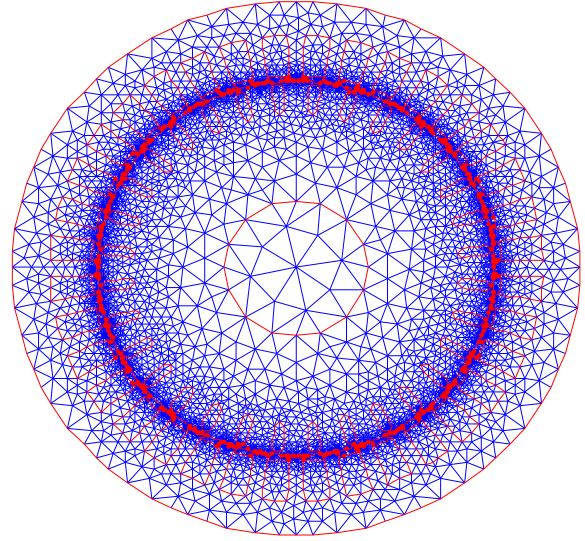


Fig. 5. Mesh of induction machine in finite elements

The magnetic potential distribution is shown in Fig. 6, we noticed that the later is enclosed and have maximum values around of the first inner current source and canalized from stator core to rotor across the air-gap where created a maximum values of magnetic flux density.

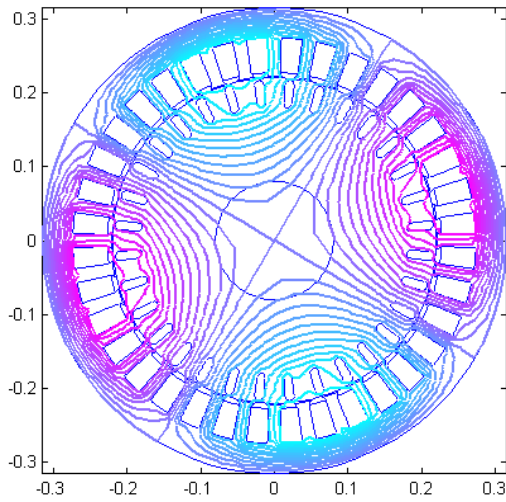


Fig. 6. Magnetic potential distribution

The result computed of magnetic forces is presented in Fig. 7; the maximal value of magnetic induction is 0.31T.

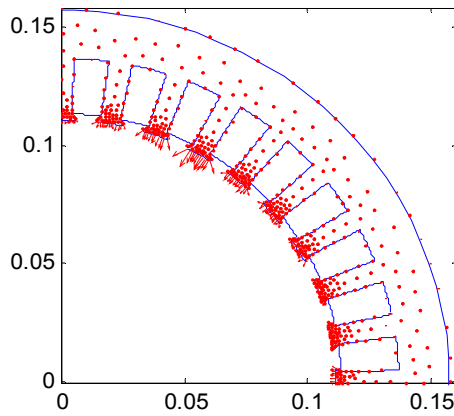


Fig. 7. Magnetic forces on stator

The high values of magnetic forces are situated at the teeth of stator near to the air-gap. The result computed of magnetostriction forces is presented in Fig.8.

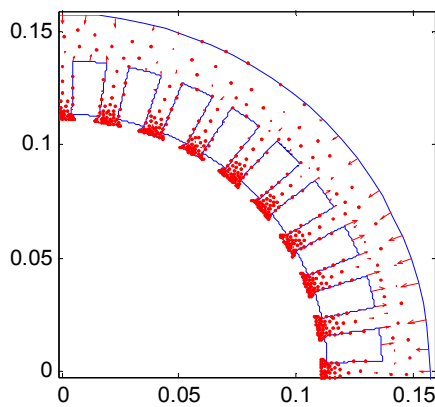


Fig. 8. Magnetostriction forces on stator

The high values of magnetostriction forces are situated at the outer diameter of stator and slots successively.

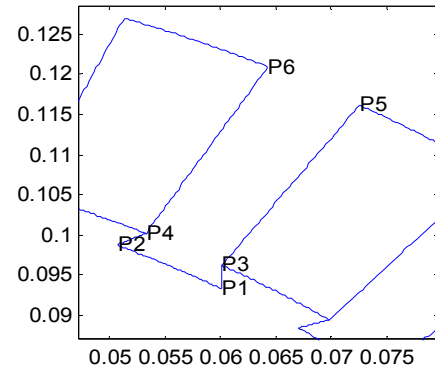


Fig. 9. Nodes under studies on one stator tooth

As an example, the magnetostriction displacement of nodes P1, P2, P3, P4, P5, and P6 situated at corner of stator tooth, shown in Fig.9, caused by magnetostriction forces are shown in Fig.10.

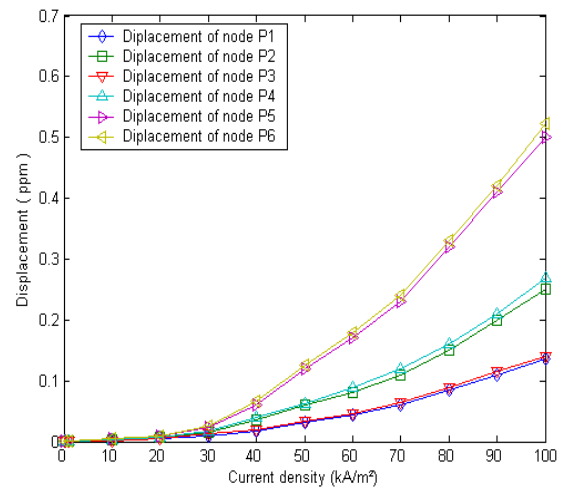


Fig. 10. Displacement versus current density

We noticed from Fig.10 that points P5 and P6 are displaced rapidly then the other points on the stator tooth.

5. Conclusion

A numerical model of magnetostriction for induction machine was presented, based on the energy variation formulation. The particular static case with formulation in term of magnetic vector potential and displacement is studied. The displacements of teeth are small compared to the machines dimension but they are significant especially at the region of the air-gap. Stator deformations are caused not only by magnetic forces, but also by magnetostriction effect on the stator iron. The results indicate that magnetostrictive forces are significant and must be accounted for studies of vibrations and noises.

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